Is a "movable hinge axis" used by the human stomatognathic system?

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This treatise deals with sagittal in vivo motions of the human mandible. The concept of a "movable hinge axis", which is commonly used in dentistry, was scrutinised theoretically and empirically. We wondered whether a "movable hinge axis"– or better a mandibularly fixed hinge axis (MFHA) – was actually used by subjects with sound temporomandibular joints. To answer this question we first showed that the assumptions of a MFHA would comprise that of the neuromuscular apparatus of the stomatognathic system piloting the mandible by solely two kinematical degrees of freedom (DOF). We spatially recorded in vivo motions of mandibles with high-precision ultrasonic devices. The subjects were asked to guide their mandibles in sagittal movements so that the lower incisal edges ran along the Posselt diagrams. The mathematical procedure is described in detail, hence a possible use of two DOF by a subject could quickly be puzzled out from a set of motions. These analyses revealed that the quasi-plane mandibular movements were approximately piloted by two kinematical DOF in subjects with sound temporomandibular joints. The grade of approximation was measured. Thus, the ensemble of possible positions of the moved body (mandible) can be described by a coordinate system, which is inherent in the stomatognathic system. Lacking precision and poor reproducibility in using only two variables for mandibular position control yield hints that the subject has clinical problems in his stomatognathic system.

Key words: human mandible, mandibular movements, movable hinge axis, degrees of freedom, inherent coordinate system, neuromuscular system

1. Introduction

Up to now axiography is a major part of instrumental analyses in clinical dental practices to evaluate functional states of the stomatognathic system. It is said to

record the path of the so-called "movable hinge axis" of the mandible. The procedure is as follows: The dental surgeons mostly guide the patient's mandible out of centric occlusion (CO) in a small movement parallel to the sagittal plane and thus produce a finite rotational axis in the region of the temporomandibular joint (TMJ). If its lateral projection looks like a point this axis is said to be the "movable hinge axis" which would remain stationary in the mandible and abount which the mandible would rotate [1]–[4]. This statement was often criticized [5]–[9]. NÄGERL et al. [10] have proved that this axiographically defined "movable hinge axis" physically makes no sense since it has no prominent kinematical significance compared with other lines connecting two mandibular points. To find definitely a mandibularly fixed axis, if such exists, the following approach has to be adopted:

Firstly the subjects should be able to perform quasi-plane sagittal mandibular movements keeping the three degrees of freedom (DOF) negligibly small belonging to lateral shift, horizontal and frontal rotation. The ensemble of possible positions of the moved body (mandible) in the reference system (maxilla) is then given by the position of an arbitrarily taken mandibular point (2 DOF), which lies within a plane domain whose margin consists of a closed curve and the mandibular rotation (1 DOF). But, if a distinct mandibular point exists, whose domain is degenerated to a pure line segment, the criterion for the existence of the "movable hinge axis" – or better mandibularly fixed hinge axis (MFHA) – would be fulfilled. Only then the subject would reduce the control of mandibular positions to 2 DOF: The position of the point
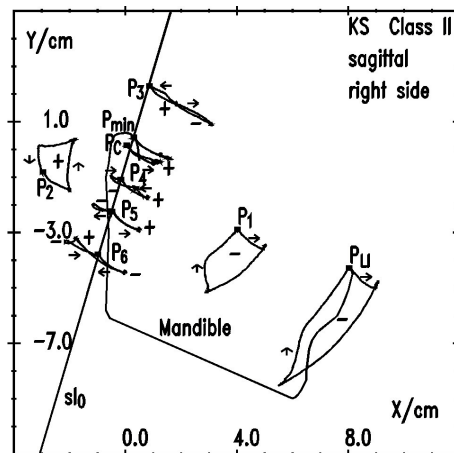

Fig. 1. Motion paths of selected mandibular points of patient KS, projected in the sagittal-vertical plane seen from the right. Point P_{LI} represented the lower incisor edge which followed the Posselt motion. Point P_C at (0, 0) is located near the condyle's centre on the right side of patient's head. Nearby this point the mandibular points ran around very small areas.

Point P_{min} moved on the same path there and back while opening and closing the mouth.
Remark the change of the sense of circulation for points above and below P_{min}

on this line segment (arc length) and the rotation angle. For any cyclic movement of the mandible this special point must move on this line segment back and forth.

We would like to demonstrate by in vivo measurements whether or not the reduction from 3 to 2 DOF of the mandible's position may actually be possible. An 11-year old girl was asked to pilot her mandible as much as possible along the borders of the domain of the mandibular positions parallel to the sagittal plane: The starting position was CO. She moved her mandible to the most anterior possible position under teeth contact, opened her mouth as far as possible, and closed her mouth in a posterior motion to reach CO again. She complied with our requests and piloted her mandible along far distant positions in a plane fashion as we could check by the spatial kinematical measurements. The paths of some mandibular points were calculated from the recorded data (figure 1). The edge of the lower incisor P_{LI} ran around the area of the well-known Posselt diagram [11]. The path of the point P_C (located near the condyle's centre) enclosed a domain just as the paths of the points P_3, P_4, P_5, P_6. For the point P_{min}, however, the domain seemed to be degenerated to a line segment on which this point ran there and back. At first sight P_{min} fulfilled the criterion of a maxillarily movable and mandibularly fixed hinge axis as described above. The girl was apparently able to pilot diverse plane movements between mandibular positions having long distances from each other by using only 2 DOF: A position i of the mandible can be described by the pair (L_i, α_i). L_i is the arc length covered by the hinge axis P_{min} along its path starting from CO ($L_0 = 0$, $\alpha_0 = 0°$), and α_i is the angle of mouth opening in relation to CO (figure 2).
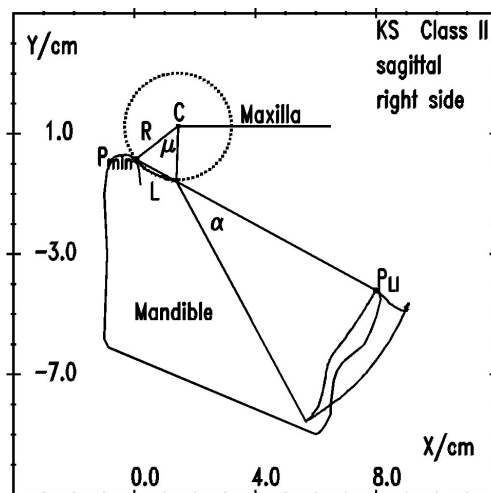
Fig. 2. The motion path of point P_{min} could closely be approximated by a circle
with the centre C and radius R. L is the arc length covered by P_{min}, α is the angle
of mouth opening in relation to centric occlusion

In the following we describe the mathematical procedure by which the possible reduction from 3 to 2 DOF of the mandibular movements can be figured out from spatial in vivo measurements and by which the grade of approximation of using a MFHA by the neuromuscular system can be estimated.

2. Material and mathematical method

2.1. Experimental apparatus

To record spatial motions of mandibles in vivo, we first used the ultrasonic device MT1602 (Dr. Hansen & Co. Bonn, Germany [12]) and later on a more precise CMS-JMA (Zebris Medizintechnik, Isny, Germany) (figure 3). Both measurement systems are able to record the spatial movement of the mandible in relation to the maxilla with 6 DOF so that the spatial path of each mandibular point can be calculated.
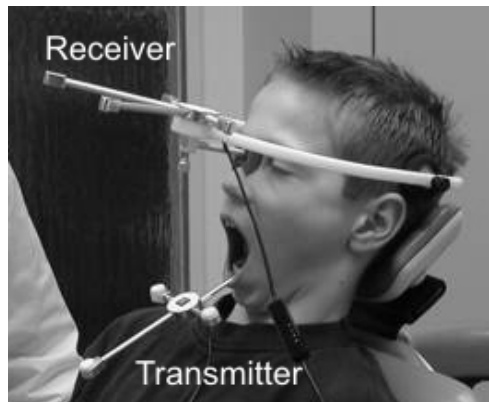

Fig. 3. The ultrasonic measurement system CMS-JMA in a young patient *in-situ*:
The transmitter antenna with three sensors was fixed to the lower incisors,
while the receiver antenna with four sensors was attached above the nose to the head

2.2. The subjects

Up to now we have measured more than 120 subjects with the MT1602 and more than 30 subjects with the CMS-JMA as described by way of introduction. To figure out the number of DOF used for mandible control we scrutinised in detail quasi-plane mandibular movements of 17 adult persons without orthodontic treatment classified as class I and 20 young and adult class-I-patients after orthodontic treatment recorded by the MT1602 as well as 28 young class-II-patients before orthodontic treatment measured by the CMS-JMA.

2.3. Search for the mandibularly fixed hinge axis (MFHA)

We took into account the shapes and values of the areas around which mandibular points drove (figure 1): The lower incisal edge P_{LI} drove clockwise along its Posselt diagram just like other mandibular points nearby (P_1) yielding mathematically negative areas A_2. The points in the posterior region (P_2), however, drove anticlockwise yielding $A_2 > 0$. The points P_C, P_3, P_4, P_5, P_6 ran along loops. Therefore the areas were composed of positive (j) and negative (k) parts: $A_2 = \sum A_{2j} + \sum A_{2k}$. The loops of the points P_3 and P_4, above and below P_C, showed opposite sense of circulation.

These observations made on all subjects suggested the following qualitative statements:

1. With regard to the sense of circulation a line l_0 must exist which separates the positive from the negative areas A_2. This line l_0 is the geometric locus of the mandibular points which run along loops whose positive and negative partial areas add up to zero: $A_2 = \sum A_{2j} + \sum A_{2k} = 0$ with $A_{2j} > 0$ and $A_{2k} < 0$. These observations correspond to the theorem of Steiner (1840) of plane kinematics: The geometric locus of the points of the moved plane (mandible), whose closed paths surround areas A_2 of the same size, forms a circle. The circles of different sizes of A_2 have the common centre S, the so-called Steiner point [13].

2. Considering the absolute areas $A_1 = \sum A_{2j} + \sum |A_{2k}|$ we arrive at the conclusion that among the points of the line l_0 there must exist a point $P_{min}(l_0)$, whose path encloses a minimal absolute area A_{1min}, since in comparison with the cranial points of the line l_0 its caudal points ran along their loops with $A_2 = 0$ in opposite sense of circulation.

3. If the absolute area A_{1min} of the point P_{min} was found to be zero ($A_{1min} = 0$) the subject has only used 2 DOF for piloting the mandible and actually adjusted a MFHA.

The calculation of the area A_2 needs less computer programming and run time than the calculation of the absolute area A_1 since for this calculation the crossing points of the loops have to be determined additionally. To find the point $P_{\min}(A_{1\min})$ the following three steps kept the computational time short.

Step 1: A point within the domain of the condyle (P_C) served as the centre for a square of $10 \cdot 10$ cm^2 in the sagittal-vertical plane (x, y). This square was subdivided by a square net with a step width of 0.5 cm. By means of the measured data the path of every net point $P(x, y)$ was calculated. According to the clock frequency of measurement the path shaped up as a series of n points defining a polygon with n vertices $CP(x_i, y_i)$. The mathematical area of this polygon was split up in triangles which added up to the area A_2:

$$A_2 = \frac{1}{2} \cdot \left[\sum_{i=0}^{i=n-2} (x_i \cdot y_{i+1} - x_{i+1} \cdot y_i) + x_{n-1} \cdot y_0 - x_0 \cdot y_{n-1} \right].$$

The calculated function $A_2(x, y)$ of the net points $P(x, y)$ was then approximated by means of the method of least squares to the plane $A_{2\mathrm{Plane}}(x, y)$. $A_{2\mathrm{Plane}}(x, y) = 0$ yielded the straight line sl_0. We settled upon this procedure because a straight line sl_0 was commonly found to be a good approximation of the circular segment line l_0 between negative and positive areas A_2.

Step 2: On the straight line sl_0 we searched for the point $P_{\min}(sl_0)$ whose path enclosed the minimal absolute area A_1. For this purpose we fitted a parabola to the function $A_1(P(sl_0))$ using Brent's method [14]. According to the *golden section search* the procedure jumped back and forth and found very fast the minimum value of $A_1(P(sl_0))$. This special procedure was considered to be very favourable because the searched point $P_{\min}$ was commonly found to be closely positioned to point $P_{\min}(sl_0)$ (see below).

Step 3: In order to find finally the point $P_{\min}$, we searched in the neighbourhood of the point $P_{\min}(sl_0)$ using Powell's method [14] in two preferred directions: Vertical to the straight line sl_0 and parallel to it. This procedure was a combination of multi-dimensional and one-dimensional minimization. Mostly it was sufficient to calculate the minimal value of the area A_1 once vertical to the straight line sl_0 and once again parallel. Rarely a second iteration was necessary.

3. Results

3.1. The straight line sl_0

In step 1 of our procedure, the function $A_2(x, y)$ was favourably fitted by the plane $A_{2Plane}(x, y)$ since $A_2(x, y)$ mostly represented nearly a plane. Figure 4 demonstrates this fact by the girl's data (figure 1). The functions $A_2(x, y = +5$ cm, 0, and -5 cm) nearly formed straight lines. Consequently the contour lines of $A_2(x, y) = $ constant nearly formed straight lines, too, which were parallel to the straight line $sl_0(A_2 = 0)$ (figure 5).
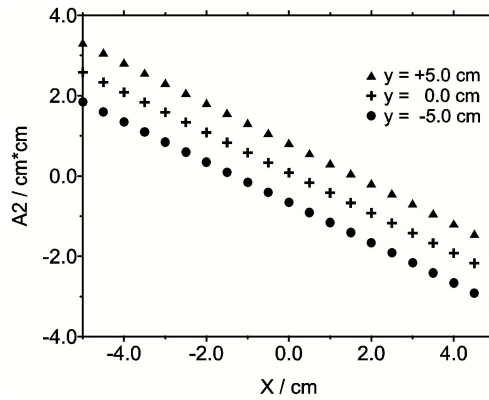

Fig. 4. Data of patient KS: Areas A_2 calculated from the paths of the net points $P(x, y = \text{const.})$ with $y \in (+5$ cm, 0, -5 cm). The graphs $A_2(x, y = \text{const.})$ almost formed straight lines
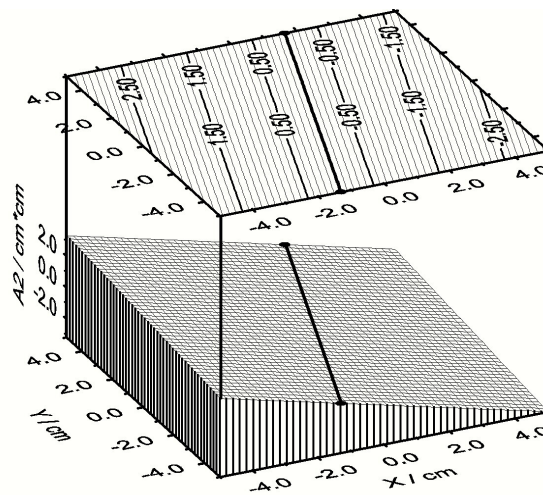
Fig. 5. Data of patient KS: A square of $10 \cdot 10$ cm^2 was laid around the point P_C.
For net points of this square (distance = 0.1 cm) the paths and the corresponding areas A_2 were calculated.
In excellent approximation, the contour line graph (upper part) yielded straight lines
including straight line sl_0 (for area $A_2 = 0$). $A_2(x, y)$ practically represented a plane (lower part)

For most of the 37 young and adult class-I-patients and the 28 young class-II-patients the contour lines for A_2 in the region of $10 \cdot 10$ cm^2 were straight lines in the demonstrated good approximation. Hence, the approximation of the parting line l_0 for the positive and negative areas A_2 by the straight line sl_0 was found to be the useful step 1 in our mathematical procedure.

3.2. The area A_{1min} of the point P_{min}

The girl's data yielded the absolute areas $A_1(sl_0)$ for the points of the straight line sl_0 (figure 6). Two minima were seen. The main minimum belonged to the point P_{min} in figure 1; the second corresponded to the point P_5.
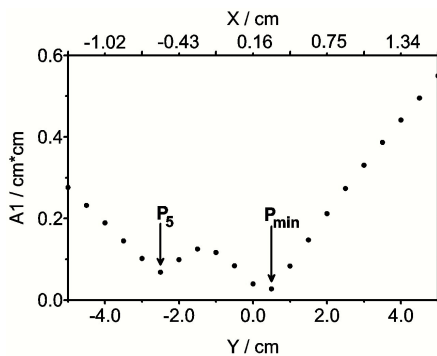

Fig. 6. Data of patient KS: The absolute areas A_1 were calculated for the paths of those points which lay on the straight line sl_0. The curve showed two minima. The main minimum corresponded to the point P_{min}, the second minimum belonged to the point P_5 in figure 1

The absolute areas $A_1(x, y)$ were plotted for the net points in the $10 \cdot 10$ cm^2 square (figure 7) as was already done in figure 5 for $A_2(x, y)$. The $A_1(x, y)$-diagrams revealed a valley where the deepest point was the searched minimum. In the contour line graph, the straight line sl_0 is added which was often found to run through the two minima as demonstrated in figure 6.
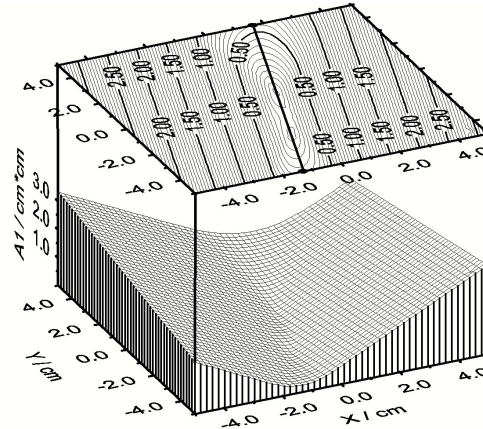
Fig. 7. Data of patient KS: A square of $10 \cdot 10$ cm^2 was laid around the point P_C.
For net points of this square (distance = 0.1 cm) the paths and the corresponding areas A_1 were calculated.
In the contour line graph (upper plot), a main minimum and a second minimum
were found through which the straight line sl_0 ran. The three-dimensional graph correspondingly
showed two hollows in the valley (lower part)

Investigating the kinematical properties of the points P_{min} and P_5 in more detail we found that: 1. In all subjects with sound TMJs, P_{min} moved exclusively forth in the course of mouth opening and exclusively back in the course of mouth closing. 2. P_{min} normally ran apparently along the same path there and back. 3. The path of P_{min} normally showed a circle-like curvature. These findings apparently checked with the expected properties of a MFHA. P_5 (the second minimum) did not show these features: 1. The path was not at all circle-like. 2. The point already ran back during anterior mouth opening (figure 1). P_5 was always caudal of P_{min} and mostly found within the $10 \cdot 10$ cm^2 square. In almost all cases, $A_1(P_5)$ was larger than $A_1(P_{min})$.

Since we had to reckon with two minima, we had to prevent the mathematical procedure of step 2 from taking the second as the searched minimum of $A_1(x, y)$. Brent's method therefore started at first at the lower end of the straight line sl_0 and then again at the upper end. The minimum positioned more cranially was always taken for the further evaluation.

In step 3, we searched the absolute minimum of $A_1(x, y)$ in the region of the point $P_{min}(sl_0)$ based on the findings that there the lines $A_1(x, y) =$ constant were almost parallel and therefore grad($A_1(x, y)$) was almost vertical to the straight line sl_0 resulting from $A_2(x, y) = 0$.

Table. The statistical values for (a) the distance d_{min} between the point $P_{min}(sl_0)$ and the point $P_{min}(A_{1min})$
and (b) the distance d_{ref} between the point P_C and the point $P_{min}(A_{1min})$

Device	(a) Distance d_{min} between $P_{min}(sl_0)$ and $P_{min}(A_{1min})$					(b) Distance d_{ref} between P_C and $P_{min}(A_{1min})$				
	Median	Mean	Std. dev.	Min	Max	Median	Mean	Std. dev.	Min	Max
	cm	cm	cm	cm	cm	cm	cm	cm	cm	cm
MT1602	0.04	0.07	0.07	0.00	0.36	1.32	1.53	0.81	0.32	3.66
CMS-JMA	0.02	0.04	0.05	0.00	0.29	0.61	0.56	0.26	0.04	1.11
			$F = 1.96$					$F = 9.71$		
			$p < 5\%$					$p < 5\%$		

The distance d_{min} between $P_{min}(A_{1min})$ and $P_{min}(sl_0)$ was found to be small in each case (table), while the distance d_{ref} between $P_{min}(A_{1min})$ and P_C was large. Hence $P_{min}(A_{1min})$ was not located in the condyle's centre.

3.3. Is the area $A_{1min}(P_{min}) = 0$?

This question should be answered with yes if $P_{min}(A_{1min})$ was a point of an actually existing MFHA. Serious problems arose: On the one hand, the area $A_{1min}(P_{min})$ was unavoidably larger than zero because of measuring errors. On the other hand, we regarded the MFHA primarily not given by anatomical mechanical constraints but produced by piloting the mandible by the neuromuscular system. Because of this the condyles can be withdrawn a little bit from the os temporale giving thus the TMJ a certain articulating space [15]. Therefore we expected that the neuromuscular system was only able to implement a MFHA with certain uncertainty.

To handle this problem we estimated the very upper limit of the error for the area $A_{1min}(P_{min})_n$ of each person n and compared the distributions of these limits with that of the devices. The limits were determined in the following way:

1. We took the confidence interval (CI) into account for determining a point: CI was 0.01 cm for the MT1602 [12] and 0.003 cm for the CMS-JMA [16].

2. We calculated the individual path length L_{pn} by adding up the distances between the vertices $CP(x_i, y_i)$ of the polygon (used for the calculation of $A_{1min}(P_{min})_n$) on the way there and back:

$$L_{pn} = \sum_{i=0}^{i=n-2} \sqrt{(x_{i+1} - x_i)^2 + (y_{i+1} - y_i)^2} \; .$$

3. The rectangle $A_{limit\ n} = CI \cdot L_{pn}/2$ was regarded as the individual upper limit of error.

4. The difference $D_n = A_{1min}(P_{min})_n - A_{limit\ n} = A_{1min}(P_{min})_n - (CI \cdot L_{pn}/2)$ was used as the test parameter to check whether the area $A_{1min}(P_{min})$ exceeded its upper limit of error.
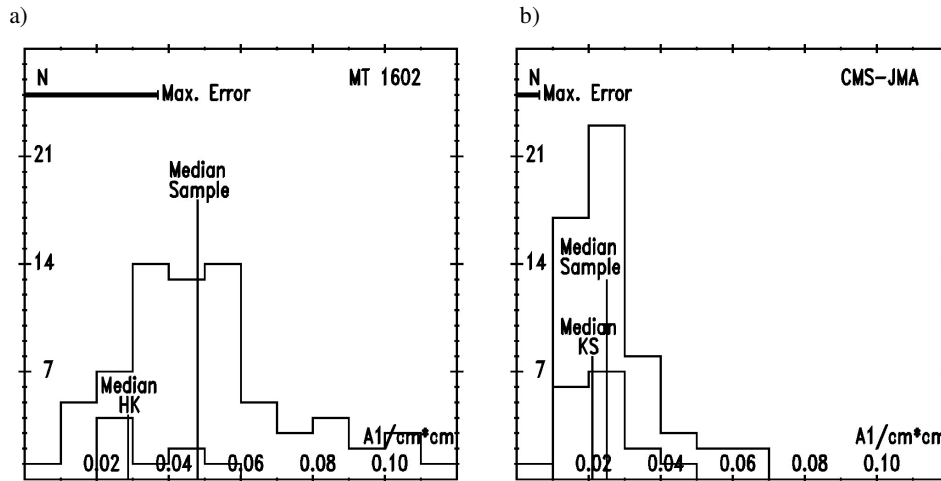
a)

b)

Fig. 8. Histograms of the $A_{1\min}$-distributions: (a) of the 37 young and adult class-I-patients measured
with the MT1602 and of the 4 movement cycles of the young class-I-patient HK and (b)
of the 28 young class-II-patients measured with the CMS-JMA and of the 8 movement cycles
of the young class-II-patient KS. The upper limit of error was smaller than the median
for the ensemble as well as for the individual

For both samples (MT1602 and CMS-JMA measurements) the median of the $A_{1\min}(P_{\min})$-distribution lay above the respective very upper limit of error (figure 8a, b). In particular, for the more precise CMS-JMA measurements all individual $A_{1\min}(P_{\min})_n$ lay above the upper limit of error. The Student-test of the variable D_n yielded: $t = 8.25$ ($t(5\%) = 1.99$) for the MT1602 measurements and $t = 12.58$ ($t(5\%) = 2.00$) for the CMS-JMA. Hence, the area $A_{1\min}(P_{\min})$ was found not to equal zero! This finding was supported by individual repeated measurements: $t = 4.68$ ($t(5\%) = 2.37$) for patient HK and $t = 8.40$ ($t(5\%) = 2.13$) for patient KS.

4. Discussion

All subjects made quasi-plane mandibular motions. The 3 DOF of horizontal and frontal rotation and lateral shift were measurable but small. We wondered whether the stomatognathic system used a further reduction from 3 to 2 DOF in a similar approximation. We could affirm this. The paths of the MFHA in mouth opening and closure almost coincided though the mandible was guided along strongly differing positions. The MT1602 and the CMS-JMA groups showed mean

residual areas of ~0.048 cm² and ~0.025 cm², respectively, and path lengths of ~3.75 cm and ~2.09 cm. Hence the spacing between back and forth (0.048 cm^2 / 3.75 cm = 0.013 cm and 0.025 cm^2 / 2.09 cm = 0.012 cm, respectively) corresponded to the breadth of a thin pencil line. Therefore the neuromuscular systems were able to differentiate the entire ensemble of open mouth positions with high precision despite using practically 2 DOF. The mandibular positions could be definitely specified by two variables, the arc length L covered by the MFHA and the rotational angle α in relation to the maxilla (figure 2). Thus an orthogonal coordinate system is obtained in which the ensemble of the possible positions of the rigid body is defined one-to-one. This system is independent of the coordinate system of the measuring apparatus and inherent in the stomatognathic system. It makes intra- and interindividual comparisons of mandibular movements possible and easy. Especially the structure of the guidance by the stomatognathic neuromuscular system can be evaluated when the mandible was brought from
a starting to a final position [17].

Since according to Steiner the ensemble of the functions $A_2(x, y)$ = constant have to represent concentric circles and since we found in the most cases that $A_2(x, y) = 0$ could be replaced by a straight line (sl_0), the Steiner centre lay far away from the mandible. In the few cases having a nearer Steiner centre, the search for $P_{min}(A_{1min})$ required additional computational steps.

The existence of $A_2(x, y) = 0$ is common to plane kinematics with 3 DOF. It does not imply that a mandibular point P_{min} must exist whose absolute area $A_1(P_{min})$ equals zero. $A_1(P_{min}) = 0$ is the criterion of plane movements with 2 DOF. The residual $A_1(P_{min})$ characterizes the approximation of the measured movements to plane movements with 2 DOF. It was found to be close.

The data of the few patients with lacking precision and poor reproducibility in using the MFHA and having high residuals $A_{1min}(P_{min})$ gave hints that these patients have hidden problems with their neuromuscular apparatus. In this regard our method to evaluate in vivo the patients' MFHA yields a novel diagnostic tool of clinical value by determining the area $A_{1min}(P_{min})$.

5. Conclusions

Though an ideal MFHA could not be found, we could show that the stomatognathic system normally adjusts and uses a MFHA with a surprisingly high precision to guide the mandible's position in the case of mouth opening and closing by using only 2 main DOF. Thus an inherent coordinate system of mandibular movements can be determined.

The mathematical procedure for finding out the inherent coordinate system can be used for quasi-plane movements of other joint systems by analogy.

Acknowledgements

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