

Comparison of two interpolation methods for empirical mode decomposition based evaluation of radiographic femur bone images

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Analysis of bone strength in radiographic images is an important component of estimation of bone quality in diseases such as osteoporosis. Conventional radiographic femur bone images are used to analyze its architecture using bi-dimensional empirical mode decomposition method. Surface interpolation of local maxima and minima points of an image is a crucial part of bi-dimensional empirical mode decomposition method and the choice of appropriate interpolation depends on specific structure of the problem. In this work, two interpolation methods of bi-dimensional empirical mode decomposition are analyzed to characterize the trabecular femur bone architecture of radiographic images. The trabecular bone regions of normal and osteoporotic femur bone images ($N = 40$) recorded under standard condition are used for this study. The compressive and tensile strength regions of the images are delineated using pre-processing procedures. The delineated images are decomposed into their corresponding intrinsic mode functions using interpolation methods such as Radial basis function multiquadratic and hierarchical b-spline techniques. Results show that bi-dimensional empirical mode decomposition analyses using both interpolations are able to represent architectural variations of femur bone radiographic images. As the strength of the bone depends on architectural variation in addition to bone mass, this study seems to be clinically useful.

Key words: anisotropy, bone mineral density, hierarchical b-spline, intrinsic mode function, radial basis function multiquadratic, trabecular soft bone, texture analysis

1. Introduction

Human bone is generally classified into two types called Cortical bone, also known as compact bone and Trabecular bone, also known as cancellous or spongy bone. These two types are classified on the basis of porosity and the unit microstructure. Cortical bone is much denser with a porosity ranging between 5% and 10%. It is found primarily in the shaft of long bones and forms the outer shell around cancellous bone at the end of joints and the vertebrae. Trabecular bone is much more porous with porosity ranging anywhere from 50% to 90%. It is found in

the end of long bones, in vertebrae and in flat bones like the pelvis.

Trabecular bone is highly metabolically active, and is therefore most susceptible to the bone loss which occurs during osteoporosis diseases. Osteoporosis-related proximal femur fractures impose a major public health risk for the elderly, as they lead to high rates of disability and complications [1], [2]. The majority of femoral fractures are sustained as a result of a lateral fall on the hip [3]. The loss of trabecular density occurs through a series of changes to the trabecular microstructure. The role of trabecular bone structure has increasingly been recognized as a significant contributory factor in the prediction of bone strength [4], [5].

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Received: February 12th, 2012

Accepted for publication: February 13th, 2013

The bone strength comprises the components of both bone density and quality. Bone mineral density (BMD) is expressed as grams of bone mineral present per area or volume and can be directly estimated by clinically well-established diagnostic procedures. Bone mass can be measured by bone mineral densitometry techniques such as ultrasound or Dual Energy X-ray Absorptiometry (DEXA). Bone mineral density as obtained from DEXA is a good predictor for fracture risk and measurements can be performed at or close to the site of interest (i.e., at the spine, the proximal femur, and the distal radius). However, there is overlap in the BMD results between individuals who have experienced fractures and those who have not [6], [7].

Methods for directly measuring trabecular architecture include invasive methods such as iliac bone biopsy, noninvasive imaging techniques such as X-ray, micro-computed tomography and high resolution magnetic resonance imaging [8]. Noninvasive femoral strength evaluation methods are valuable tools for diagnosing bone disorders, monitoring treatment and facilitating drug developments [2].

Conventional radiographs are commonly used to quantify trabecular texture patterns in human femur specimens. Trabecular bone structure is visible in great detail on standard radiographs and the significant parts of the information that are available in 3D images are also available in the conventional radiographs [9]. Bone structure can be estimated by observing the change of trabecular pattern in proximal femur radiograph [10]. It has also been shown that texture analysis of X-ray radiographs could be a useful complementary tool in the investigation of bone microarchitecture and is also correlated to trabecular histomorphometry [11], [12].

Computer-based texture analysis of digital images concerns the utilization of algorithms capable of quantifying the textural properties of an image [13]. In medical imaging, texture analyses are used for the extraction of diagnostically meaningful information by means of textural features that are not easily perceivable [14]. However, the texture analysis methods are less sensitive to changes in spatial variation.

Empirical mode decomposition is a new multi-scale analysis method proposed by Huang to extract texture features at multiple scales or spatial frequencies [15]. The empirical mode decomposition used to analyze the two-dimensional signals is called Bi-dimensional Empirical Mode Decomposition (BEMD). BEMD is better than Fourier, wavelet and other decomposition algorithms in extracting intrinsic

components of textures because of its data driven property. BEMD method has been successively used for texture analysis, edge detection, texture classification, segmentation, compression and content based medical image retrieval [16]. The performance of this method depends on detection of extrema points and the interpolation of the scattered extrema points.

In this work, various interpolation methods of bi-dimensional empirical mode decomposition are employed to characterize apparent compressive and tensile strength of normal and osteoporotic femur bones using conventional radiographic images.

2. Materials and methods

Forty pelvis images are recorded using standard clinical X-ray unit and anteroposterior view is used to image all subjects considered for this study.

Auto threshold binarization algorithm is then employed to recognize the presence of mineralization in the digitized images. This process minimizes the information loss and is suitable for trabecular images [17].

The compressive and tensile regions of interest in images of proximal femur are identified and delineated based on the method proposed by Singh et al. [18], [19]. The qualitative analysis is also performed on the delineated images to derive apparent porosity which is defined as the ratio of void area to the total area.

BEMD method decomposes a given image into finite number of two-dimensional Intrinsic Mode Functions (IMFs) based on the local frequency or oscillation information. The IMF is defined as a function with equal number of extrema and zero crossings, or at most differed by one, and the envelopes as defined by all the local maxima and minima being symmetric with respect to zero. At any point of a 2D IMF, the mean value of the upper and lower envelopes, defined by the local maxima and the minima, is ideally zero. Additionally, the IMFs are locally orthogonal. The local maxima and minima points of the image are detected using neighbor location method [20]. Once the extrema are identified, various interpolation methods are used to generate 2D envelope by connecting maxima points and minima points, respectively.

In this work, Radial basis function multiquadratic and hierarchical b-spline methods are used to interpolate the extrema points of radiographic femur bone

images. Radial basis function (RBF) based interpolation methods are examples of global interpolation methods for scattered data points. RBF multi-quadratic impose fewer restrictions on the geometry of the interpolation centers and are suited to problems where the interpolation centers do not form a regular grid as in the case of local maxima or minima maps appearing in the BEMD process [21]. B-splines are the basic building blocks for splines. Their usefulness stems from the fact that they are compactly supported for shortest possible polynomial splines. Multilevel or hierarchical b-spline interpolation is well suited to any application that can be cast as a surface fitting problem, like image reconstruction, image warping and object reconstruction. The basic idea is to subdivide the global approximation problem adaptively into several local problems where only a comparatively small part of the data has to be taken into account. These local problems will lead to linear systems of small size which can be solved efficiently. Multi-level b-splines have been used during the 2D-sifting process [22].

The local mean is estimated by averaging the upper and lower envelopes. The difference between the

data and the local mean is designated as the first IMF component of the data. This process separates the finest local mode from the image based only on the characteristic multiscale. This sifting procedure is repeated again to all the subsequent IMFs. The stopping criterion for this procedure is accomplished by limiting the size of the standard deviation computed from the two consecutive sifting results. The first IMF contains the highest local frequencies of oscillation or the highest local spatial scales. The second IMF corresponds to the medium pattern and the residue to the lowest [23], [24]. The decomposed IMFs are utilized to extract texture features which include kurtosis, skewness and entropy. Skewness is a measure of asymmetry of an image. Kurtosis measures the peakedness and entropy measures the disorder of an image. The values of these features are further analyzed in normal and abnormal images.

3. Results

Typical binary forms of planar radiographic images of normal and abnormal femur trabecular

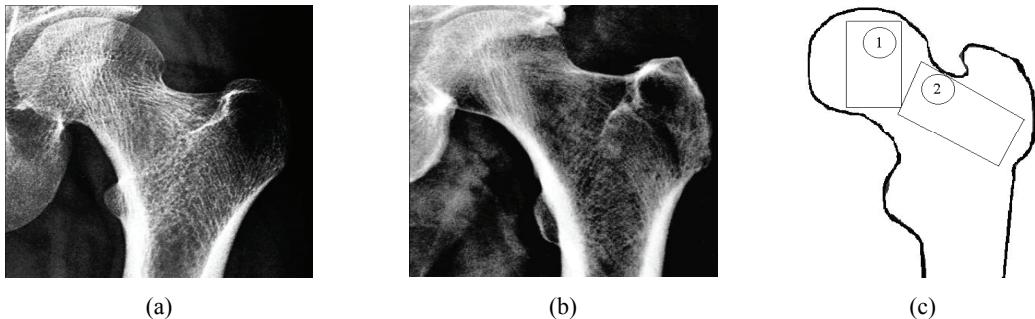


Fig. 1. (a) normal bone, (b) abnormal bone, and (c) representative strength region
The images are initially processed with Singh index delineation method to identify the compressive (1) and tensile (2) regions [18], [19] as shown in Fig. 1(c).

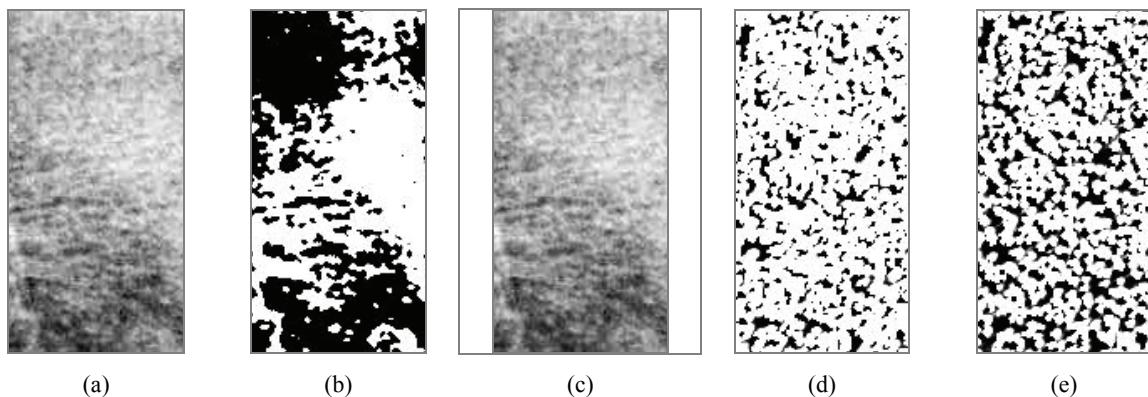


Fig. 2. Representative normal Compressive region: (a) Original, (b) IMF1, (c) IMF2 of RBF multiquadratic, and (d) IMF1 and (e) IMF2 of hierarchical b-spline

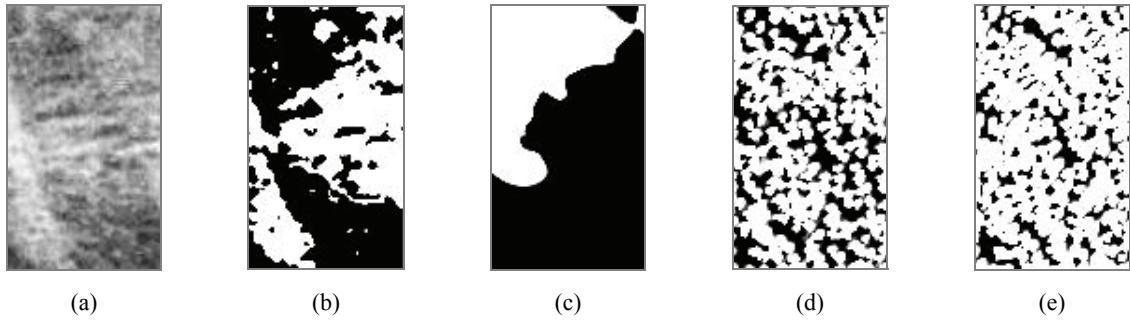


Fig. 3. Representative abnormal Compressive region: (a) Original, (b) IMF1, (c) IMF2 of RBF multiquadratic, and (d) IMF1 and (e) IMF2 of hierarchical b-spline

bones are shown in Figs. 1(a) and 1(b), respectively. The trabecular patterns are distinct and are closely arranged in normal images. In abnormal samples, trabecular spacing is large with high discontinuities. The overlap between trabeculae is found to be less in abnormal when compared to normal images.

In this study, regions of interest corresponding to the compressive and tensile regions are cropped using windows of constant size of 300×150 [19]. The qualitative analyses are performed on the delineated images to derive apparent porosity.

The apparent porosity is found to be minimum for normal subjects in tensile region when compared to other regions. In the case of abnormal subjects, the apparent porosity is always high for all the strength regions.

Two modes of decompositions are obtained for both compressive and tensile regions using RBF multiquadratic and b-spline interpolation techniques. The representative compressive region of interest and their decomposed intrinsic mode functions of normal and abnormal images are shown in Figs. 2 (a)–(e) and Figs. 3 (a)–(e), respectively. In both interpolation methods, it is found that first IMF contains largest spatial variations compared to the second and subsequent IMFs. Distinct variations are observed among the different modes of decomposition of normal and abnormal images. The spatial frequency component of these modes decreases with increase of intrinsic mode function. Similar observation is found for tensile regions as well [25].

First and higher order texture parameters such as entropy, kurtosis and skewness are extracted from IMFs of normal and abnormal images. The variations in values of entropy for the observed apparent porosity are shown in Figs. 4a–4d. The scattergram showing the variations of entropy values with porosity for IMF1 of normal subjects is shown in Fig. 4a. The values of entropy derived from compressive region

using two interpolation techniques are less scattered at low values of porosity. In b-spline interpolation

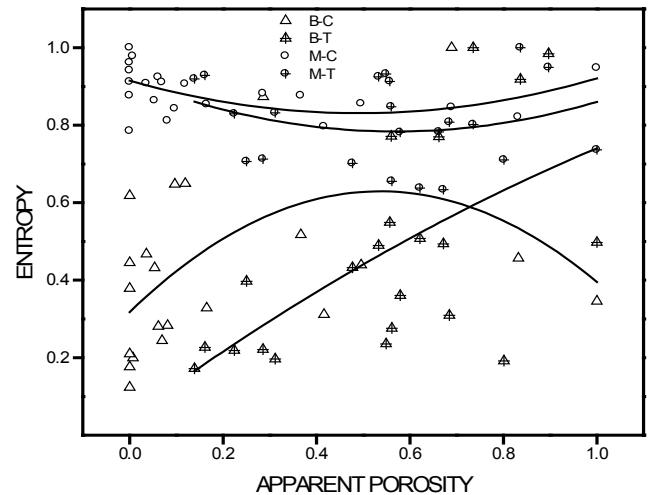


Fig. 4a. Variations of entropy values with porosity for IMF1 of normal subjects

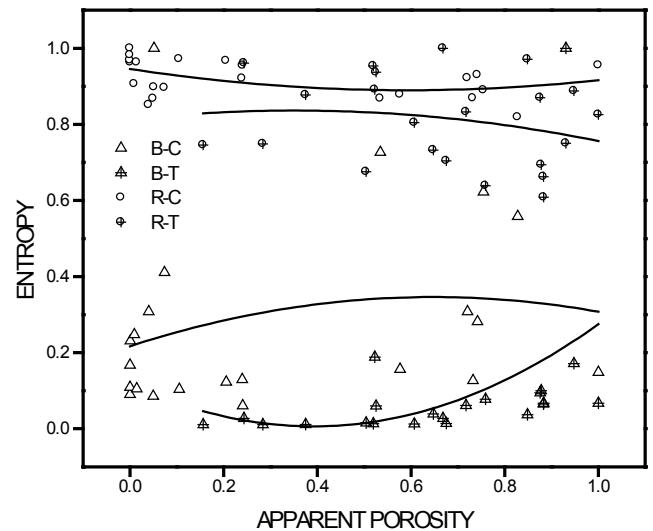


Fig. 4b. Variations of entropy values with porosity for IMF1 of abnormal subjects

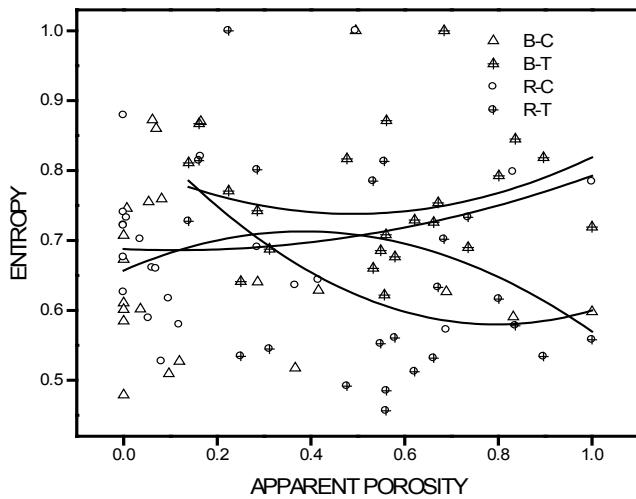


Fig. 4c. Variations of entropy values with porosity for IMF2 of normal subjects

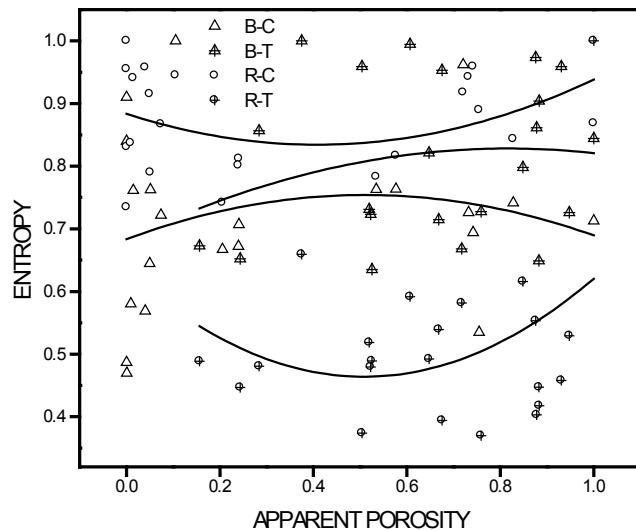


Fig. 4d. Variations of entropy values with porosity for IMF2 of abnormal subjects

method, high correlation is observed for both the compressive and tensile regions whereas multiquadratic RBF could achieve good correlation only in compressive region. The values of entropy derived from tensile region using b-spline increase with increase in porosity and have better correlation. Entropy values derived using multiquadratic interpolation do not vary distinctly with porosity. This demonstrates the ability of the b-spline interpolation method to extract heterogeneity and anisotropic nature of trabecular structures in femur bones.

The scattergram showing the variations of entropy values with porosity for IMF1 of abnormal subjects is shown in Fig. 4b. The values of entropy derived from compressive region using two interpolation techniques appear clustered at low values of porosity and those

derived from tensile region appear clustered at high values of porosity. In multiquadratic interpolation method, the values of entropy are high and have good correlation with porosity for compressive region. B-spline interpolation method could achieve better correlation for tensile region.

The scattergram showing the variations of entropy values with porosity for IMF2 of normal subjects is shown in Fig. 4c. The values of entropy derived using two interpolation techniques are more clustered at low values of porosity for compressive region and highly scattered at high values of porosity for tensile region. In multiquadratic interpolation method, high correlation is observed only in tensile region.

In both interpolation techniques, high values of entropy are obtained for IMF2 of abnormal subjects as shown in Fig. 4d. This could be due to heterogeneous nature of abnormal trabecular bone. The values of entropy show distinct variation with increase in porosity for tensile region.

The values of kurtosis and skewness for the observed apparent porosity are shown in Figs. 5a and 5b.

The scattergram showing the variations of kurtosis values with porosity for IMF1 of normal subjects is shown in Fig. 5a. The values of kurtosis derived from compressive region using two interpolation techniques appear clustered at low values of porosity and those derived from tensile region appear clustered at high values of porosity. The values of kurtosis are high for the IMF1 of normal subjects and correlation seems to be better. This could be due to presence of high values of mineralization in normal bones. Kurtosis values derived from b-spline and RBF multiquadratic interpolation method seems to correlate better with porosity for compressive region.

The scattergram showing the variations of skewness values with porosity for both interpolation methods is shown in Fig. 5b. High correlation is observed for both interpolation methods in compressive region, whereas less correlation is found for tensile region. The values of skewness derived from compressive region using two interpolation techniques appear clustered at low values of porosity and those derived from tensile region appear clustered at high values of porosity. The values of skewness derived using b-spline are high and have better correlation with porosity of compressive region.

The feature values extracted from first two intrinsic mode functions of compressive and tensile regions of femur bone images using different interpolations of BEMD are compared using p-test significance and is presented in Table 1. It is found that variations in the values of kurtosis and skewness of IMF2 derived using

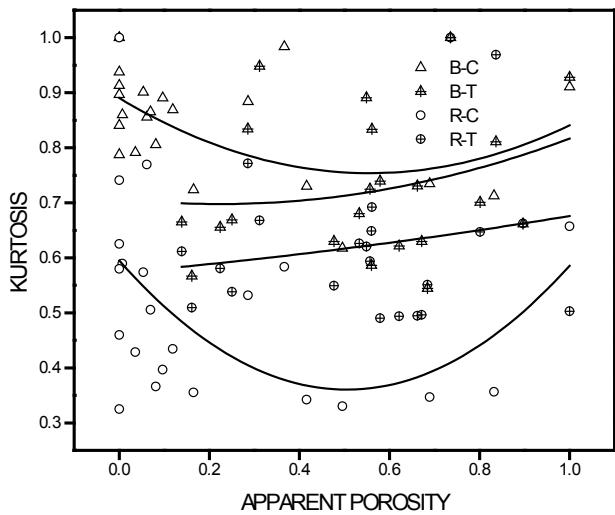


Fig. 5a. Variations of kurtosis values with porosity for IMF1 of normal subjects

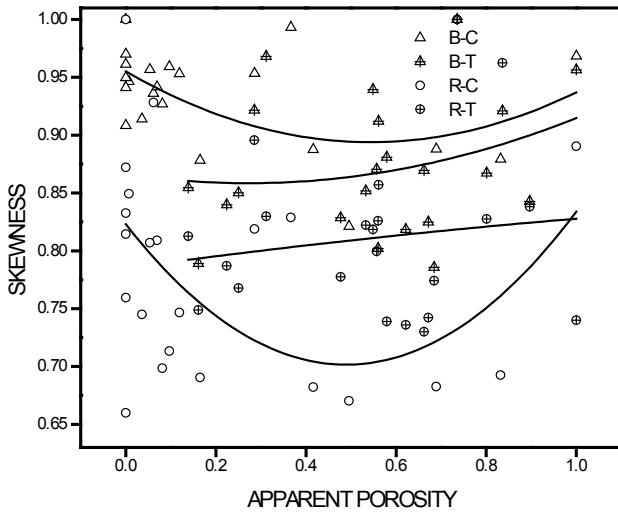


Fig. 5b. Variations of skewness values with porosity for IMF1 of normal subjects

b-spline interpolation are statistically highly significant ($p < 0.05$) for both compressive and tensile regions. Similarly, variations in these parameters derived using multiquadratic interpolation are statistically highly significant ($p < 0.05$) for IMF1 of tensile region. However, these features are not significant in IMF1 of b-spline and IMF2 of RBF multiquadratic interpolation for both compressive and tensile regions.

The variations in values of entropy of IMF2 for multiquadratic interpolation are highly significant in both the compressive and tensile region. Similarly, variation in this feature derived using b-spline appears to be statistically significant for IMF1 of tensile region. However, variation in this feature is not significant in IMF1 and IMF2 of b-spline interpolation for compressive region. Pathological conditions induce marked changes in the structures in abnormal samples which might lead to the reduction of load bearing capacity of the bone. IMF1 of multiquadratic interpolation and IMF2 of b-spline interpolation are able to extract these changes and help in differentiating normal and abnormal subjects. This demonstrates the ability of the interpolation methods to represent large variations in bone mass in the compressive and tensile regions.

4. Discussion

Osteoporosis, osteomalacia, osteopenia and osteoarthritis, are the bone disorders that lead to loss of bone strength and consequent fracture risk.

Although bone mineral density is widely used clinically, it has been found to be insensitive to alterations of trabecular architecture that are likely to play an important role in skeletal fragility associated with osteoporosis [26], [27]. It has been shown that a non-invasive method to characterize trabecular structure is used to predict bone strength [4], [28].

Nikodem [5] used multifactorial analysis obtained the high correlations between individual structural and mechanical properties both for normal tissue and pathological cases.

Texture analyses of X-ray radiographs have been performed in some earlier studies to characterize the changes in trabecular bone and seem to be a suitable approach for automated analysis of 2D bone micro architecture. Smyth et al. [29] have tested feature-based techniques on 25 cadaver femoral radiographs and showed advantages of this technique over the Singh index, while obtaining comparable perform-

Table 1. p -test significance for compressive and tensile regions

Interpolation of BEMD	IMF Number	Compressive region			Tensile region		
		Entropy	Kurtosis	Skewness	Entropy	Kurtosis	Skewness
Hierarchical b-spline	Imf1	$p < 0.03$	$p < 0.40$	$p < 0.29$	$p < 0.000008$	$p < 0.149$	$p < 0.076$
	Imf2	$p < 0.31$	$p < 0.00035$	$p < 0.00003$	$p < 0.109$	$p < 0.00036$	$p < 0.0032$
RBF multiquadratic	Imf1	$p < 0.04$	$p < 0.0005$	$p < 0.34$	$p < 0.381$	$p < 0.0056$	$p < 0.00001$
	Imf2	$p < 0.000001$	$p < 0.11$	$p < 0.311$	$p < 0.00061$	$p < 0.497$	$p < 0.17$

ance. Geraets et al. [30] have examined the use of geometric and directional parameters in analyzing the femoral radiographic pattern in hip fracture patients and normal controls and indicated that 58% of cases predicted fracture risk, gender and Singh index correctly, and 78% predicted at least two of the three.

Texture analysis of standard radiographs using the fast Fourier transform yield variables that are significantly associated with fracture but has not been significantly correlated with age, body mass index or Neck-BMD [31]. Lin et al. have found that when the fractal parameters examined were added to BMD in a multivariate regression analysis, additional predictive power for predicting biomechanical properties is obtained [32]. A study comparing the properties of calcaneus, distal femur, proximal femur and vertebrae on human specimens have proved that anisotropy of trabecular bone is different according to the skeletal sites: Majumdar et al. [33] have found the highest anisotropy of trabecular bone at the calcaneus followed by distal femur and proximal femur, vertebrae constituting the least anisotropic site. Barbara Brunet [34] has proposed a new index using fast fourier transform to assess the degree of anisotropy on radiographic images comprising from highly anisotropic structure to the almost isotropic and was ranging from 1.3 to 3.8, respectively. While Fourier techniques may be able to discern orientation differences, they lack the ability to identify local structural features.

Trabecular structure and its mechanical strength distribution on human femur bone are analyzed using multiresolution wavelet analysis. The parameters mean and median computed at the output of Haar wavelet at level 4 decomposition is found to be a useful predictor to discriminate the normal and the abnormal groups [35]. These multiscale features try to characterize textures by filter responses directly. However, one difficulty of the multiresolution analysis is its non-adaptive nature since it uses filtering schemes. A method that can extract spatial variations at multiple scales is required to characterize the microstructural patterns of the trabecular bone.

Spatial decomposition based BEMD method is a fully data driven method, does not use any pre-estimated filter or wavelet functions [22] and affords local characterization of structural features at multiple spatial frequencies. BEMD has been widely used in multi scale analysis of images for the extraction of diagnostically meaningful information from them [22]. Surface interpolation of local maxima and minima points of image is a crucial part of the BEMD method

and the choice of the appropriate interpolation method depends on the specific structure of the problem [21]. In this work, the compressive and tensile strength regions of normal and abnormal femur bone images are delineated and comprehensively analyzed using BEMD analysis. Two different interpolation methods namely, hierarchical b-spline and RBF multiquadratic are used to derive intrinsic mode functions and texture features are extracted from them. Hierarchical b-spline interpolation is able to represent characterization of heterogeneous objects based on linear combination of weighted basis function. RBF multiquadratic interpolation demonstrates that high-fidelity reconstruction is possible from a selected set of sparse and irregular pattern. The quantitative analyses are also performed on the delineated images to derive apparent porosity.

Results show that BEMD analysis using RBF multiquadratic and b-spline is able to represent architectural variations in compressive and tensile regions of femur bone radiographic images. In IMF2, the variation in the values of entropy derived using RBF multiquadratic interpolation and variations in values of kurtosis and skewness extracted using b-spline interpolation are found to be statistically more significant for both the compressive and tensile region. This shows the ability of IMF2 to represent large variance of femur trabecular bone. The parameters derived from IMF using hierarchical b-spline show more correlation and spatial anisotropic architectural variations that are not extractable by other texture methods are observed in this method. As automated analysis of trabecular architecture is important for mass screening and monitoring of osteoporosis like diseases, this study seems to be clinically highly relevant.

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