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Studies on constitutive equation that models bone tissue

MAREK PAWLIKOWSKI[1, *], MARIAN KLASZTORNY[2], KONSTANTY SKALSKI[1]

[1] Institute of Mechanics and Printing, Warsaw University of Technology, Warszawa, Poland,
e-mail: mpawlik1@wip.pw.edu.pl, kskalski@wip.pw.edu.pl
[2] Department of Mechanics and Applied Computer Science, Military University of Technology, Warszawa, Poland,
e-mail: m.klasztorny@wme.wat.edu.pl

A method for identifying viscoelastic constitutive equations for bone is developed. In the method, anisotropy, non-homogeneity and bone remodelling are taken into consideration. The equations correspond to monotropic rheological model of bone. In order to calculate the material parameters of elasticity and viscosity, a new algorithm is developed, in which the results of creep compression and shear tests are used. The way of determining the material constants of elasticity and viscoelasticity for bone in the areas that are crucial in strain and stress analysis is shown a well. The strength experiments (tests) and creep tests were performed on bone samples extracted from the femur of calf.

The method of bone modelling in terms of rheology is the following: bone samples are properly prepared from biological material. For given bone samples, which have more or less the same density, three independent short-term creep tests are carried out. The results of the creep tests permit us to determine five elastic constants and viscoelastic constants of a monotropic material [1]. In order to describe rheological processes in terms of structural models, fractional exponential functions and normal exponential functions are used. The constitutive equations are formulated in the compliance form. The computer program that executes the algorithm of elastic and viscoelastic constant determination is used.

Key words: *bone, constitutive equations, rheology, viscoelastic constants*

1. Introduction

The main problem, which often occurs after hip arthroplasty, is relatively low durability of prosthesis. The most common phenomenon is atrophy of bone tissue in the vicinity of prosthesis stem in the proximal part of femur [2]–[7]. Bone atrophy, i.e. a decrease in bone density, is a result of stress shielding phenomenon that takes place in bone after prosthesis implantation. Under normal conditions, non-implanted bone carries a load itself. After prosthesis implantation the load is carried both by bone and prosthesis. In consequence, stresses in the proximal part of femur are lower than those in non-implanted femur. According to Wolff's law a stress reduction in bone compared to natural state leads to bone functional adaptation, i.e. bone density decrease (internal remodelling) and/or bone volume decrease (external remodelling) [8].

Internal remodelling is the most dangerous phenomenon, which also occurs most often after prosthesis implantation. The phenomenon may lead to prosthesis loosening (stem loosening in femur or acetabulum loosening in pelvis). Low durability of prosthesis, which is revealed by early prosthesis loosening due to lack of its firm biofixation in the bone, leads to revision arthroplasty that is usually more difficult than primary arthroplasty.

Correct modelling of rheological processes in femur–prosthesis–pelvis system is one of the ways of enhancing the durability of hip joint prosthesis. Thus, it is essential to design an adequate rheological model of bone and to formulate and identify its viscoelastic

* Corresponding author: Marek Pawlikowski, Institute of Mechanics and Printing, Warsaw University of Technology, ul. Narbutta 85, 02-524 Warszawa, Poland. E-mail: mpawlik1@wip.pw.edu.pl

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constitutive equations with functional adaptation taken into account.

Mathematically, bone functional adaptation is described by kinetic equations that are usually formulated in terms of rate. Other ways of mathematical descriptions of bone internal remodelling are also known. They may be based on strain [9], stress [10] and energy forms [11]. Such descriptions represent the most popular approach to bone remodeling, i.e. mechanistic approach. There are also other mathematical models of the phenomenon, such as mechano-biological models or opimisation models [12].

In prosthesis design, kinetic equations and equations relating bone material constants and bone density allow bone remodelling phenomenon to be taken into consideration [13], [14]. The implementation of this way of modelling bone structure changes in stress and strain analyses by means of finite element method makes it possible to observe bone remodelling processes as an effect of coupled interactions of such a mechanical stimulator as the stress state in bone–prosthesis system [15]. The modification of constitutive equations and continuum equations of equilibrium is more advanced approach. Usually when bone remodelling phenomenon is described by modified constitutive equations, the equations are formulated for elastic continuum [16]–[19]. The results of the creep tests make it possible to build more advanced mathematical model of bone adaptation that includes not only bone density change, but also constitutive equations for viscoelastic continuum.

Mechanical properties of bone depend on race, sex, age, physical activity and general health of a patient. Other factors that influence the mechanical properties of bone are: the region of specimen extraction, the magnitude and shape of specimens, the way of sample storing and the technique of measurement. Rheological studies completed so far concerned mainly the cortical bone of femur and trabecular bone of femoral head. They indicate that cortical bone may be modelled as a monotropic material [20]–[22]. The tensile creep tests are performed under the load corresponding to 0.2–0.7 of tensile strength and last 100 minutes. There have been already proposed viscoelastic constitutive equations for bovine cortical bone [23]. In those equations, exponential factorial functions and exponential functions were used. The results of viscoelastic studies on trabecular bone in hip joint that correspond to compression relaxation are presented in [24].

2. Methods

The results of the creep tests were used to build a rheological model of bone tissue. The model applied describes short-term and moderate-term strain history with high accuracy. The most generic rheological model of bone (model HRK) applied in the studies consisted of three elements, i.e. Hooke's element corresponding to elastic strains, Rabotnov's element [25] corresponding to viscoelastic strains that might be described by fractional exponential functions, and Kelvin's element corresponding to viscoelastic strains described by normal functions. The following assumptions were made:

- bone is homogeneous and orthotropic,
- static isothermal elastic processes are taken into consideration,
- bone reaction corresponds to linear viscoelastic material behaviour,
- the Boltzmann superposition principle is valid,
- short- and moderate-term rheological processes described by retardation times are taken into account.

The viscoelastic compliance of an orthotropic material can be formulated as follows [26]:

$$\widetilde{S}_{ij}(t) = S_{ij}\left\{1 + \int_0^t [\omega_{0,ij}F_0(\vartheta;\tau_0) + \omega_{1,ij}F_1(\vartheta;\tau_1)]\mathrm{d}\vartheta\right\}, \quad (1)$$

where: i, j are the indices of compliance tensor elements in orthogonal coordinate system, and

$$F_0(t;\tau_0) = \frac{1}{\tau_0}\int_0^\infty \exp\left(-\frac{ut}{\tau_0}\right)u\Lambda(u)\mathrm{d}u,$$
$$\Lambda(u) = \frac{1}{\pi\sqrt{u}(1+u)}, \quad (2)$$
$$F_1(t;\tau_1) = \frac{1}{\tau_1}\exp\left(-\frac{t}{\tau_1}\right).$$

In formulae (1) and (2), the following denotations are used: t – time variable, $S_{ij}(t)$ – elastic compliance, $F_0(t;\tau_0)$ – fractional exponential function, $F_1(t;\tau_1)$ – normal exponential function, τ_0, τ_1 – retardation times $(\tau_0 << \tau_1)$, $\omega_{0,ij}, \omega_{1,ij}$ – long-term creep coefficients. It is assumed that all viscoelastic compliances are described by common retardation times τ_0, τ_1. The viscoelastic compliance is described by two functions, i.e. $F_0(t;\tau_0)$ that defines short-term and moderate-term rheological processes and $F_1(t;\tau_1)$ that influences only moderate-term rheological processes.

The standard constitutive equation representing linear viscoelaticity of an orthotropic material has the following form:

$$\boldsymbol{\varepsilon}(t) = \widetilde{\boldsymbol{S}}(\mathrm{t}) \otimes \boldsymbol{\sigma}(t) \equiv \int_{-\infty}^{t} \widetilde{S}(t-\tau)\frac{\partial \sigma(\tau)}{\partial \tau}\mathrm{d}\tau\,, \quad (3)$$

where:

$\boldsymbol{\sigma}(t) = \mathrm{col}(\sigma_x(t), \sigma_y(t), \sigma_z(t), \sigma_{yz}(t), \sigma_{xz}(t), \sigma_{xy}(t))$,

$\boldsymbol{\varepsilon}(t) = \mathrm{col}(\varepsilon_x(t), \varepsilon_y(t), \varepsilon_z(t), \gamma_{yz}(t), \gamma_{xz}(t), \gamma_{xy}(t))$,

$\otimes$ – the Stjelties convolution symbol.

Viscoelastic orthotropic material is described by 9 independent elastic constants and 20 independent viscoelastic constants, i.e. 18 short- and long-term viscoelastic coefficients $\omega_{0,ij}, \omega_{1,ij}$, i, j = 11, 22, 33, 12, 13, 23, 44, 55, 66 and 2 retardation times τ_0, τ_1 $(\tau_0 << \tau_1)$.

Creep tests were performed in order to verify whether it is possible to formulate constitutive equations of bone on the basis of HRK rheological model or wheter simplified rheological model (HK or HR) can be applied. In the latter case, the number of viscoelastic constants to identify drop to 11.

In the algorithm of viscoelastic constant identification, it is originally assumed that rheological properties of bone are described by the generic rheological model HRK. The compliance matrix elements are defined by normal and fractional exponential functions. It can be noticed that the HRK model comprises two simplified models, i.e. HR and HK models. The HR model is described by 9 elastic constants and 10 short-term viscoelastic constants $\omega_{0,ij}$, ij = 11, 22, 33, 12, 13, 23, 44, 55, 66, τ_0. The HK model is described by 9 elastic constants and 10 long-term viscoelastic constants $\omega_{1,ij}$, ij = 11, 22, 33, 12, 13, 23, 44, 55, 66, τ_1.

The orthotropic directions in bone are defined as follows: x – longitudinal direction of the bone, y – radial direction, z – circumferential direction.

Both selection of the adequate rheological model of bone (HRK, HR or HK) as well as determination of elastic and viscoelastic constants of bone were made by analysing the gradients of creep functions obtained from six classical creep tests (CT): CT1, CT2, CT3 – compression in the x-, y-, z-directions, respectively; CT4, CT5, CT6 – shear in the xy, yz, xz planes, respectively.

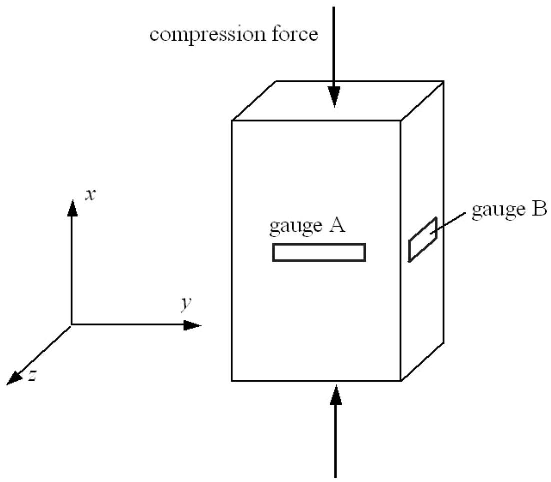

Fig. 1. Pattern of gauges applied in compression creep test: CT1: $\alpha = x$, $\beta = y$, $\gamma = z$, CT2: $\alpha = y$, $\beta = z$, $\gamma = x$, CT3: $\alpha = z$, $\beta = x$, $\gamma = y$

In compression tests, bone samples were rectangular prisms. Strains in the compression direction α were measured by means of strain gauge staple. Strains in the two perpendicular directions were

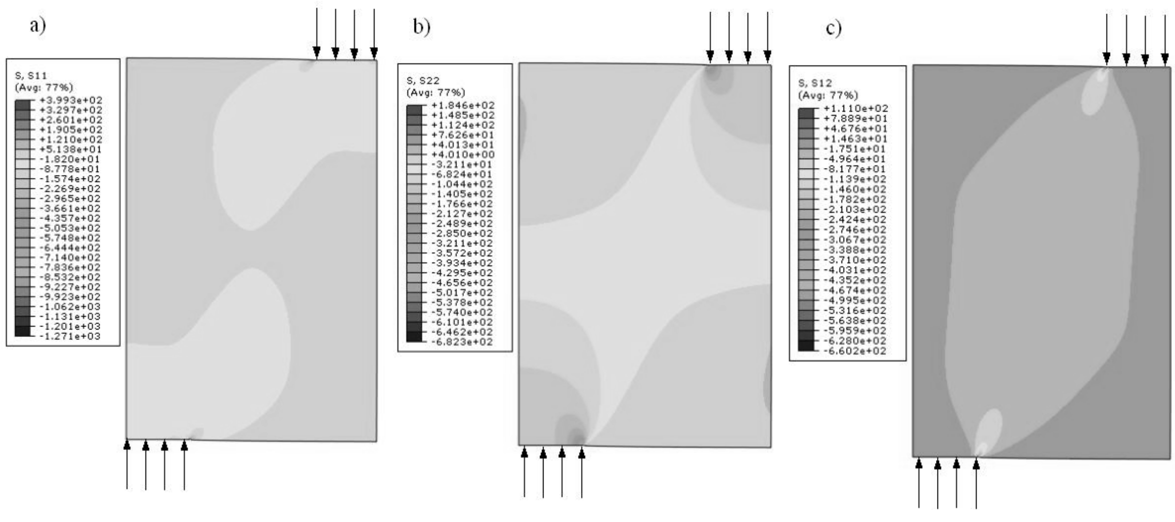

Fig. 2. Stress distribution in bone sample: a) stresses in horizontal direction (MPa), b) stresses in vertical direction (MPa), c) shear stresses (MPa)

measured by means of strain gauges that were stuck to bone sample surfaces in the proper directions (figure 1). The compression force applied was such that the stresses in the samples were 40 MPa. As for shear tests, it had to be decided first how many strain gauges and in what configuration had to be used in order to measure proper stains. Numerical analyses were, thus, made to verify whether or not in the sample there is a pure shear state. The analyses were carried out on a planar model (plane stress state) defined as an elastic orthotropic material. The results of numerical analyses (figure 2) showed that in the sample there is no pure shear state during shear test. Thus, a strain rosette was used to measure strains in shear test (figure 3). In the shear tests, quasi-uniform shear is applied in $\alpha\beta$ plane. The value of the load applied was determined such that the stresses in a bone sample did not exceed 50% of bone shear strength.

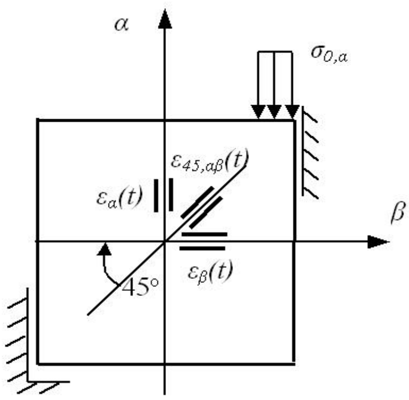

Fig. 3. Pattern of gauges in shear creep tests in plane $\alpha\beta$: CT4: $\alpha\beta = xy$, $\alpha\gamma = xz$, CT5: $\alpha\beta = yz$, $\alpha\gamma = yx$, CT6: $\alpha\beta = zx$, $\alpha\gamma = zy$

The load $\sigma_{0,\alpha}$ (compression test) or $\sigma_{0,\alpha\beta}$ (shear test) was applied for a relatively short period of time, i.e., 2÷5 s. The time t was measured from the moment of applying a full load.

The bone samples were extracted from calf femur. Once they were prepared they were kept in a liquid to prevent them from drying up. The surfaces on which strain gauges were to be stuck had to be dry out in order to ensure best surface conditions for gauge mounting [25]. The dimensions of a single sample were 10 × 6 × 3 mm. The smallest available gauges were applied in the creep tests. Their dimensions were 3 × 2.3 × 0.06 mm.

The elastic constants of orthotropic model of bone were identified for $t = 0$ s (elastic strains). In the further rheological tests, the following constitutive formulae were obtained:

CT1:

$$\begin{cases} \varepsilon_x(0) = S_{11}\sigma_{0,x} \\ \varepsilon_y(0) = S_{12}\sigma_{0,x} \end{cases} \Rightarrow \begin{cases} \varepsilon_x(0) = \dfrac{1}{E_x}\sigma_{0,x} \\ \varepsilon_y(0) = -\dfrac{\nu_{yx}}{E_x}\sigma_{0,x} \end{cases}$$

$$\Rightarrow E_x = \frac{\sigma_{0,x}}{\varepsilon_x(0)}, \nu_{yx} = -\frac{\varepsilon_y(0)}{\varepsilon_x(0)}, \tag{4}$$

CT2:

$$\begin{cases} \varepsilon_y(0) = S_{22}\sigma_{0,y} \\ \varepsilon_z(0) = S_{23}\sigma_{0,y} \end{cases} \Rightarrow \begin{cases} \varepsilon_y(0) = \dfrac{1}{E_y}\sigma_{0,y} \\ \varepsilon_z(0) = -\dfrac{\nu_{zy}}{E_y}\sigma_{0,y} \end{cases}$$

$$\Rightarrow E_y = \frac{\sigma_{0,y}}{\varepsilon_y(0)}, \nu_{zy} = -\frac{\varepsilon_z(0)}{\varepsilon_y(0)}, \tag{5}$$

CT3:

$$\begin{cases} \varepsilon_z(0) = S_{33}\sigma_{0,z} \\ \varepsilon_x(0) = S_{13}\sigma_{0,z} \end{cases} \Rightarrow \begin{cases} \varepsilon_z(0) = \dfrac{1}{E_z}\sigma_{0,z} \\ \varepsilon_x(0) = -\dfrac{\nu_{zx}}{E_x}\sigma_{0,z} \end{cases}$$

$$\Rightarrow E_z = \frac{\sigma_{0,z}}{\varepsilon_z(0)}, \nu_{xz} = -\frac{\varepsilon_x(0)}{\varepsilon_z(0)}, \nu_{zx} = \nu_{xz}\frac{E_x}{E_z}, \tag{6}$$

CT4:

$$\gamma_{xy}(0) = S_{66}\sigma_{0,yx} \Rightarrow \gamma_{xy}(0) = \frac{1}{G_{xy}}\sigma_{0,yx}$$

$$\Rightarrow G_{xy} = \frac{\sigma_{0,yx}}{\gamma_{xy}(0)}, \tag{7}$$

CT5:

$$\gamma_{yz}(0) = S_{44}\sigma_{0,zy} \Rightarrow \gamma_{yz}(0) = \frac{1}{G_{yz}}\sigma_{0,zy}$$

$$\Rightarrow G_{yz} = \frac{\sigma_{0,zy}}{\gamma_{yz}(0)}, \tag{8}$$

CT6:

$$\gamma_{xz}(0) = S_{55}\sigma_{0,xz} \Rightarrow \gamma_{xz}(0) = \frac{1}{G_{xz}}\sigma_{0,xz}$$

$$\Rightarrow G_{xz} = \frac{\sigma_{0,xz}}{\gamma_{xz}(0)}. \tag{9}$$

In the creep test CT1, one obtains:

$$\begin{cases} \varepsilon_x(t) = \widetilde{S}_{11}(t)\sigma_{0,x}, \\ \varepsilon_y(t) = \widetilde{S}_{12}(t)\sigma_{0,x}, \end{cases}$$

and according to equations (3) and (1):

$$\begin{cases} \varepsilon_x(t) = S_{11}\left[1+\omega_{0,11}\varphi_0\left(\frac{t}{\tau_0}\right)+\omega_{1,11}\varphi_1\left(\frac{t}{\tau_1}\right)\right]\sigma_{0,x} \\ \varepsilon_y(t) = S_{12}\left[1+\omega_{0,12}\varphi_0\left(\frac{t}{\tau_0}\right)+\omega_{1,12}\varphi_1\left(\frac{t}{\tau_1}\right)\right]\sigma_{0,x} \end{cases}$$

$$\Rightarrow \begin{cases} \varepsilon_x(t) = \left[1+\omega_{0,11}\varphi_0\left(\frac{t}{\tau_0}\right)+\omega_{1,11}\varphi_1\left(\frac{t}{\tau_1}\right)\right]\varepsilon_x(0), \\ \varepsilon_y(t) = \left[1+\omega_{0,12}\varphi_0\left(\frac{t}{\tau_0}\right)+\omega_{1,12}\varphi_1\left(\frac{t}{\tau_1}\right)\right]\varepsilon_y(0). \end{cases} \quad (10)$$

In the farther rheological tests, one analogically arrives at:

CT2:

$$\begin{cases} \varepsilon_y(t) = \left[1+\omega_{0,22}\varphi_0\left(\frac{t}{\tau_0}\right)+\omega_{1,22}\varphi_1\left(\frac{t}{\tau_1}\right)\right]\varepsilon_y(0), \\ \varepsilon_z(t) = \left[1+\omega_{0,23}\varphi_0\left(\frac{t}{\tau_0}\right)+\omega_{1,23}\varphi_1\left(\frac{t}{\tau_1}\right)\right]\varepsilon_z(0), \end{cases} \quad (11)$$

CT3:

$$\begin{cases} \varepsilon_z(t) = \left[1+\omega_{0,33}\varphi_0\left(\frac{t}{\tau_0}\right)+\omega_{1,33}\varphi_1\left(\frac{t}{\tau_1}\right)\right]\varepsilon_z(0), \\ \varepsilon_x(t) = \left[1+\omega_{0,13}\varphi_0\left(\frac{t}{\tau_0}\right)+\omega_{1,13}\varphi_1\left(\frac{t}{\tau_1}\right)\right]\varepsilon_x(0), \end{cases} \quad (12)$$

CT4:

$$\gamma_{yx}(t) = \left[1+\omega_{0,66}\varphi_0\left(\frac{t}{\tau_0}\right)+\omega_{1,66}\varphi_1\left(\frac{t}{\tau_1}\right)\right]\gamma_{yx}(0), \quad (13)$$

CT5:

$$\gamma_{yz}(t) = \left[1+\omega_{0,44}\varphi_0\left(\frac{t}{\tau_0}\right)+\omega_{1,44}\varphi_1\left(\frac{t}{\tau_1}\right)\right]\gamma_{yz}(0), \quad (14)$$

CT6:

$$\gamma_{xz}(t) = \left[1+\omega_{0,55}\varphi_0\left(\frac{t}{\tau_0}\right)+\omega_{1,55}\varphi_1\left(\frac{t}{\tau_1}\right)\right]\gamma_{xz}(0). \quad (15)$$

Equations (10)÷(15) have the analogical structure. Thus, it is sufficient to formulate the algorithm of viscoelastic constant identification using one generic formula:

$$\varepsilon(t) = \left[1+\omega_0\varphi_0\left(\frac{t}{\tau_0}\right)+\omega_1\varphi_1\left(\frac{t}{\tau_1}\right)\right]\varepsilon(0). \quad (16)$$

The retardation times τ_0, τ_1 were determined from one creep test, e.g. CT1. In the next tests, the hypothesis of the same retardation times for all the strain components is verified. The constants $\omega_0, \omega_1, \tau_0, \tau_1$ were determined from the condition of two curve fitting, i.e. the theoretical curve representing the change of $\varepsilon(t)$ with time for a given rheological model and the experimental curve representing the change of $\widetilde{\varepsilon}(t)$ with time.

The algorithm of identification of viscoelastic constants is as follows [26]:

a) determination of change range of τ_0, τ_1 values on the basis of the experimental creep function,

b) determination of change range of ω_0, ω_1 values on the basis of experimental creep function,

c) estimation of ω_0, τ_0 in the time range $t \in [0, T_0]$, assuming that HRK model is simplified to HR model based on the condition of the closest fitting of function

$$\varepsilon(t) = \left[1+\omega_0\varphi_0\left(\frac{t}{\tau_0}\right)\right]\varepsilon(0)$$

and function

$$\widetilde{u}(t) = \omega_0\varphi_0\left(\frac{t}{\tau_0}\right),$$

d) estimation of ω_1, τ_1 in the time range $t \in [0, T_1]$ from the condition of the closest fitting of function

$$\varepsilon(t) = \left[1+\omega_0\varphi_0\left(\frac{t}{\tau_0}\right)+\omega_1\varphi_1\left(\frac{t}{\tau_1}\right)\right]\varepsilon(0)$$

and function

$$\widetilde{u}(t) = \omega_0\varphi_0\left(\frac{t}{\tau_0}\right)+\omega_1\varphi_1\left(\frac{t}{\tau_1}\right),$$

e) calculation of function fitting error δ.

The creep tests were performed on a stand of our own design and construction (figure 4). A bone sample is subjected to compression or shear by means of cantilever. Sample is placed in a special unit, being different for compression tests and shear tests.

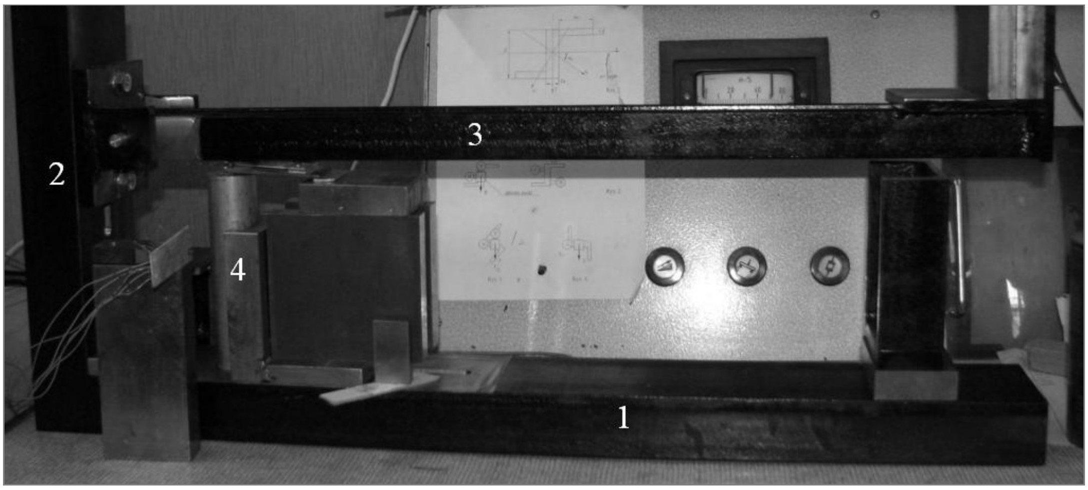

Fig. 4. Stand for creep tests: 1 – basis, 2 – vertical post, 3 – cantilever, 4 – shear and compression unit where sample is placed

3. Results

The identification of elastic and viscoelastic constants will be shown in detail for the rheological test CT1. In this test, three creep curves were obtained in three perpendicular directions (figure 5). The Young modulus in a given direction can be calculated using equation

$$E_\alpha = \frac{\sigma_{0,\alpha}}{\varepsilon_\alpha(0)},$$

where $\alpha = x, y, z$. The respective values of the Young modulus (E) and the Poisson ratio (ν) are as follows: E_x = 5.95 GPa, $\nu_{yx} = -0.43$ (equation (4)), E_y = 13.6 GPa, $\nu_{zy} = -0.1$ (equation (5)), E_z = 1.24 GPa, $\nu_{xz} = -0.2$ (equation (6)). Figure 6 shows the strain history only in the direction of compression in logarithmic scale of time. In the time range $t \in$ [4000″, 70 000″], the creep gradient indicates a normal exponential function as a function of load history. The history of the creep process in the ranges of short-term and long-term creep makes it possible to modify the identification algorithm, which consists in reading off the time retardation. From the experimental creep function the following retardation times were determined:

$$\tau_0 = 800'', \quad \tau_1 = 30\,000''.$$

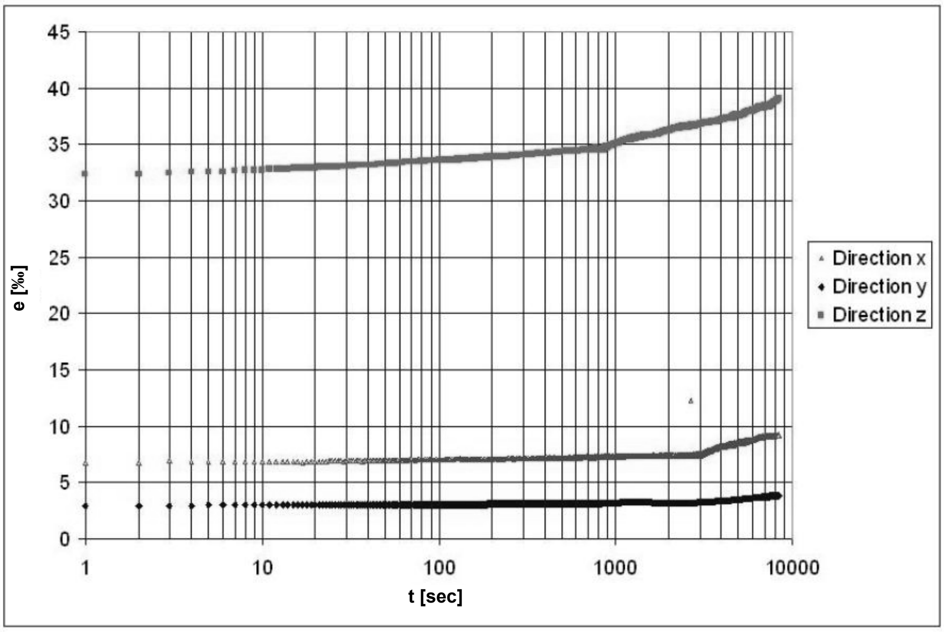

Fig. 5. Experimental creep curves in: longitudinal (x), radial (y), circumferential (z) directions obtained from CT1

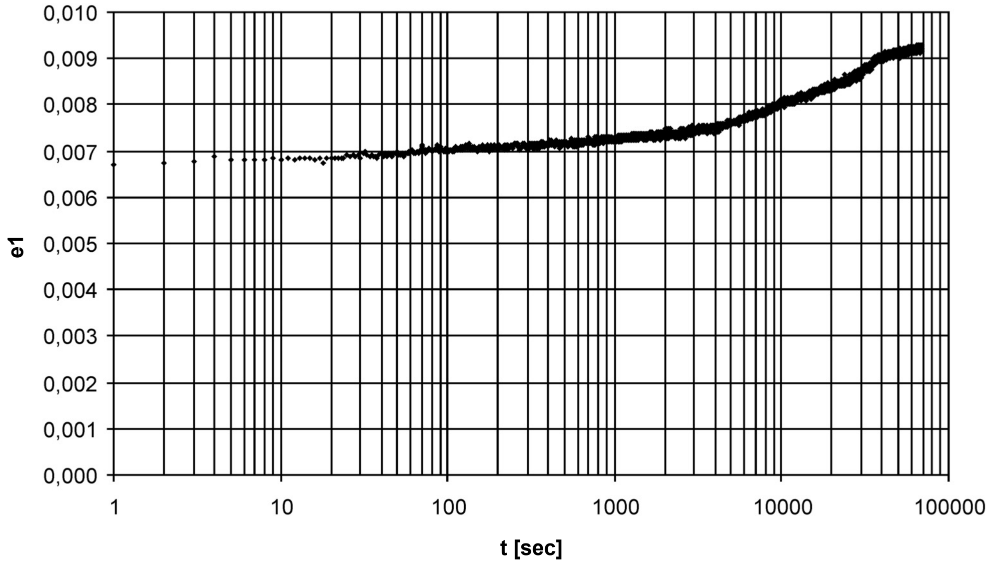

Fig. 6. Experimental creep curve in direction of compression force (CT1)

The viscoelastic constants were identified by means of the algorithm described above:

- for HRK model

$$\omega_{0,11} = 0.15, \qquad \tau_0 = 800'', \qquad \delta = 0.38\%,$$
$$\omega_{1,11} = 0.24, \qquad \tau_1 = 30000'', \qquad \delta = 0.81\%;$$

- for HK model

$$\omega_{1,11} = 0.36, \qquad \tau_1 = 13000', \qquad \delta = 2.25\%.$$

In figures 7 and 8, theoretical creep functions for HRK model and HK model are shown, respectively. It is the creep function for HRK model that the experimental creep function fits best. The HK creep function shows no strain gradients in short-term creep, thus it is not adequate to describe rheological properties of bone.

The other elastic properties of bone, i.e. shear strain $\gamma_{\alpha\beta}$ and the Kirchhoff modulus $G_{\alpha\beta}$, where $\alpha, \beta = x, y, z$, can be calculated on the basis of the results obtained from shear creep tests. For instance, the results from shear creep test in the *xz* plane prove that the shear strain of bone γ_{xz} equals 0.00275, and the Kirchhoff modulus of bone G_{xz} is 3.6 GPa.

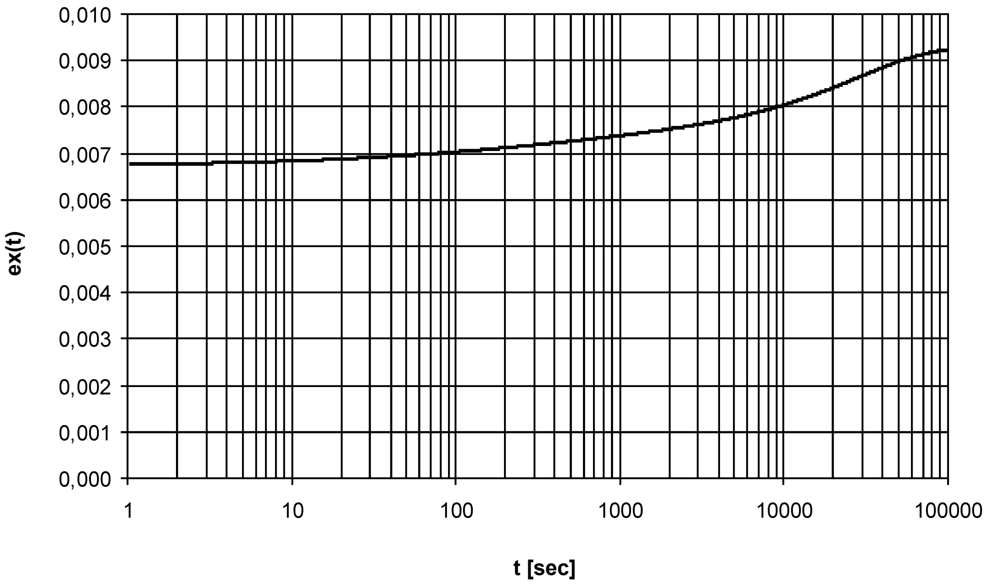

Fig. 7. Theoretical creep curve for HRK model (CT1)

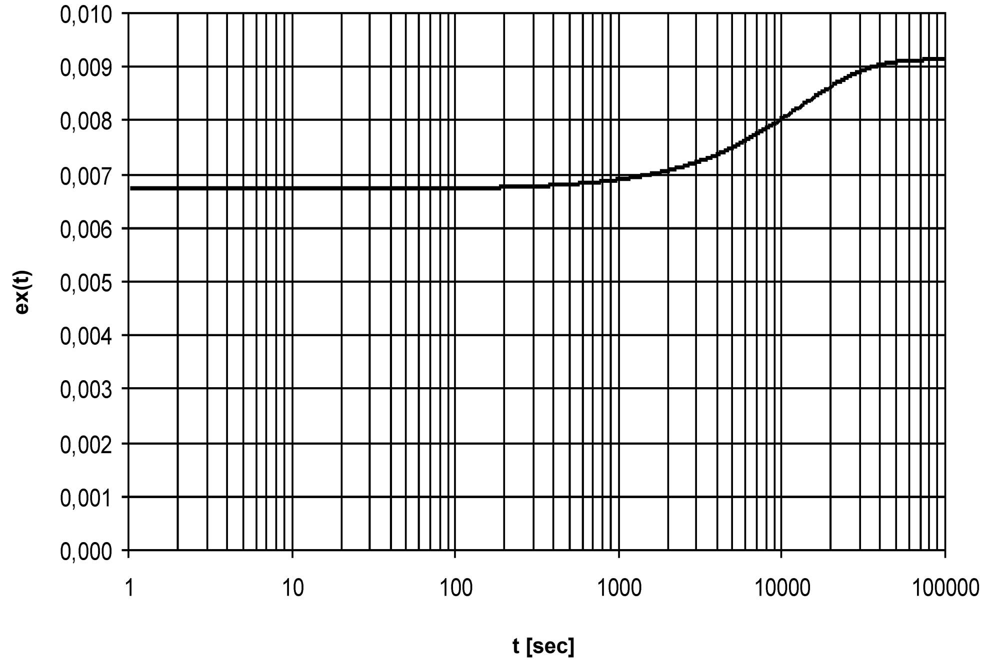

Fig. 8. Theoretical creep curve for HK model (CT1)

4. Conclusion

The preliminary studies on bone rheological properties and bone constitutive equations indicate that the HRK (Hooke–Rabotnov–Kelvin) model leads to an accurate simulation of real creep process of bone (see figures 6 and 7). The error of the relative fitting of theoretical creep function to experimental one is equal to 0.38% for short-term creep and 0.81% for long-term and moderate-term creep. Short-term creep gradients can be described by fractional exponential functions, while moderate-term creep gradients – by normal exponential functions. The classical HK (Hooke–Kelvin) model leads to significant qualitative errors in the range of short-term creep, i.e. there are too high discrepancies in the initial phase of creep between the HK model theoretical creep history and experimental creep history (see figures 6 and 8). Quantitative error is also significant and equals 2.25%.

Our studies on bone constitutive equations have a preliminary character. Their results will make it possible to simulate numerically the behaviour of bone in a more realistic way. The numerical simulation will next be included in the process of a custom-made prosthesis design. This will make the process more effective by yielding prostheses of higher durability. However, one cannot yet firmly state that the identified elastic and viscoelastic constants determine real rheological behaviour of bone. Also the relationship between bone sample density and viscoelastic constants is of a significant importance. Once this relationship is known one can include bone remodelling phenomenon, which can be defined as bone density change with time, into bone behaviour simulation and consequently into a custom-made prosthesis design process. The creep tests described above were done on calf bone samples extracted from femur. In the future, the authors intend to carry out similar rheological tests on human bone samples also extracted from femur and/or from pelvis. The results of those tests will be then included into the process hip joint prosthesis design. It is obvious that different bones have different material properties; however, the whole methodology of rheological constant identification can be applied also to other bones. This will allow us to simulate also the behaviour of other bones and to utilise this in the process of design of different joint prostheses.

For more proper formulation of bone constitutive equations it is indispensable to perform more creep tests on samples taken from various areas of the same bone.

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