

Analysis of different material theories used in a FE model of a lumbar segment motion

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In this study, a nonlinear poroelastic model of intervertebral disc as an infrastructure was developed. Moreover, a new element was defined consisting a disc (Viscoelastic Euler Beam Element) and a vertebra (Rigid Link) as a unit element. Using the new element, three different viscoelastic finite element models were prepared for lumbar motion segment (L4/L5). Prolonged loading (short-term and long-term creep) and cyclic loading were applied to the models and the results were compared with results of *in vivo* tests. Simplification of the models by using the new element leads to reduction of the runtime of the models in dynamic analyses to few minutes without losing the accuracy in the results.

Key words: creep, dynamic loading, Finite Element Method (FEM), lumbar motion segment, poroelastic, viscoelastic

1. Introduction

Finite element method, which is reputable, fast and inexpensive, was used for analysis of spine issues invasively. Finite elements models of spine can be divided into two types, i.e., complicated models and simplified models.

Complicated finite element models were prepared for a region of the spine as lumbar [1], [2] or cervical [3], [4] or for a segment of motion or disc [5]–[10]. The complicated models, which were usually used for static analysis consist of the vertebrae with posterior parts, the discs with two different material properties (annulus and nucleus), and the ligaments.

For modeling the thoracolumbar region [11] and upper body [12], simplified finite element models were used. In the simplified models of spine, either vertebra

was modeled without posterior part or as an elastic beam element. Either disc was modeled in two parts (annulus and nucleus part with elastic behavior), or the disc was modeled as a damping unit [11]. In some models, ribcage and pelvis finite element models were also added to the spine finite element model [12].

Although running a complicated motion segment model is possible for long-time dynamic loadings, the models for a region of spine or whole spine, which enjoy complicated material behaviors and geometry, cannot be easily run for long-term dynamic loading because of having too long runtimes. Therefore, we need to use more simplified elements and material methods in these models. It is important that the simplified models be able to predict long-time dynamic loadings without losing accuracy when compared with the complicated model predictions. Therefore, it is necessary to compare the results of complicated and

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simplified models with each other. At first, the comparison can be carried out between complicated and simplified models for a segment of motion.

Finite element models of lumbar motion segments have taken multiple directions: These early studies used relatively simple representations of geometry and material properties. Belytschko et al. [13] assumed an axisymmetric geometry, allowing a single two dimensional slice to represent the entire motion segment. They used linear elastic materials, combined the annulus fibers and ground substance into a single element, and assumed an incompressible nucleus.

A more complex, but still linear, model was developed by Shirazi-Adl et al. [8]. A full 3D representation of the motion segment was developed, and the annulus was divided into separate elements for the fibers and ground substance. Kim et al. [14] added nonlinear ligaments, and loss of fluid in the disc was studied by Shirazi-Adl et al. [11] by changing the disc volume. Time dependent characteristics of the intervertebral disc were modeled with viscoelastic elements by Wang et al. [9], [10]. A lot of experimental observations suggest that it would be more realistic to consider intervertebral disc as a multiphase model [15], [6].

In recent years, poroelastic theory has been applied to model the intervertebral disc. The studies using these models suggest that the biphasic nature of the disc is an important factor in load transfer and stress distribution. On the basis of the Biot theorem, Simon et al. [16] considered the soft tissues in the spinal motion segment as poroelastic material which was later extended by Wu et al. [17], Lee et al. [18], Laible et al. [19], Iatridis et al. [20], Williams et al. [21], Ehlers et al. [22], and Schmidt et al. [1].

In the present study, the finite element models were analyzed regarding different material models and complexity of the elements. At first, a nonlinear poroelastic model of intervertebral disc was developed as an infrastructure to enhance the understanding of the role of different loading regimes on the creep behavior of a lumbar motion segment. In the next step, a new element was defined to simplify motion segment models and three models of a lumbar segment motion (L4/L5)

were developed by it. The models had the same geometry but three different viscoelastic models (Kelvin, Prony series first order and Burger's model) were used for them. Prolonged loading (short-term and long-term creep) and cyclic loading were applied to the models mentioned and the global intervertebral disc responses (displacement versus time) were compared with the results of *in vivo* tests.

2. Materials and methods

2.1. Poroelastic model of intervertebral disc

A commercial finite element package (ABAQUS v6.9; ABAQUS Inc., Pawtucket, RI, USA) was used to formulate axisymmetric nonlinear finite element model of an intervertebral disc from the L4-L5 disc level. Because symmetry towards transversal plane is assumed, only half of the intervertebral disc was modeled (Fig. 1). Drained elastic material properties as well as poroelastic properties of the components for the nucleus pulposus (NP), annulus fibrosus (AF), and cartilaginous endplates were taken from literature [17] (Table 1). It was assumed that there was free fluid flow and no sliding, across the interface between nucleus and annulus. Along the transversal symmetry plane, no axial displacement and no fluid flux across the plane were assumed. To simulate the swelling pressure, a boundary pore pressure of 0.3 MPa was imposed at external surfaces of intervertebral disc. 8-node axisymmetric elements with quadratic interpolation of

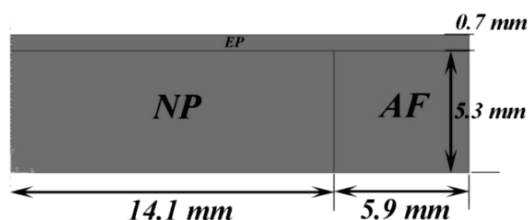


Fig. 1. Axisymmetric model of intervertebral disc

Table 1. Material properties of different tissues used in the poroelastic finite element model

Region	Coefficient			
Nucleus pulposus	$E = 1 \text{ MPa}$	$\nu = 0.1$	Porosity: 0.78	Permeability: $3\text{E-}16 \text{ m}^4/\text{Ns}$
Annulus fibrosus	$E = 2.5 \text{ MPa}$	$\nu = 0.1$	Porosity: 0.71	Permeability: $3\text{E-}16 \text{ m}^4/\text{Ns}$
Cartilaginous endplates	$E = 20 \text{ MPa}$	$\nu = 0.1$	Porosity: 0.80	Permeability: $7\text{E-}15 \text{ m}^4/\text{Ns}$

displacement fields and linear interpolation of pore pressure were used for different components of IVD model. The sensitivity analysis was investigated to optimize the mesh density and finally the whole model was developed with 12400 elements [24].

2.2. New element

In the present study, a new element consisting a disc (Viscoelastic Euler Beam Element) and a vertebra (Rigid Link) as a unit element was defined. Three lumbar motion segment models (L4/L5) were made by using the new element (MATLAB Commercial Software). Three different viscoelastic models (Kelvin model, Prony series order one and Burger's model) were used in the models.

2.2.1. Viscoelastic Euler beam element

The disc was modeled as a Viscoelastic Euler Beam Element. Each node of the element has 6 DOF. It bends around two axes (y and z directions), and has torsion (x direction) and axial deformation (x direction). The equations for bending, torsion and axial loading of these elements were extracted by using virtual method. The equations extracted for viscoelastic Euler beam with Kelvin model for bending, torsion and axial loading, respectively, are the following

$$IE[k_e]\{d_e\} + I\eta[k_e]\{\dot{d}_e\} = \{F_e\}, \quad (1)$$

$$JG[k_e]\{d_e\} + J\eta[k_e]\{\dot{d}_e\} = \{F_e\}, \quad (2)$$

$$AE[k_e]\{d_e\} + A\eta[k_e]\{\dot{d}_e\} = \{F_e\}, \quad (3)$$

where: A – cross section, I – moment of inertia, E and η – parameters of the Kelvin model, G – shear modulus, $\{F_e\}$ – nodal force vector, $\{d_e\}$ – nodal displacement vector and $\{K_e\}$ – stiffness matrix.

The equations extracted for viscoelastic Euler beam with first order Prony series model for bending, torsion and axial loading, respectively, are as follows

$$I[k_e]\{\dot{d}_e\} + Iq_0[k_e]\{d_e\} = p_1\{\dot{F}\} + p_0\{F\}, \quad (4)$$

$$J[k_e]\{\dot{d}_e\} + Jq_0^*[k_e]\{d_e\} = p_1^*\{\dot{F}\} + p_0^*\{F\}, \quad (5)$$

$$A[k_e]\{\dot{d}_e\} + Aq_0[k_e]\{d_e\} = p_1\{\dot{F}\} + p_0\{F\}, \quad (6)$$

where

$$q_0 = \frac{E_1 E_0}{(E_1 + E_0)\eta_1}, \quad (7)$$

$$p_1 = \frac{1}{(E_1 + E_0)}, \quad (8)$$

$$p_0 = \frac{E_1}{(E_1 + E_0)\eta_1}, \quad (9)$$

where E_0 , E_1 and η_1 are parameters of the first order Prony series model. In equation (9), q_0^* , p_1^* and p_0^* were defined by replacing G_1 and G_2 instead of E_1 and E_2 , respectively in equations (7)–(9).

The equations extracted for viscoelastic Euler beam with Burger's model for bending, torsion and axial loading, respectively, are as follows

$$I[k_e]\{\ddot{d}_e\} + Iq_0[k_e]\{\dot{d}_e\} = p_2\{\ddot{F}\} + p_1\{\dot{F}\} + p_0\{F\}, \quad (10)$$

$$J[k_e]\{\ddot{d}_e\} + Iq_1^*[k_e]\{\dot{d}_e\} = p_2^*\{\ddot{F}\} + p_1^*\{\dot{F}\} + p_0^*\{F\}, \quad (11)$$

$$A[k_e]\{\ddot{d}_e\} + Aq_1[k_e]\{\dot{d}_e\} = p_2\{\ddot{F}\} + p_1\{\dot{F}\} + p_0\{F\}, \quad (12)$$

where

$$p_0 = \frac{E_1}{\eta_1 \eta_2}, \quad (13)$$

$$p_1 = \frac{1}{\eta_1} \left(1 + \frac{E_1}{E_2} + \frac{\eta_1}{\eta_2} \right), \quad (14)$$

$$p_2 = \frac{1}{E_2}, \quad (15)$$

$$q_1 = \frac{E_1}{\eta_1}, \quad (16)$$

where E_1 , η_1 , E_2 and η_2 are parameters of Burger's model. In equation (12), q_1^* , p_2^* , p_1^* and p_0^* were defined by replacing G_1 and G_2 instead of E_1 and E_2 , respectively in equations (13)–(16).

2.2.2. Rigid link

To create the finite element model of spine, the disc modeled as a viscoelastic Euler beam and the body modeled as rigid link should be assembled (Fig. 2).

The cinematic of the rigid link make a relation between deformations of nodes 2 and 3 (Eq. (17)).

$$\begin{bmatrix} u_3 \\ v_3 \\ w_3 \end{bmatrix} = \begin{bmatrix} u_2 \\ v_2 \\ w_2 \end{bmatrix} + \begin{bmatrix} -\alpha_2 l_z + \beta_2 l_y \\ -\beta_2^* l_x \\ \alpha_2^* l_x \end{bmatrix}, \quad (17)$$

where α_2 – slope of second node around y axis, β_2 – slope of second node around z axis, u , v , w – dis-

placements in x , y , z directions respectively for each node, l_x , l_y and l_z – the rigid link length in direction of x , y , z , respectively.

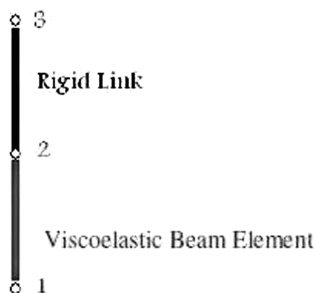


Fig. 2. Illustration of the disc as a Viscoelastic Euler Beam and vertebra as Rigid Link

By using equation (17), the rigid link and viscoelastic Euler beam can be assumed as an element (Fig. 3). For the new element, one just needs to modify the viscoelastic Euler beam stiffness matrix.

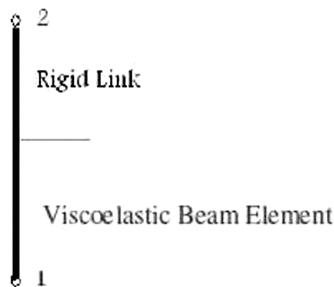


Fig. 3. Illustration of the new element. A disc and a vertebra were considered as an element

In the codes written by MATLAB Commercial Software, the stiffness matrix (12*12) was defined for the new element in global system. Then, the extracted equations were solved by using central difference method for each viscoelastic model. Time step to solve equations was defined to be one second. In each time step, the displacements of element nodes, which were considered small deformations, were calculated.

By using the new defined element, three different models of a lumbar segment of motion (L4/L5) were prepared and consequently three different viscoelastic models (Kelvin model, Prony series order one and Burger's model) were used in the models. The parameters of the viscoelastic models were extracted by fitting response of each model to the averaged experimental results of eight motion segments by Wang et al. [9].

2.3. Long-term creep

A model-response validation was determined by comparing model predictions with the results of circadian variation measured *in vivo* by Tyrrell et al. [25]. In *in vivo* tests, disc was compressed by 850 N for 16 hr (creep test) and was unloaded to 400 N for 8 hr (relaxation test) [21].

2.4. Short-term creep

The validated FE models were subjected to short-term creep loading and their predictions were compared with *in vivo* results (Tyrrell et al. [25]). In the short-term creep study conducted by Tyrrell et al. [25] a normal subject held a 40 kg barbell across his shoulders for 20 min, at which time the barbell was removed for a 10 min recovery period. Therefore, a pressure equivalent to 400 N was applied as a preload followed by an additional 400 N for 20 min on the superior surface of L4, after which the load was reduced to 400 N for 10 min for recovery [21].

2.5. Cyclic loading

The validated FE models were employed to investigate the biomechanical response of the intervertebral disc during daily cyclic loading and unloading. Loading regime for a day was simulated with an eight-hour resting period followed by a sixteen-hour period for diurnal activities. For diurnal activities, three different loading regimes were applied to the models: The resting period was the same in all the three different cases involving an eight-hour period under a constant compressive load of 350 N [1].

3. Results

3.1. Long-term creep and models validation

Distribution of the percent loss of total stature predicted by the models was closely compared to *in vivo* results throughout loading as well as unloading (Fig. 4). The models predictions of total stature change at the end of loading (16 hr) were 3.92% for Kelvin model, 1.75% for Prony series first order, 6.27% for Burger's model and 8.12% for poroelastic model of the corresponding *in vivo* measurement and the total stature loss at the end of creep unloading also matched the *in vivo* results [25].

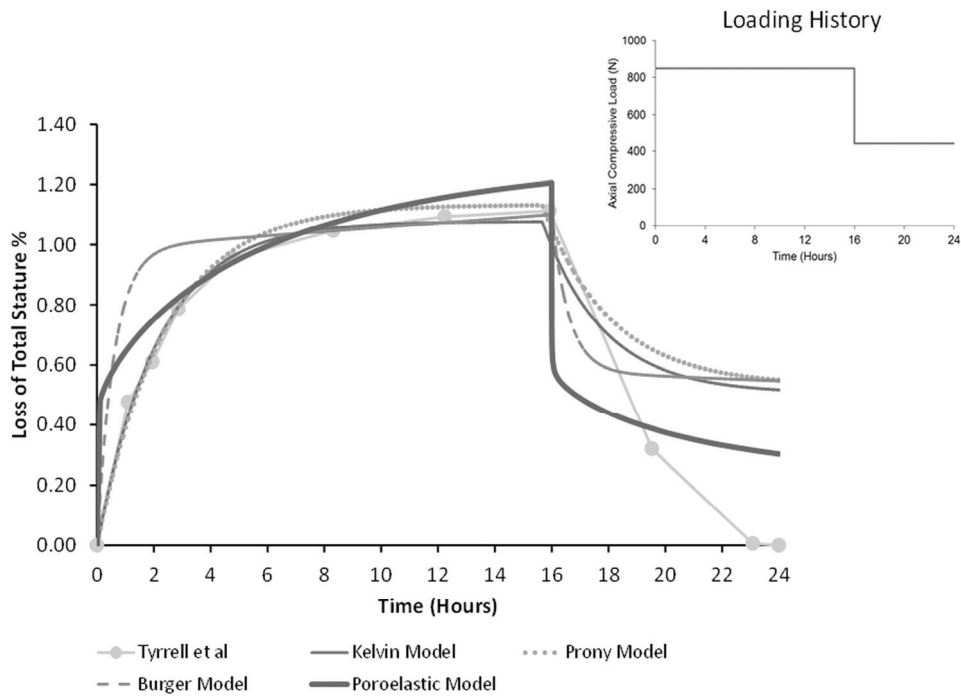


Fig. 4. Comparison between *in vivo* data (Tyrrell et al. [25]) and results of the viscoelastic finite element models subjected to long-term creep. The total time equals 24 hr (16 hr creep by 850 N and 8 hr the load reduced to 400 N)

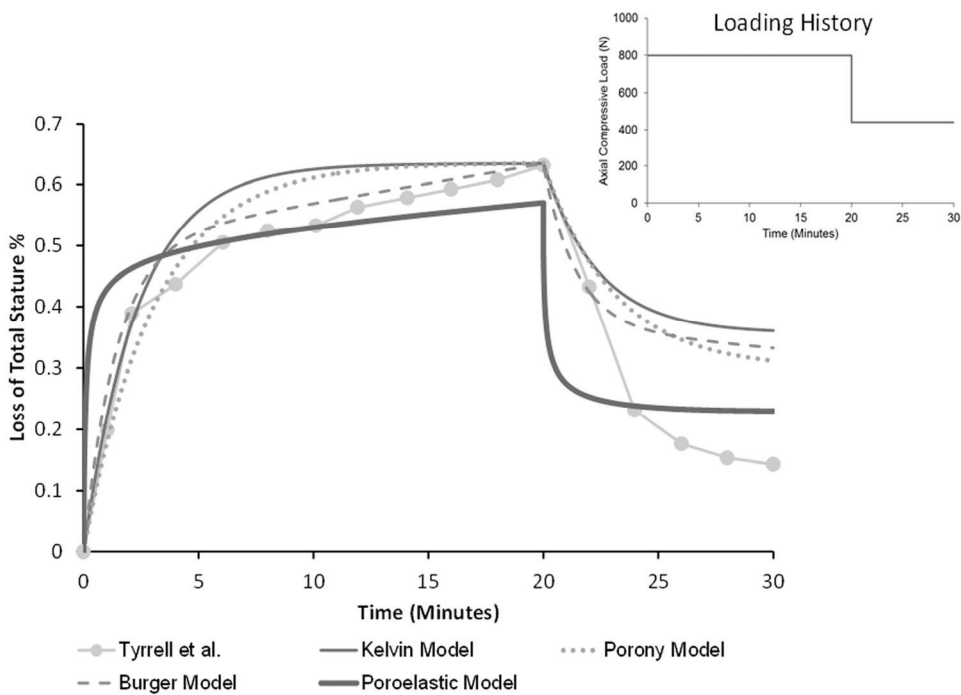


Fig. 5. Comparison between *in vivo* data (Tyrrell et al. [25]) and results of the viscoelastic finite element models subjected to short-term creep. Total time equals 30 min (20 min creep by 800 N and 10 min the load reduced to 400 N)

3.2. Short-term creep

The models predictions of total stature change (Fig. 5) at the end of loading (20 min) were 0.44% for

Kelvin model, 0.55% for Prony series first order, 0.44% for Burger’s model and 6.58% for poroelastic model of the corresponding *in vivo* measurement and the total stature loss at the end of creep unloading also matched the *in vivo* results [25].

3.3. Cyclic loading

The responses of FE models subjected to the three loading regimes mentioned are shown in Figs.

6–9. Experimental measurements by Adams et al. indicated that the intervertebral disc height loss were 1.2 ± 0.3 mm [26] after three hours under a constant prolonged compression (1000 N). Our FE models predictions were 1.26 mm for Kelvin model (Fig. 6),

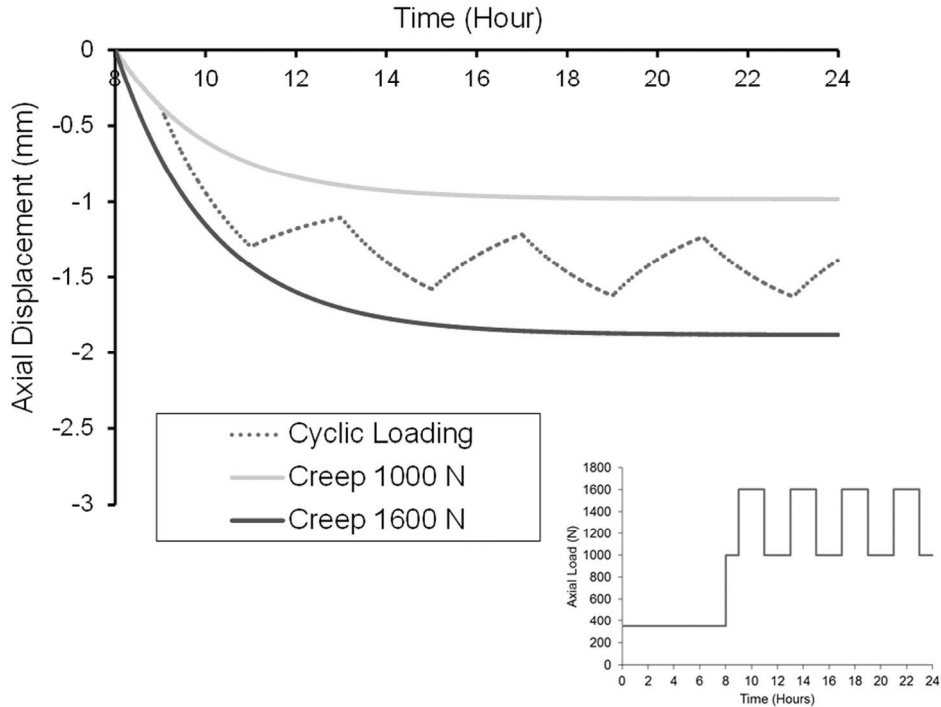


Fig. 6. Comparison of Kelvin model response subjected to cyclic loading (grey line) with related prolonged loadings (green line for 1000 N creep test and red line for 1600 N creep test)

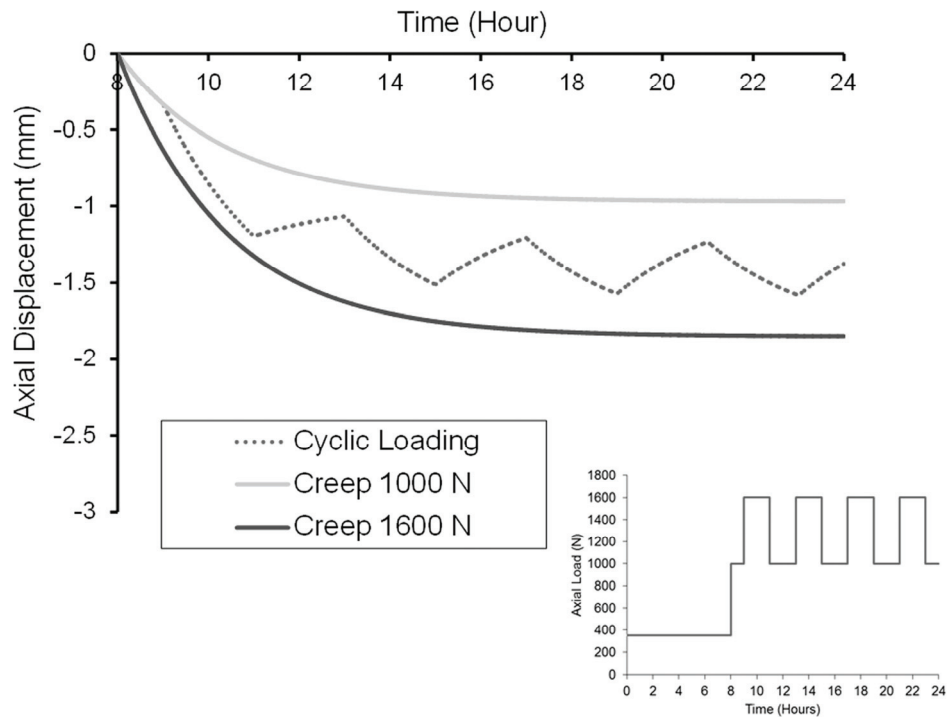


Fig. 7. Comparison of Prony series first order model response subjected to cyclic loading with related prolonged loadings (1000 N creep test and 1600 N creep test)

1.33 mm for Prony series first order (Fig. 7), 1.39 mm elastic model (Fig. 9) which are in the reported range. for Burger's model (Fig. 8) and 1.16 mm for poro-

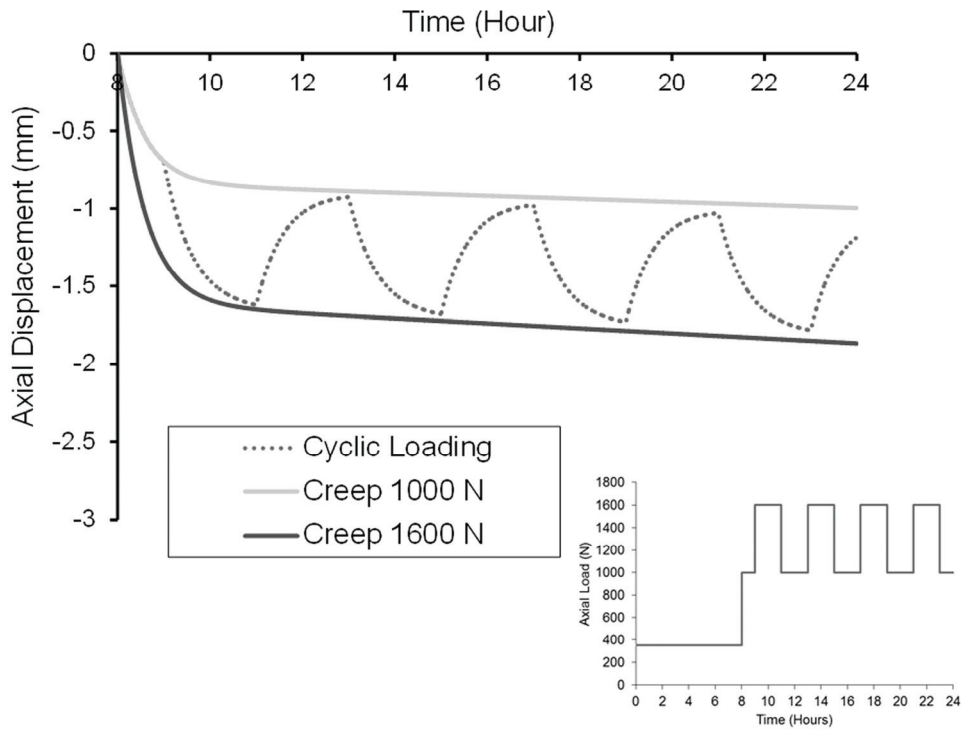


Fig. 8. Comparison of Burger's Model response subjected to cyclic loading with related prolonged loadings (1000 N creep test and for 1600 N creep test)

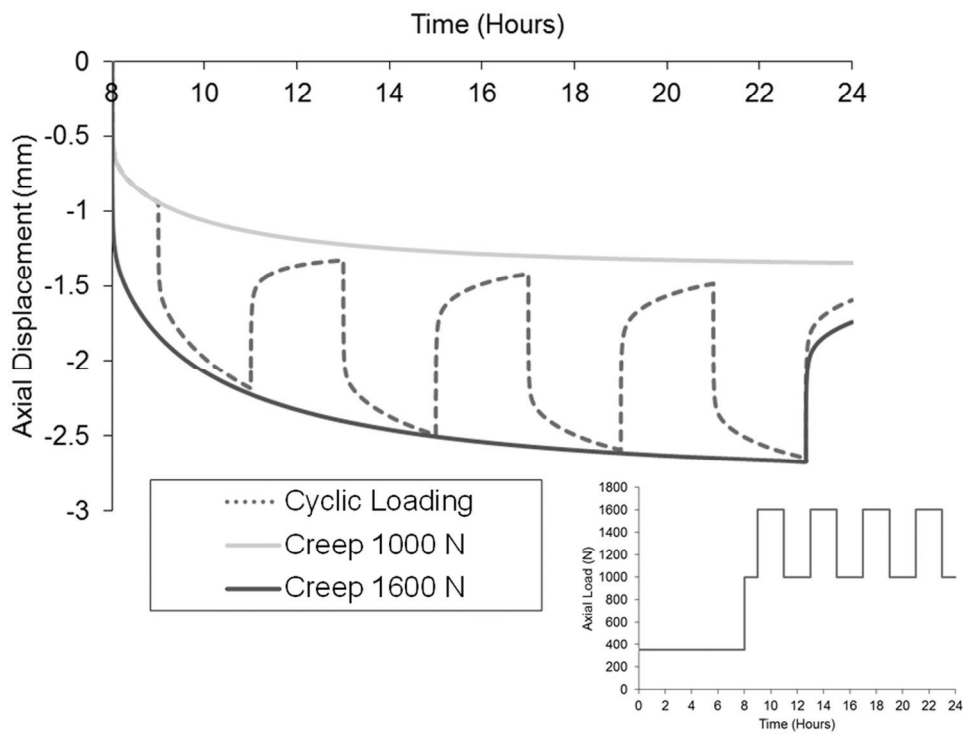


Fig. 9. Comparison of poroelastic model response subjected to cyclic loading with related prolonged loadings (1000 N creep test and for 1600 N creep test)

4. Discussion

The viscoelastic finite element models of lumbar segment motion (L4/L5) were defined by using the new element. The three different viscoelastic models (Kelvin model, Prony series first order, Burger's model) were used to define viscoelastic properties of disc in the models. Another model of lumbar segment motion was prepared with poroelastic properties for disc.

To validate models, the long term creep results were compared with *in vivo* results by Tyrrell et al. [25]. All the models represented results with acceptable accuracy. Although the viscoelastic models had more accurate responses in creep zone, the poroelastic model result in the relaxation zone is nearer to the results of *in vivo* tests by Tyrrell et al. [25].

Although the results of FE models did not completely show the recovery of intervertebral disc investigated in Tyrrell *in vivo* tests [25]. The FE model predictions had the general trends of the long-term creep responses. The more advanced work of Williams et al. [21] also failed to correctly predict the final phase recovery and this poses a further challenge for our future models.

The validation models were loaded for short-term creep test and represented results with acceptable accuracy. The trends of the models responses in the short creep test were the same with their responses in the long-term creep. Compared with the long-term creep test results, the short-term creep test results in relaxation time are closer to the results of *in vivo* tests by Tyrrell et al. [25].

Cyclic loadings defined for 24-hour activities by Williams et al. [21] were exerted to the models and the result revealed that all the model predictions were in the range published by Adams et al. [26].

Although the predictions of the models were close to each other, the runtimes of the viscoelastic models were less than that of the poroelastic model. The poroelastic model had complicated geometry and was created by a lot of elements. Therefore, running the model for dynamic loadings was time consuming. Via employing the new element in the model of a motion segment, the needed element for preparing a segment motion model was reduced to only an element. Therefore, for the viscoelastic models, runtime for dynamic loadings took just some minutes.

In the finite element models of a segment of motion, although details of geometry complexity and complicated material properties make our models more realistic, the complicated models have long run-

time for dynamic loadings. Therefore, simplification of models by using the new element, leads to reduction of the runtime of the model for dynamic loading to few minutes without losing accuracy.

In future, finite element models of lumbar region, thoracolumbar region and whole spine are going to be prepared by the new element. These models will have the minimum number of the elements and enjoy non-linear viscoelastic properties for discs. The models can easily analyze dynamic loadings with personal computers in some minutes. Using the new element, we can simulate the responses of whole spine for a day or longer which is not applicable with conventional elements. For example, the models can be used to predict wearing brace results on patients having spine disorders. During several months, brace forces are exerted to patients' body. Using models made by the new element can be practicable to analyze the effects of braces during a one-month period by finite element method.

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