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Biomechanical model of the human cervical spine

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The paper describes the biomechanical model simulating dynamic phenomena in the human cervical spine caused by assumed acceleration and external impact on head and trunk, especially in car accident. The model assumptions, general equations describing dynamic equilibrium of spine members and detailed analysis of important elements are presented. The method of solving obtained set of equations is described.

Keywords: cervical spine, mathematical modelling, crash biomechanics

1. Introduction

During car crashes the head and human cervical spine are the two parts of body mostly exposed to injuries, which result in permanent disability or even death.

In many research centres all over the world the experiments are carried out in the field of crash biomechanics to learn the mechanism of injury. This knowledge is used to increase the safety of road users [8].

Experimenting on peoples is usually impossible because of its dangerous character. As a result, crash tests are carried out on dummies and cadavers [8]. Another way to learn the behaviour of the human body during car crashes is the mathematical modelling, using the newest numerical methods [4]. Unfortunately, because of the problem complexity, only simplified models for specific crash cars are constructed, which disregard some important parameters influencing the body movement [1; 5–7].

This paper aims at creating the model describing the behaviour of the cervical vertebrae during the collision, including the influence of muscles, ligaments and intervertebral discs and interaction with head and trunk [2, 3].

2. Model assumptions

The constructed biomechanical model of cervical spine consist of detailed dynamical models of cervical vertebrae with intervertebral discs, ligaments, neck muscles and facet joints and of simplified model of neighbouring members – head and trunk. Scheme of the modelled system is presented in Fig. 1. Physical model of typical vertebrae (excepting an atlas and axis) is shown in Fig. 2, along with used co-ordinate systems.

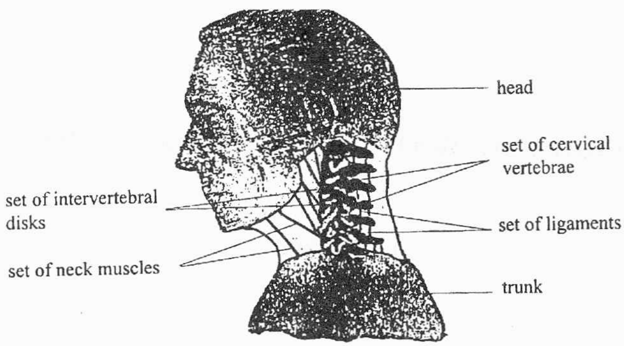

Fig.1. Scheme of modelled cervical spine

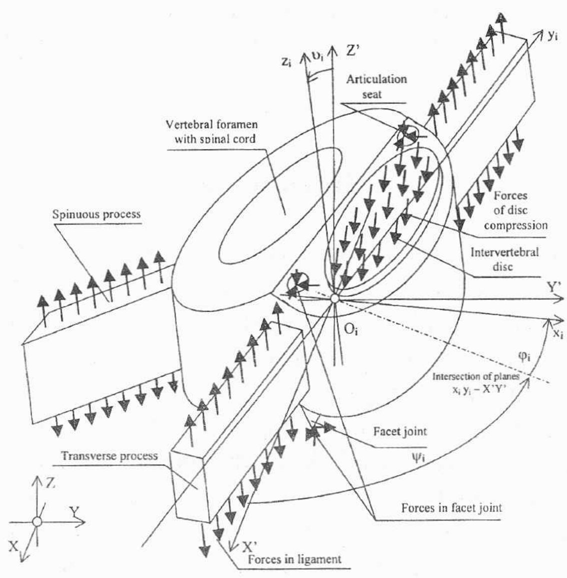

Fig. 2. Scheme of typical cervical vertebra

Modelling and numerical simulation includes motion of all members and changes of forces in ligaments, discs and joints (with regarding effects of possible fractures) caused by assumed external accidental impact, in the form of accelerations or forces, acting on defined points of body. Basic detailed assumptions are listed below.

Vertebrae, head and trunk are treated as stiff bodies, with defined mass and inertial moments. All 6 degrees of freedom are considered, including possibility of some displacements in joints, in directions other than normal rotation, caused by elasticity or fracture of ligaments and cartilage, which may be very important when simulating short-lasting shocks. The constraints, resulting from contact with neighbouring members, are determined as forces in joints, ligaments, discs and muscles, depending on mutual position of members.

Ligaments are modelled as weightless elements, acting on joined vertebrae with compressing forces, depending on actual elongation. In the model, wider ligaments are divided into parallel strips, with independently calculated elongation. The lines connecting the conventional points of sticking of ligament strips to vertebral processes were treated as a direction of forces. The dependence of force on elongation may be strongly non-linear. The force is zeroed when shortening occurs. When the force exceeds dynamic strength of ligament the fracture of appropriate strip is simulated by zeroing its compressing force.

Intervertebral discs are treated as elastic elements, with mass added to neighbouring vertebrae. They are divided into segments, acting on vertebrae with the force nonlinearly dependent on actual compression – change of distance between centres of conventional points of sticking of segments to neighbouring discs.

Neck muscles are modelled as elements with negligible mass, acting on connected members with compressing force, having assumed time-dependent value and direction resulting from actual position of conventional fixing points.

Operation of joints is represented by the system of forces, resulting from elasticity of cartilage and ligaments, constraining mutual displacement of joined members, nonlinearly dependent on mutual displacements of joined members in the directions constrained by the joint – other than normal revolution. Additive forces appear when limitation of joint revolution is exceeded, resulting from the contact of a suitable process with resisting surface. They are defined as dependent on calculated penetration of process into resisting surface, and act perpendicularly to it.

External excitations are determined in the form of assumed time-dependent forces or accelerations of elastic striking objects acting on definite points of modelled cervical members. Gravity forces are added to the sum of external forces, acting in the direction of vertical axis Z.

3. General equations of motion of model member

Position of every stiff body i considered in the model (one of vertebrae, trunk or head) is described by introduction of local, central, movable co-ordinate system $O_i x_i y_i z_i$, coupled with the member i. Relative position of this system in absolute, immovable system $OXYZ$ is described by the vector $\overline{X}_{oi}$, defining the distance between the position of the mass centre O_i and the origin of immovable system O, and by a set of Euler's angles $\overline{\theta}_i$. The co-ordinate systems are shown in Fig. 2.

Relations between the vector $\overline{X}_p$ describing the position of a selected point P in absolute system $OXYZ$ and the vector $\overline{x}_{pi}$ defining this point in the movable system $O_i x_i y_i z_i$, have the form:

$$\overline{X}_p = \overline{\overline{A}}_i \cdot \overline{x}_{pi} + \overline{X}_{oi}, \qquad \overline{x}_{pi} = \overline{\overline{A}}_i^{-1} \cdot \left(\overline{X}_p - \overline{X}_{oi}\right), \qquad \overline{\overline{A}}_i^{-1} = \overline{\overline{A}}_i^{T}, \tag{1}$$

where

$$\overline{X}_{oi} = \left[X_{oi}, Y_{oi}, Z_{oi}\right]^T, \qquad \overline{X}_p = \left[X_p, Y_p, Z_p\right]^T, \qquad \overline{x}_{pi} = \left[x_{pi}, y_{pi}, z_{pi}\right]^T,$$

$$\overline{\theta}_i = \left[\vartheta_i, \psi_i, \varphi_i\right]^T$$

Elements of rotation matrix $\overline{\overline{A}}_i$ are equal to cosines of angles formed by axes of movable and absolute system

$$\overline{\overline{A}}_i = \begin{bmatrix} \cos(Xx_i) & \cos(Xy_i) & \cos(Xz_i) \\ \cos(Yx_i) & \cos(Yy_i) & \cos(Yz_i) \\ \cos(Zx_i) & \cos(Zy_i) & \cos(Zz_i) \end{bmatrix}. \tag{2}$$

Direction cosines and rotation matrix may be written using the set of Euler's angles:

$$\overline{\overline{A}}_i = \begin{bmatrix} \cos\psi_i \cos\varphi_i - \cos\vartheta_i \sin\psi_i \sin\varphi_i & -\cos\psi_i \sin\varphi_i - \cos\vartheta_i \sin\psi_i \cos\varphi_i & \sin\vartheta_i \sin\psi_i \\ \sin\psi_i \cos\varphi_i + \cos\vartheta_i \cos\psi_i \sin\varphi_i & -\sin\psi_i \sin\varphi_i + \cos\vartheta_i \cos\psi_i \cos\varphi_i & -\sin\vartheta_i \cos\psi_i \\ \sin\vartheta_i \sin\varphi_i & \sin\vartheta_i \cos\varphi_i & \cos\vartheta_i \end{bmatrix} \tag{3}$$

Instantaneous motion of the member i is defined by the velocity of its mass centre $\overline{V}_{oi}$ in absolute co-ordinate system and by angular velocity $\overline{\omega}_i$ describing rotation about momentary axis of revolution. Angular velocity is analysed in local co-ordinate system. Its components on axes of the local system ω_{xi}, ω_{yi}, ω_{zi} are only apparent velocities and values of angles cannot be obtained by their integration.

The equations of dynamic equilibrium of forces, including the inertial forces and the sum ΣF_i of external excitations and forces in muscles, ligaments and joints, acting on the member, are considered in absolute co-ordinate system. The moment equations are written in local system coupled with the member. The obtained set of equations has the Newton–Euler's form [2, 8]:

$$\frac{d\overline{V}_{oi}}{dt} = \sum \frac{\overline{F}_i}{m_i}, \qquad \frac{d\overline{\omega}_i}{dt} = \overline{\overline{T}}_i^{-1}\left\{-\overline{\omega}_i \times \left(\overline{\overline{T}}_i \cdot \overline{\omega}_i\right) + \sum \overline{M}_i\right\}, \tag{4}$$

where

$$\overline{V}_{oi} = \left[V_{Xoi}, V_{Yoi}, V_{Zoi}\right]^T, \qquad \sum \overline{F}_i = \left[\sum F_{Xi}, \sum F_{Yi}, \sum F_{Zi}\right]^T,$$

$$\overline{\omega}_i = \left[\omega_{xi}, \omega_{yi}, \omega_{zi}\right]^T, \qquad \sum \overline{M}_i = \left[\sum M_{xi}, \sum M_{yi}, \sum M_{zi}\right]^T.$$

Tensor of inertia comprises inertial moments, computed in central local co-ordinate system $O_i x_i y_i z_i$, coupled with the member:

$$\overline{\overline{T}}_i = \begin{bmatrix} I_{xixi} & -I_{xiyi} & -I_{xizi} \\ -I_{xiyi} & I_{yiyi} & -I_{yizi} \\ -I_{xizi} & -I_{yizi} & I_{zizi} \end{bmatrix}. \tag{5a}$$

For symmetric members, having the plane of symmetry $O_ix_iz_i$, the inverse of tensor $\overline{\overline{T}}_i^{-1}$ is calculated by the formula:

$$\overline{\overline{T}}_i^{-1} = \begin{bmatrix} I_{xixi} & 0 & -I_{xizi} \\ 0 & I_{yiyi} & 0 \\ -I_{xizi} & 0 & I_{zizi} \end{bmatrix}^{-1} = \begin{bmatrix} I_{zizi}/a & 0 & I_{xizi}/a \\ 0 & 1/I_{yiyi} & 0 \\ -I_{xizi}/a & 0 & I_{xixi}/a \end{bmatrix}, \tag{5b}$$

where

$$a = I_{xixi} I_{zizi} - I_{xizi}^2 .$$

The relations coupling positions and velocities of the mass centre O_i of the member i have the obvious form:

$$\frac{d\overline{X}_{oi}}{dt} = \overline{V}_{oi}. \tag{6}$$

Formula describing the dependence of changes of Euler's angles in time on angular velocities can be obtained from geometrical analysis [2]:

$$\begin{bmatrix} d\vartheta_i/dt \\ d\psi_i/dt \\ d\varphi_i/dt \end{bmatrix} = \frac{1}{\sin\vartheta_i} \begin{bmatrix} \sin\vartheta_i\cos\varphi_i & -\sin\vartheta_i\sin\varphi_i & 0 \\ \sin\varphi_i & \cos\varphi_i & 0 \\ -\cos\vartheta_i\sin\varphi_i & -\cos\vartheta_i\cos\varphi_i & \sin\vartheta_i \end{bmatrix} \cdot \begin{bmatrix} \omega_{xi} \\ \omega_{yi} \\ \omega_{zi} \end{bmatrix}. \tag{7a}$$

For the values of $\vartheta_i \approx k\pi$ ($k = 0, 1,...,$) formula (7a) is badly conditioned and has to be replaced by a simplified relation:

$$\begin{bmatrix} d\vartheta_i/dt \\ d\psi_i/dt \\ d\varphi_i/dt \end{bmatrix} = \begin{bmatrix} \sqrt{\omega_{xi}^2 + \omega_{yi}^2} \begin{cases} \operatorname{sign}(\omega_{xi}/\cos\varphi_i) & \text{for } \varphi_i \neq (2k+1)\pi/2 \\ \operatorname{sign}(\omega_{yi}/sin\varphi_i) & \text{for } \varphi_i = (2k+1)\pi/2 \end{cases} \\ 0 \\ \omega_{zi} \end{bmatrix}. \tag{7b}$$

The system of equations, determining the time-dependent changes of 12 parameters (V_{oXi}, ..., ω_{xi}, ..., X_{oi}, ..., ϑ_i, ψ_i, φ_i) and defining linear and angular displacements and velocities of the member i, consists of six relations that describe equilibrium of forces and moments (4) and of six kinematic formulas (6), (7). Complete system of non-linear ordinary equations for whole modelled system can be written in a general matrix form, provided that forces in muscles, ligaments and joints depend on displacements of connected members. Vector $\overline{F}_{\text{external}}$ includes values of all assumed excitations and forces in muscles.

$$\frac{d\overline{X}}{dt} = \Phi\left(\overline{X}, \overline{F}_{\text{external}}\right), \tag{8}$$

where

$$\overline{X} = \left[\overline{X}_1, ..., \overline{X}_i, ...,\right]^T, \qquad \overline{X}_i = \left[V_{oXi}, V_{oYi}, V_{oZi}, \omega_{xi}, \omega_{yi}, \omega_{zi}, X_{oi}, Y_{oi}, Z_{oi}, \vartheta_i, \psi_i, \varphi_i\right]^T.$$

4. Forces in intervertebral ligaments

Establishing the relations between actual position of members and forces acting on them is the most arduous part of derivation of the set of model Eqs. (8). It consists in determination of the position of adequate points of cervical members in absolute co-ordinate system, calculation of their mutual displacements and components of resulting elastic forces, and evaluation of moments in local co-ordinate system. Establishing is performed with the use of general formulae, binding co-ordinates in different systems. The methods of defining forces in intervertebral ligaments and in joint are presented as examples.

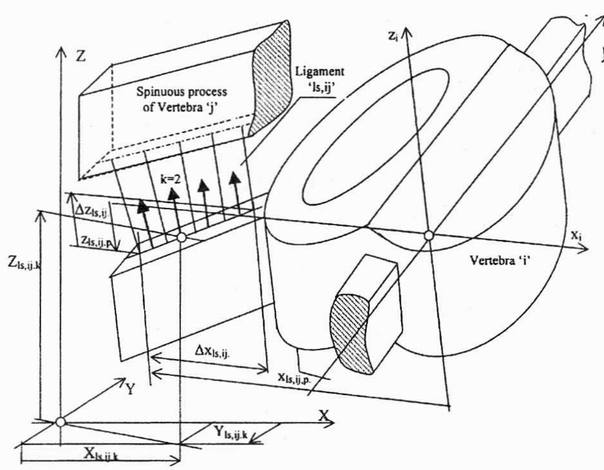

Fig. 3. Scheme of displacements and forces in ligament

Position of conventional fixing points of ligament *lsij*, connecting spinuous processes of vertebrae *i* and *j*, on vertebra *i*, in local co-ordinate system, coupled with this vertebra, is defined by four co-ordinates: $x_{ls,ij,p}$, $\Delta x_{ls,ij}$, $z_{ls,ij,p}$, $\Delta z_{ls,ij}$, shown in Fig. 3. Wide ligament is divided into strips ($k = 1, ..., n_{ls,ij}$), and co-ordinates of fixing points of every strip are determined by geometric formulas:

$$\begin{bmatrix} x_{ls,ij,k} \\ y_{ls,ij,k} \\ z_{ls,ij,k} \end{bmatrix} = \begin{bmatrix} x_{ls,ij,p} + \Delta x_{ls,ij} (2k-1)/n_{ls,ij} \\ 0 \\ z_{ls,ij,p} + \Delta z_{ls,ij} (2k-1)/n_{ls,ij} \end{bmatrix}. \tag{9a}$$

In an absolute system, the positions of fixing points of ligament strips on processes of vertebrae i and j are computed after calculation of rotation matrices $\overline{A}_i$ and $\overline{A}_j$ (3):

$$\begin{bmatrix} X_{ls,ij,k} \\ Y_{ls,ij,k} \\ Z_{ls,ij,k} \end{bmatrix} = \overline{\overline{A}}_i \begin{bmatrix} x_{ls,ij,k} \\ y_{ls,ij,k} \\ z_{ls,ij,k} \end{bmatrix} + \begin{bmatrix} X_{oi} \\ Y_{oi} \\ Z_{oi} \end{bmatrix}, \qquad \begin{bmatrix} X_{ls,ji,k} \\ Y_{ls,ji,k} \\ Z_{ls,ji,k} \end{bmatrix} = \overline{\overline{A}}_j \begin{bmatrix} x_{ls,ji,k} \\ y_{ls,ji,k} \\ z_{ls,ji,k} \end{bmatrix} + \begin{bmatrix} X_{oj} \\ Y_{oj} \\ Z_{oj} \end{bmatrix}. \tag{9b}$$

Actual length $L_{ls,ij,k}$ of the strip k of ligament $lsij$ and its elongation $\Delta L_{ls,ij,k}$, calculated as difference between actual length and initial length $L_{ls,ij,k,0}$, are described by formulas:

$$L_{ls,ij,k} = \sqrt{\left(X_{ls,ij,k} - X_{ls,ji,k}\right)^2 + \left(Y_{ls,ij,k} - Y_{ls,ji,k}\right)^2 + \left(Z_{ls,ij,k} - Z_{ls,ji,k}\right)^2},$$
$$\Delta L_{ls,ij,k} = L_{ls,ij,k} - L_{ls,ji,k,0}. \tag{9c}$$

The forces in ligament depend on actual elasticity constant $C_{ls,ij}$, being assumed the function of elongation. The components of forces in the strip k in ligament $lsij$, acting on the vertebra i in absolute co-ordinate system, are defined by the relation:

$$\begin{bmatrix} F_{ls,ij,k,X} \\ F_{ls,ij,k,Y} \\ F_{ls,ij,k,Z} \end{bmatrix} = \begin{bmatrix} \left(X_{ls,ij,k,x} - X_{ls,ji,k}\right) \Delta L_{ls,ij,k} C_{ls,ij} \left(\Delta L_{ls,ij,k}\right) / L_{ls,ij,k} \\ \left(Y_{ls,ij,k,x} - Y_{ls,ji,k}\right) \Delta L_{ls,ij,k} C_{ls,ij} \left(\Delta L_{ls,ij,k}\right) / L_{ls,ij,k} \\ \left(Z_{ls,ij,k,x} - Z_{ls,ji,k}\right) \Delta L_{ls,ij,k} C_{ls,ij} \left(\Delta L_{ls,ij,k}\right) / L_{ls,ij,k} \end{bmatrix}. \tag{9d}$$

Components of forces acting on the vertebra j have the same values and opposite signs. These components are included in the sum of the forces ΣF in Eqs. (4).

Moments of forces in ligament, included in the sum of the moments ΣM in Eqs. (4), are calculated in local co-ordinate system, as vector product of distance of fixing point from system centre and components of forces, computed in local system:

$$\begin{bmatrix} F_{ls,ij,k,x} \\ F_{ls,ij,k,y} \\ F_{ls,ij,k,z} \end{bmatrix} = \overline{\overline{A}}_i^T \begin{bmatrix} F_{ls,ij,k,X} \\ F_{ls,ij,k,Y} \\ F_{ls,ij,k,Z} \end{bmatrix}, \qquad \begin{bmatrix} M_{ls,ij,k,x} \\ M_{ls,ij,k,y} \\ M_{ls,ij,k,z} \end{bmatrix} = \begin{bmatrix} x_{ls,ij,k} \\ y_{ls,ij,k} \\ z_{ls,ij,k} \end{bmatrix} \times \begin{bmatrix} F_{ls,ij,k,x} \\ F_{ls,ij,k,y} \\ F_{ls,ij,k,z} \end{bmatrix}. \tag{9e}$$

Moments acting on the vertebra j are calculated by sign change and multiplication by the product of the rotation matrices $\overline{\overline{A}}_j^T \cdot \overline{\overline{A}}_j$.

5. Rotational joint

Scheme of forces acting in right rotational articulation rar,ij, joining typical vertebrae i and j, is presented in Fig. 4. Components of forces are analysed in the local system x_{rai}, y_i, x_{rai}, with the axis z_{rai} perpendicular to the conventional plane of joint seat.

Position of the tip of joint in the co-ordinate system connected with the vertebra j ($x_{rar,ji}$, $y_{rar,ji}$, $z_{rar,ji}$) is defined by assumed geometry of vertebra. Position of the same tip in co-ordinate system connected with the plane of joint seat on the vertebra i ($x_{rar,ji,rai}$, $y_{rar,ji,rai}$, $z_{rar,ji,rai}$) is calculated with the use of rotation matrices and formulae (1), (3):

$$\begin{bmatrix} x_{rar,ji,rai} \\ y_{rar,ji,rai} \\ z_{rar,ji,rai} \end{bmatrix} = \overline{\overline{A}}_{rai}^T \left\{ \overline{\overline{A}}_i^T \left(\overline{\overline{A}}_j^T \begin{bmatrix} x_{rar,ji} \\ y_{rar,ji} \\ z_{rar,ji} \end{bmatrix} + \begin{bmatrix} X_{oj} - X_{oi} \\ Y_{oj} - Y_{oi} \\ Z_{oj} - Z_{oi} \end{bmatrix} \right) \right\},$$

$$\overline{\overline{A}}_{rai} = \begin{bmatrix} \cos\alpha_{rar,i} & 0 & -\sin\alpha_{rar,i} \\ 0 & 1 & 0 \\ \sin\alpha_{rar,i} & 0 & \cos\alpha_{rar,i} \end{bmatrix}. \tag{10a}$$

Initial position of the tip on the plane of joint seat ($x_{rar,ji,rai,0}$, $y_{rar,ji,rai,0}$, $z_{rar,ji,rai,0}$) is calculated by the same formula for neutral, not loaded position of neighbouring vertebrae.

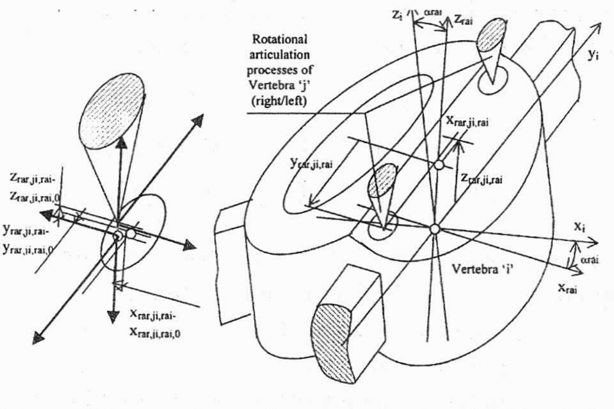

Fig. 4. Scheme of forces acting on the rotational articulation in typical vertebra

Components of forces are proportional to displacements of conventional tip of joint. They depend on elasticity of ligaments. The component acting during compression in the direction z_{rai} is also dependent on the elasticity of the cartilage. In local co-ordinate system $O_i x_i y_i z_i$ the components are described by formulae:

$$\begin{bmatrix} F_{rar,ij,x} \\ F_{rar,ij,y} \\ F_{rar,ij,z} \end{bmatrix} = \overline{\overline{A}}_{rai} \cdot \begin{bmatrix} C_{rax} \cdot \left(x_{rar,ji,rai} - x_{rar,ji,rai,0} \right) \\ C_{ray} \cdot \left(y_{rar,ji,rai} - y_{rar,ji,rai,0} \right) \\ C_{raz} \cdot \left(z_{rar,ji,rai} - z_{rar,ji,rai,0} \right) \end{bmatrix}$$

Components of moments and forces in absolute co-ordinate system are calculated from the formulae similar to (9e).

6. Method of solving model equations

System (8) with assumed initial conditions is solved numerically, using the fourth order Runge–Kutta method with automatic correction of time step.

Adequate initial conditions for simulated accident are obtained by preliminary quasi-static simulation. Model equations are solved taking into account only initial muscular and supporting external forces with zeroing velocities after every step and preserving only displacements. Initial simulation starts with an assumed initial position of motionless neck members. Future research program provides designing a computer program, identification of system parameters and performing the simulation research.

The algorithm of program consists of the operations listed below:

- Reading geometric parameters of modelled system consisting of dimensions of all members (treated as sets of regular solids) with determination of fixing points of muscles and positions of joints.
- Reading parameters describing elasticity and maximal strengths of ligaments, intervertebral discs and cartilage, and density of stiff bodies.
- Reading characteristics of external accelerations, exciting and muscular forces.
- Initial geometric calculations of masses, inertial moments, and positions of mass centres.
- Calculations for succeeding time steps (divided into four parts), at first for quasi-static and then for dynamic simulations:

computation of exciting and muscular forces and elements of the vector $\overline{X}$ in actual part of time step according to Runge–Kutta's formulae,

computation of vector of the derivatives $d\overline{X}/dt$, including calculation of the rotation matrices $\overline{A}_i$, positions and mutual displacements of every fixing points of muscles, ligaments and intervertebral discs, and components of forces and moments in local or absolute co-ordinate systems (8),

computation of changes of the vector $\overline{X}$ according to Runge–Kutta's formulae and starting next time step with new value of this vector or repetition of time step with shortened length.

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References

[1] BĘDZIŃSKI R., *Biomechanika* inżynierska, *zagadnienia wybrane*, Oficyna Wydawnicza Politechniki Wrocławskiej, Wrocław, 1997.

[2] BLAJER W., *Metody dynamiki układów wieloczłonowych*, Zakład poligraficzny Politechniki Radomskiej, Radom, 1998.

[3] BOCHENEK A., REICHER M., *Anatomia człowieka I*, Wyd. Lekarskie PZWL, Warszawa, 1990.

[4] DENG Y. C., GOLDSMITH W., *Response of a human head neck upper-torso replica to dynamic loading. II. Analytical-numerical model*, Journal of Biomechanics, 1988.

[5] DIETRICH M., ZAGRAJEK T., *Problemy modelowania w biomechanice kręgosłupa człowieka*, Zeszyty Naukowe WSI, Zielona Góra, 1988.

[6] GRZESIKIEWICZ W., *Modelowanie biomechanicznych obciążeń głowy i kręgosłupa szyjnego*, Materiały XV Szkoły Biomechaniki, Kokotek k. Lublińca, 1998.

[7] KĘDZIOR K., WOJTYRA M., ZAGRAJEK T., *Model dynamiczny układu mięśniowo-szkieletowego człowieka*, Biomechanika'95, Materiały Ogólnopolskiej Konferencji Naukowej, Zeszyty Naukowe AWF, Kraków, 1995.

[8] WISMANS J., JANSSEN E., BEUSENBERG M., *Injury Biomechanics – course notes*, Eindhoven University of Technology, Eindhoven, 1994.