

Geometrical aspects of growth plate modelling using Carter's and Stokes's approaches

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Development of the skeleton is a complex mechanobiological process. Shape and size of the majority of bone elements are the result of endochondral growth and ossification occurring during childhood and adolescent period. The influence of mechanical loading acting in the skeletal system on bone development is known since the 19th century, but understanding of such phenomenon seems to be still insufficient. Traditionally accepted Hueter–Volkman law claims that increased pressure acting on a growth plate retards bone growth and, conversely, reduced pressure or even tension accelerates it. Stokes's approach is directly based on this theory. Carter's model seems to be slightly more complex because takes into account three-dimensional stress state.

The subject of the research was to evaluate the mechanobiological condition of endochondral bone growth occurring within the growth cartilage where different geometrical structures (8 models) of the growth plate and various loading conditions (5 variants) were considered. Simulations were made using the finite element method and both Stokes's and Carter's models were used to estimate mechanical stimulation of bone growth.

Results indicate non-uniformity of the growth conditions occurring within the growth cartilage when its layer is located between two bone blocks. Non-axial loadings result in dissymmetry of mechanical stimulation of bone growth. In general, its minimum is located in the regions of the cartilage to which maximal loadings were directed. Carter's approach is, however, more sensitive to interrelation between growth plate geometrical structure and loading direction, compared to Stokes's model. Obtained results indicate the necessity of realistic modelling of the growing bone geometrical structure, including the elaboration of custom-made models.

Further research is necessary to elaborate the new formula describing mechanical influences on bone growth, taking into account the cyclic loading of a constant direction. In this way it will be possible to overcome the still existing problems with the explanation of numerous clinical phenomena.

Key words: bone, cartilage, geometry, growth plate, mechanobiology, modelling

1. Introduction

The growth plate is the structure responsible for longitudinal bone growth [1]. It consists of three tissue types: the growth cartilage, the newly formed trabecular bone of the metaphysis and the fibrous tissue surrounding the cartilage (the ring of Lacroix). The influence of mechanical loadings on its activity is known since the 19th century when the Hueter–Volkman law was formulated [2], [3], [4]. According to this theory, increased pressure acting on a growth

plate retards bone growth and, conversely, reduced pressure or even tension accelerates it. More recently, Stokes has applied Hueter–Volkman law to describe the relationship between loadings and bone growth velocity [5], [6]. Both Hueter–Volkman law and Stokes's research reduce loadings acting on the growth plate only to uniaxial tension-compression. Mechanical state occurring in the real anatomical object during growth is, however, much more complex and always has a three-dimensional nature. The main reasons are complex geometry of bone elements and the three-dimensional, variable loadings acting in the

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skeletal system. The complex structure of bone elements, especially non-homogeneity of the growth plate, is also very important [7].

A theory which allows to analyse growing bone mechanobiology taking into consideration three-dimensional stress state occurring in a growth plate was formulated by Carter and Wong, basing on the well known Sine's criteria used for predicting fatigue crack initiation in metals [8], [9]. Stevens et al., following this basic theory and additionally using Mikic proposal [10], assumed that endochondral growth is dependent on the cartilage maturation rate being the function of octahedral shear stresses and hydrostatic stresses occurring within the cartilage [11]. Taking into account cyclic loadings of various directions acting on the growing bones, authors have assumed that maturation rate should be calculated as a linear combination of the maximal octahedral shear stress and the minimal hydrostatic stress within the cartilage, occurring during the complete loading cycle. Such theory was used by Carter and Wong to analyse the development of diarthrodial joints as the process driven by cyclic loadings composed of five distinct loading cases [8]. In a similar manner, Heegaard et al. modelled joint morphogenesis occurring during fetal life [12]. The mechanical loading of variable direction, caused by joint motion, was identified as a factor taking part in forming the shape of particular bones. Shefelbine et al. [13] used such methodology to analyse the influence of mechanical factors on the forming of a femoral bicondylar angle. Summing up all abovementioned research, it can be stated that loadings of variable direction in relation to growing bone geometry play an important role in the skeleton forming process.

In the growing skeleton, loads acting on the growth plates have various value and direction. Additionally, growth plates have also different shapes. Very often development of the bone is conducted by more than one growth plate. The role of both capital epiphysis and epiphysis of greater trochanter in the forming of the proximal part of the femoral bone was experimentally proved by Salenius and Videman [14]. An indisputable fact is that forming of the real, multi-form bones is performed in a very geometrically complex environment.

The majority of above mentioned examples concern the problem of interrelation between loading condition and geometry of the growing bones in the process of their physiological development. There are numerous cases, however, when various abnormalities occur in the skeleton geometry during its growth. In such situations, the role of improper loading condi-

tions is often discussed. Shefelbine and Carter analysed the influence of loadings occurring in cerebral palsy on increased femoral anteversion [15]. A similar research tried to explain the disturbances of the femoral morphology in the case of developmental hip dysplasia [16]. Models used in both mentioned studies correspond to the structure of 1-year-old child's femur, and the whole part of epiphysis placed proximally to the growth region was modelled as a cartilage. According to the results presented by Piszczatowski [7] such attempt could not be used for older children when ossification occurred within the epiphysis. It should be mentioned that such process (formation of a secondary ossification centre) in the proximal part of the femur occurs approximately in the second half of the first year of life [17].

Apart from situations when improper loadings seem to be the reason of skeletal deformity, also some cases are known when the spontaneous improvement of bone abnormal geometry was explained on the biomechanical background. Rauch has discussed the case of mild genu varum occurring in toddlers [18]. In such situation the medial part of the growth plate is compressed more than the lateral part. According to Hueter-Volkman law, if compression inhibited growth, the medial part of the growth plate should grow slower than the lateral one and worsening of the varus deformity should be observed. In this way, any slight deviation of the lower limb axis should result in a progress of pathology leading to catastrophic deformity, but such phenomenon has not been observed in clinical practice. On the contrary, more intensive growth occurring in the more compressed, medial part of the growth plate usually leads to spontaneous straightening of the limb. A similar observation was presented by Pauwels [19]. He described the case of mild coxa vara with straightening of the femoral neck resulting from more intensive growth occurring on the more compressed side of the growth plate. Pauwels stated that addition of bending stresses to pure compression within the growth plate causes more growth on a compressed side where the greater stresses act, and in this way undesirable bending stresses are removed. Finally, straightening of the anatomical object occurs. Such clinical observations lead to a supposition that Hueter-Volkman law, and in consequence the Stokes's theory, is not capacious enough to encompass all clinical and biomechanical situations occurring during skeleton development. As a consequence, endochondral bone growth cannot be described as being strictly dependent on the compressive stresses resulting from statically applied loadings. In this situation the open question remains if Carter's

theory is able to explain the abovementioned cases of spontaneous improvement of bone abnormal geometry. Earlier analyses have shown, however, that for a simple geometrical model of the growth plate (flat layer of growth cartilage placed between two layers of trabecular bone, under axial compression) Carter's approach showed considerable similarities to Stokes's approach [7].

Numerous *in vivo* and *in vitro* experiments have shown that growth and ossification of the cartilage depends not only on the loading direction but also on the loading frequency, varying even in the case of unidirectional cyclic loading [20], [21], [22]. A similar effect during forming of bone regenerate in the process of bone lengthening was observed by Filipiak et al. [23].

Summing up all research discussed above, it can be stated that spatial arrangement of the growth plate in relation to loading conditions varying in time and space strongly influences the process of bone development. In order to utilize that knowledge in bioengineering, its appropriate mathematical models need to be elaborated. The first approach, based on Stokes's research, is used mainly for analyses of bone growth under axial, static loadings [6], [24], [25]. The second, based on the Carter's theory, is used for three-dimensional analyses of growing bones under loadings of various directions [8], [9], [26], [27]. However, to the best of author's knowledge, there was no application of this theory in analyses of a growth plate in a situation when secondary ossification centre is present and the growth cartilage is placed between two bone layers, with the exception of the first part of this research [7]. The subject of presented analyses was to continue previous analyses taking into consideration different spatial configuration of the growth plate – the loading conditions system. The main aim was to evaluate biomechanical conditions of the endochondral bone growth, obtained using both Carter's and Stokes's theories, for various shapes of the growth plate and for various loading conditions. The answer to the question whether any of discussed theories is able to explain such clinical situation like pathological deformity of the skeleton occurring in numerous diseases (e.g. in cerebral palsy [28], [29], [30], [31]) and spontaneous straightening of the bone element (e.g. in mild genu varum [18] or mild coxa vara [19]) will be one of the aims of the analyses. Such knowledge seems to be very important for better understanding of mechanical influences on the skeleton growth and for the development of effective methodology of pathological bone deformity numerical simulation.

2. Materials and methods

Modelling and numerical simulations were performed using the ANSYS package (ANSYS, Inc.) based on the finite element method. All analyses were made using a single-phase, elastic model of the cartilage. Such assumption is strictly forced by the fact that both Carter's and Stokes's models, being the object of research, are based on the single-phase model of growth cartilage. Material properties of particular parts of the growth plate were taken in reference to the basic model used in the earlier part of research [7] as follows:

a) growth cartilage: elastic modulus $E_{GC} = 6$ MPa, Poisson's coefficient $\nu_{GC} = 0.495$ (cartilage was treated as a nearly incompressible material, analyses were performed using the mixed $u-P$ formulation [32]);

b) trabecular bone: elastic modulus $E_B = 345$ MPa, Poisson's coefficients $\nu_B = 0.3$;

c) fibrous tissue: elastic modulus $E_F = 10$ MPa, Poisson's coefficients $\nu_F = 0.3$.

2.1. Geometrical models of the growth plate

The geometrical structure of growing bones is very complex. However, the shape of growth plates is quite regular. A growth cartilage layer can be compared to a disk, but in various anatomical situations it is shaped flat, convex or concave [1]. Obviously, during growth process, especially in case of any pathology, numerous deformities of the growth cartilage could appear. The full set of three-dimensional axially symmetric, geometrical models used in presented analyses consists of eight variants, designated M1–M8 (Fig. 1).

Flat, axially symmetric disk of the growth cartilage was modelled in M1 and M5 variants. Convex layer of growth cartilage was used in variants M2 and M6, whilst concave in variants M3 and M7. The most complex, wavy shape of the cartilage layer was used in the variants M4 and M8. The main difference between models M1–M4 compared to M5–M8 is the thickness (h) of the growth cartilage layer. For the first set of models M1–M4 the cartilage thickness was equal $3/8$ of its diameter ($h = 15$ mm, $d = 40$ mm). Such relation could reflect the growth plate structure at the early stage of bone development, when cartilage layer is relatively thick [1]. Models M5–M8, where the thickness of growth cartilage is three times lower ($h = 5$ mm, $d = 40$ mm, $h/d = 1/8$), better reflect the structure of growth plate present in the older, but still growing bones.

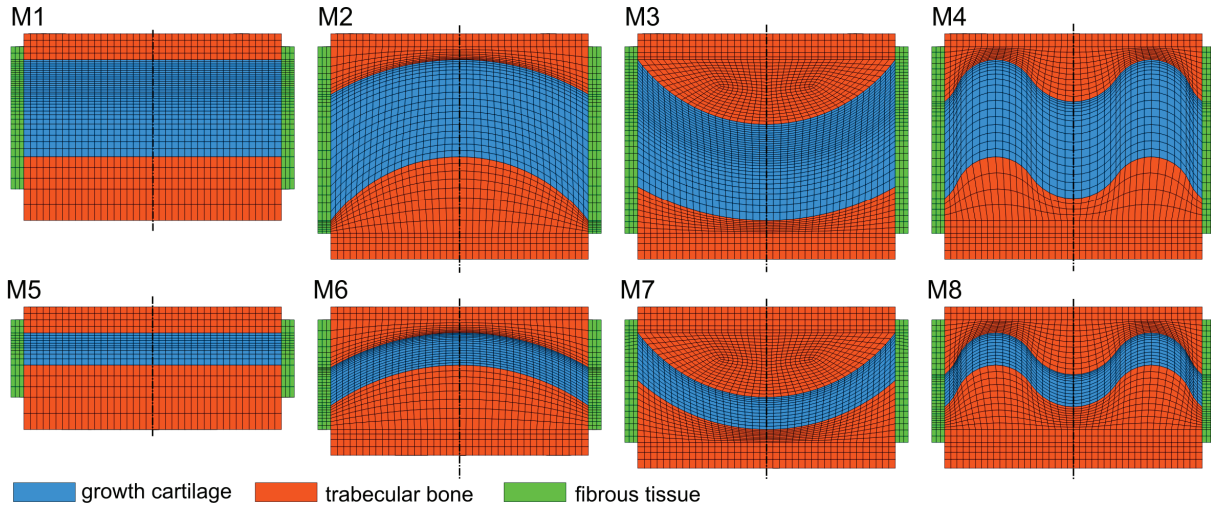


Fig. 1. Geometrical models of growth plate (axial cross-section).
Models M1–M4 with thicker growth cartilage layer and models M5–M8 with thinner growth cartilage layer

Two layers of trabecular bone were modelled, too. The first (distal) was placed under, whilst the second (proximal), over the growth cartilage. It should be noted that, as a result of various shapes of growth cartilage, the thickness of bone layer varies in particular variants of the geometrical model. A ring of fibrous tissue was modelled around the cartilage and bone layers to simulate the ring of La-croix. A finite element model was made in ANSYS preprocessor with the use of 3D 20-node structural elements (SOLID186). Particular parts of the model were strictly connected to each other on its common nodes.

2.2. Loadings and constraints

The model was fully constrained on its distal nodes, which reflects the ideal connection of the growth plate with metaphysis. Loadings were applied on the proximal bone plate in five different ways (Fig. 2).

In the first variant (L1), pressure was applied uniformly to the proximal surface of the model. The main distinction used in the L2–L5 variants is non-uniform pattern of the compressive loads with centrally located maximum obtained due to usage of the nodal forces instead of surface pressure. Compressive loads of uniform value were applied to all nodes attached to the proximal surface, but due to variable quantity of nodes per area unity (increase toward the model axis), the compressive loads became hyperbolic pattern (Fig. 2). Forces used in the L2 model were directed along the longitudinal axis of the model. L3 and L4 variants are distinguished by the slope of loading direction towards the model axis

with the angle of slope $\alpha = 20^\circ$ and $\alpha = 45^\circ$, respectively. The last variant (L5) is similar to the L2 variant, but loadings were applied only on the right half of the proximal surface. Such model tries to reflect a situation similar to the case of proximal part of tibia in mild genu varum [18], when increased loadings act on one half of the growth plate.

Resultant value of the applied loads in all variants was equal 200 N [7]. This value was taken arbitrary. It seems, however, that similar value of compressive forces could appear in several-year-old children's femur or tibia.

2.3. The indexes used for the analysis of mechanical stimulation of a growth plate

Stress pattern within the cartilage is the base used both in Carter's and Stokes's approaches to estimate mechanical stimulation of bone growth. Finite element analysis of all 40 variants of growth plate model (8 geometrical models \times 5 models of loading conditions) allowed determining all necessary stress components within all finite elements. Based on these data, the three alternatives of "growth index" (GI), expressing the intensity of mechanical stimulation of the endochondral bone growth, were calculated for particular finite elements representing the cartilage:

- a) using the Carter's approach:
 - for single load cases [8]:

$$GI_1^S = \sigma_S + a\sigma_H \text{ [N/m}^2\text{]}, \quad (1)$$

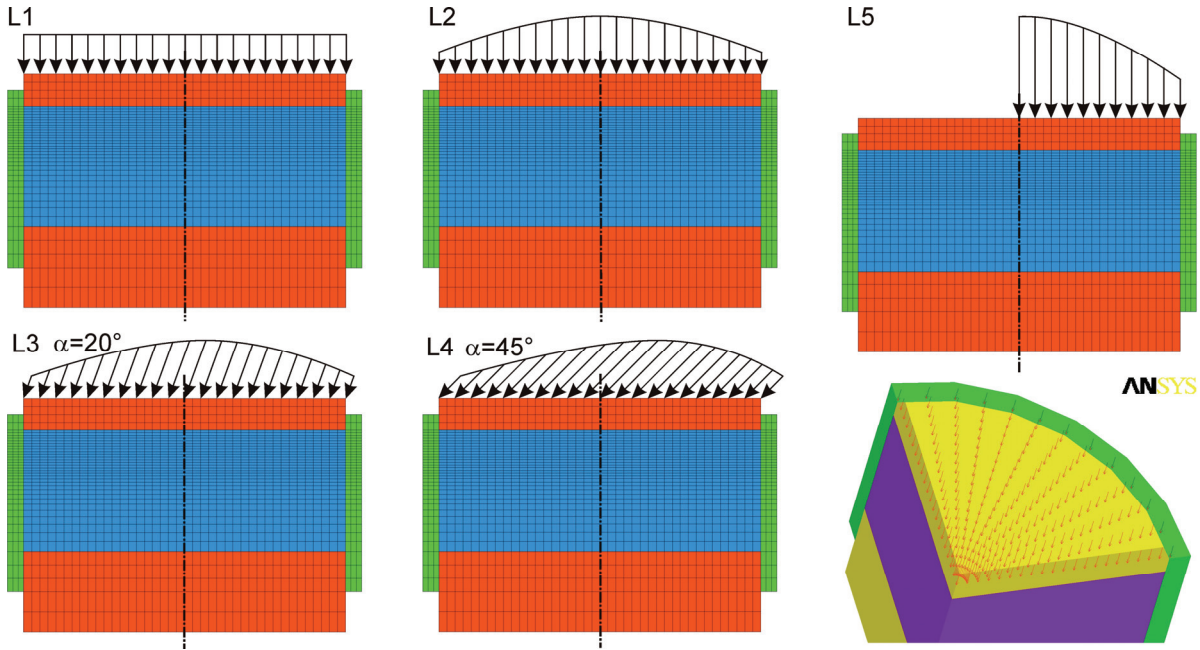


Fig. 2. Loading condition models used in the analyses. Geometrical model M1 was used to present loading conditions, but identical loadings were used for all other geometrical models.

Non-uniform pattern of the compressive loading used in M2–M5 variants was the result of the variable nodal forces density over the proximal surface of the model (maximum in the centre)

where:

σ_S – octahedral shear stress (always positive, increase the value of the GI_1^S , accelerate cartilage ossification and growth),

σ_H – hydrostatic stress (negative in compression and positive in tension),

a – weighting factor (dilatational parameter);

– for multiple load cases [11]:

$$GI_1^M = \max(\sigma_S) + b \times \min(\sigma_H) \text{ [N/m}^2\text{]}, \quad (2)$$

where:

$\max(\sigma_S)$ and $\min(\sigma_H)$ – respectively maximum of octahedral shear stress and minimum of hydrostatic stress occurring during the whole loading cycle,

b – weighting factor (dilatational parameter);

b) using the Stokes's approach [6]:

$$GI_2 = \sigma_z \text{ [N/m}^2\text{]}, \quad (3)$$

where σ_z – axial stress (z -axis represents the axial direction perpendicular to the proximal surface of the model).

Two values of the dilatational parameter a used in the formula (1) were taken into analyses: $a = 0.5$ and 1.7 [7]. By analogy, for multiple loading cases, three various values of parameter b occurring in formula (2) were taken into consideration: $b = 0.35$ [11], $b = 1.0$ and $b = 1.7$ [7]. During multiple loading analysis the

set of loadings composed of three variants L2–L4 was considered. In this way the variability of loading direction in the range 0° – 45° was modelled.

3. Results

Patterns of the growth indexes calculated using both Carter's and Stokes's concepts (GI_1^S , GI_2), obtained for variants L1 and L2 with axial loadings were presented in Fig. 3.

There are conspicuous similarities of results obtained for both loading cases, especially for models with higher thickness of cartilage layer (M1–M4). Patterns of particular growth indexes are not uniform within the volume of cartilage. This phenomenon is caused by inhomogeneity of the growth plate and it was discussed by the author earlier [7]. All patterns are symmetrical in relation to the central plane of the model and reach the minimum in the centre of growth cartilage. Non-uniformity of GI_1^S and GI_2 patterns is greater for L2 loadings. More distinct effect of the hyperbolic pattern of compressive loading (L2 variant) is visible for models with lower thickness of cartilage layer (M5–M8). Higher non-uniformity, both Carter's and Stokes's indexes, with a more distinctive minimum in the central part of the

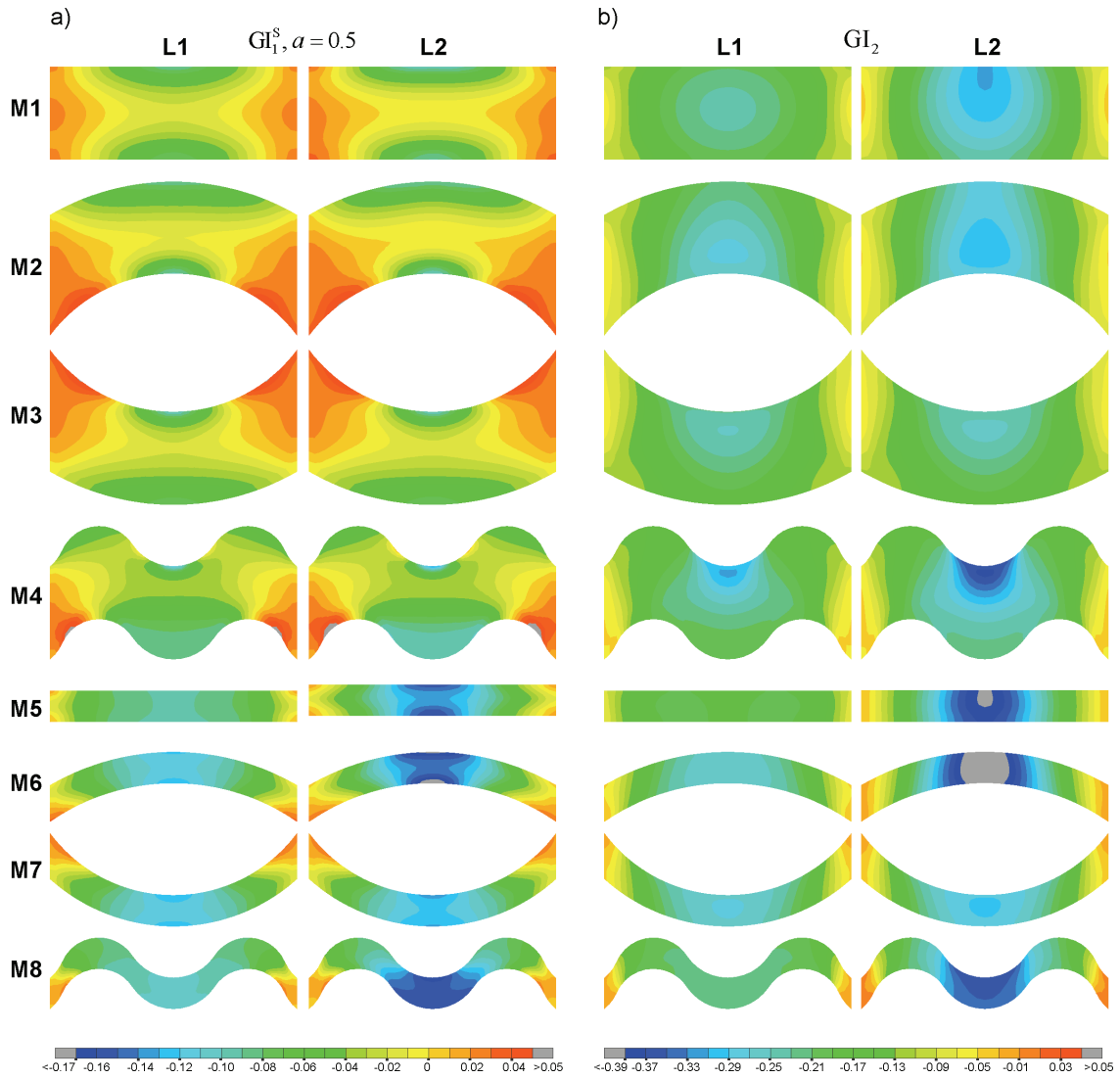


Fig. 3. Comparison of mechanical stimulation of growth cartilage obtained for uniform pressure (variant L1) and hyperbolic pattern of loading (variant L2): a) patterns of the index GI_1^S [MPa] for $a = 0.5$; b) patterns of the index GI_2 [MPa]. View on the axial cross-section of the growth cartilage model; bone and fibrous parts of the model were intentionally omitted

cartilage could be observed for these cases. Such result was not obtained only for concave cartilage layer (model M7) where the greater thickness of the bone layer, separating cartilage from the place of force application, could have blurred the effect of relatively higher loadings acting in the central part of growth plate in the L2 variant.

Patterns of the growth index GI_1^S , calculated on the basis of Carter's theory and using dilatational parameter $a = 0.5$, obtained for L2, L3 and L4 loading variants were presented in Fig. 4. The main distinctiveness of results obtained for the sloped loading directions (L3 and L4 variants) is the dissymmetry of growth index patterns in relation to the central plane of the model. The non-uniformity as well as dissymmetry of GI_1^S patterns increases for a greater angle of loading inclination (L4). Results obtained for thinner

cartilage layer (M5–M8) are, however, more unambiguous and easier to analyse than those obtained for thicker cartilage layer (M1–M4), but in general both sets of models lead to similar conclusions. The GI_1^S index reached the minimum in such regions of the cartilage to which maximal loadings were directed, whilst maximum of GI_1^S is located always on the right side of particular patterns, out of maximal loading direction. It should be noted, however, that Carter's approach seems to be sensitive to interrelation between loading direction and cartilage shape. It is visible when analysing results obtained for the wavy shape of cartilage layer (M4 and M8). For such model geometry, the greatest non-uniformity of index GI_1^S patterns within the cartilage volume for various loading directions can be observed. The minimum appears in these regions where the line of maximal loadings

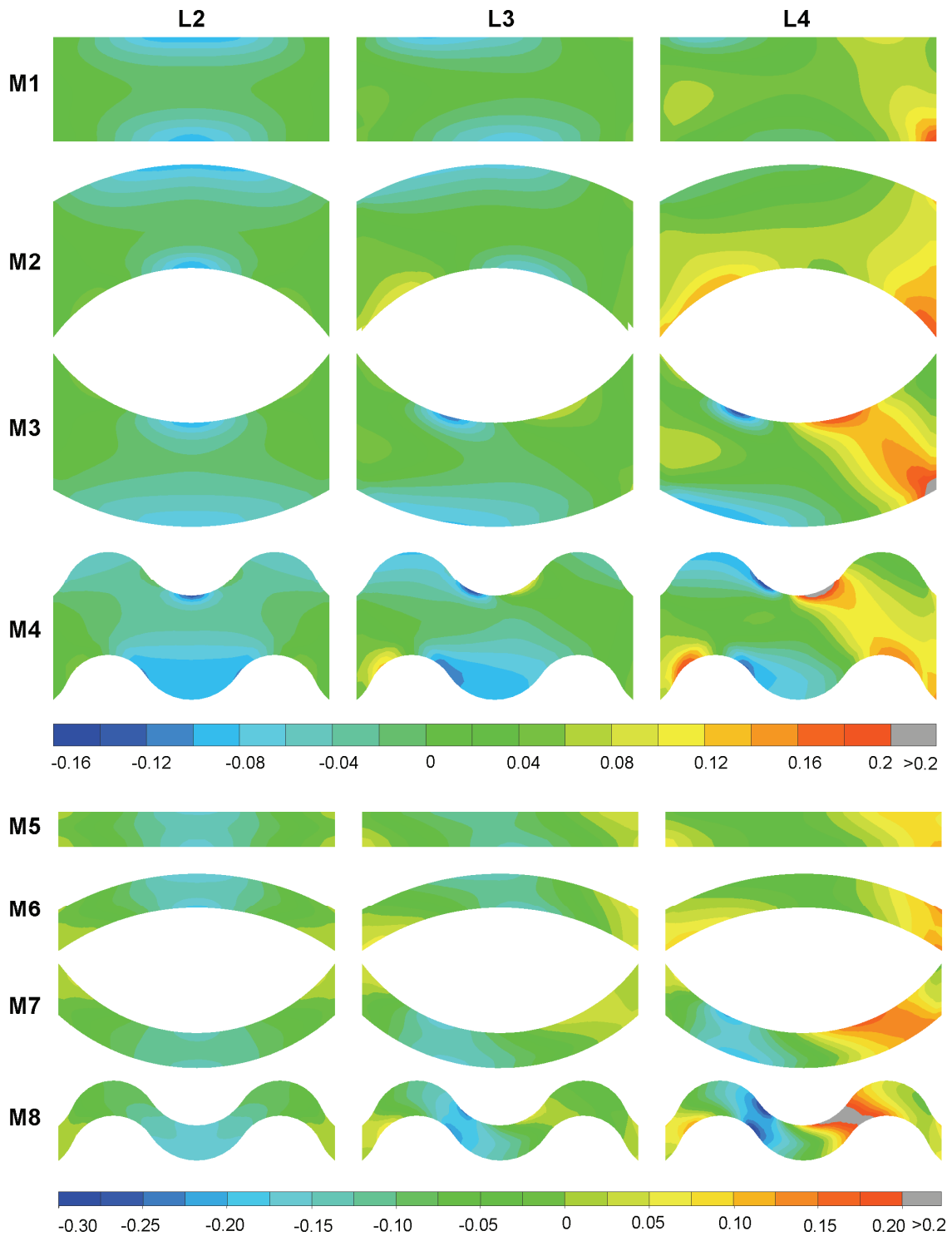


Fig. 4. Patterns of the index GI_1^S [MPa] calculated for models M1–M8 using $a = 0.5$ under loadings L2 ($\alpha = 0$), L3 ($\alpha = 20^\circ$) and L4 ($\alpha = 45^\circ$). View on the axial cross-section of the growth cartilage model; bone and fibrous parts were intentionally omitted

crosses the cartilage and concurrently the proximal surface of growth cartilage is almost perpendicular to loading direction (e.g. M8 model, medial part of the left wave for L3 and L4 loading variants). The maximum of GI_1^S appears in these parts of the cartilage model which are out of maximal loading direction and

the cartilage surface is almost parallel to loading direction. A quite similar effect is observed also for concave models (M3 and M7). A bit more complex results were obtained for convex cartilage layer (M2 and M6). At the superior surface of the cartilage model, the minimum of GI_1^S index can be found, in

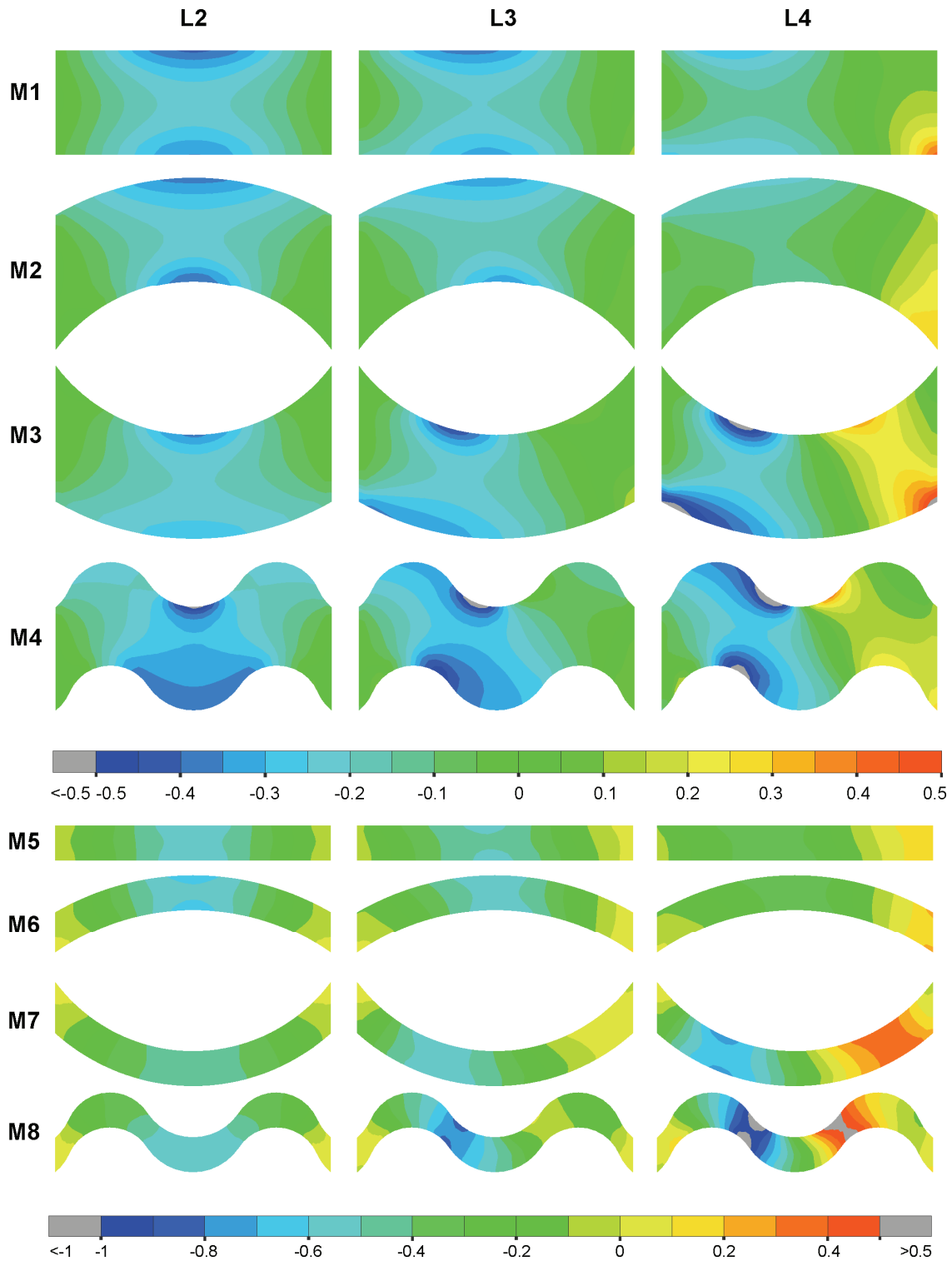


Fig. 5. Patterns of the index GI_1^S [MPa] calculated for models M1–M8 using $a = 1.7$ under loadings L2 ($\alpha = 0$), L3 ($\alpha = 20^\circ$) and L4 ($\alpha = 45^\circ$). View on the axial cross-section of the growth cartilage model; bone and fibrous parts were intentionally omitted

these cases, rather in the left part of the model, in compliance with maximal loading direction. Looking at more interiorly situated part of cartilage model, it is possible to observe that the zone of minimum value of GI_1^S index turns to the right, in the direction opposite

to the line of applied loadings. A quite similar effect is visible also for flat models (M1 and M5). It seems that relation between negative hydrostatic stresses occurring in the compressed part of the cartilage and positive shear stresses is strongly dependent on the geo-

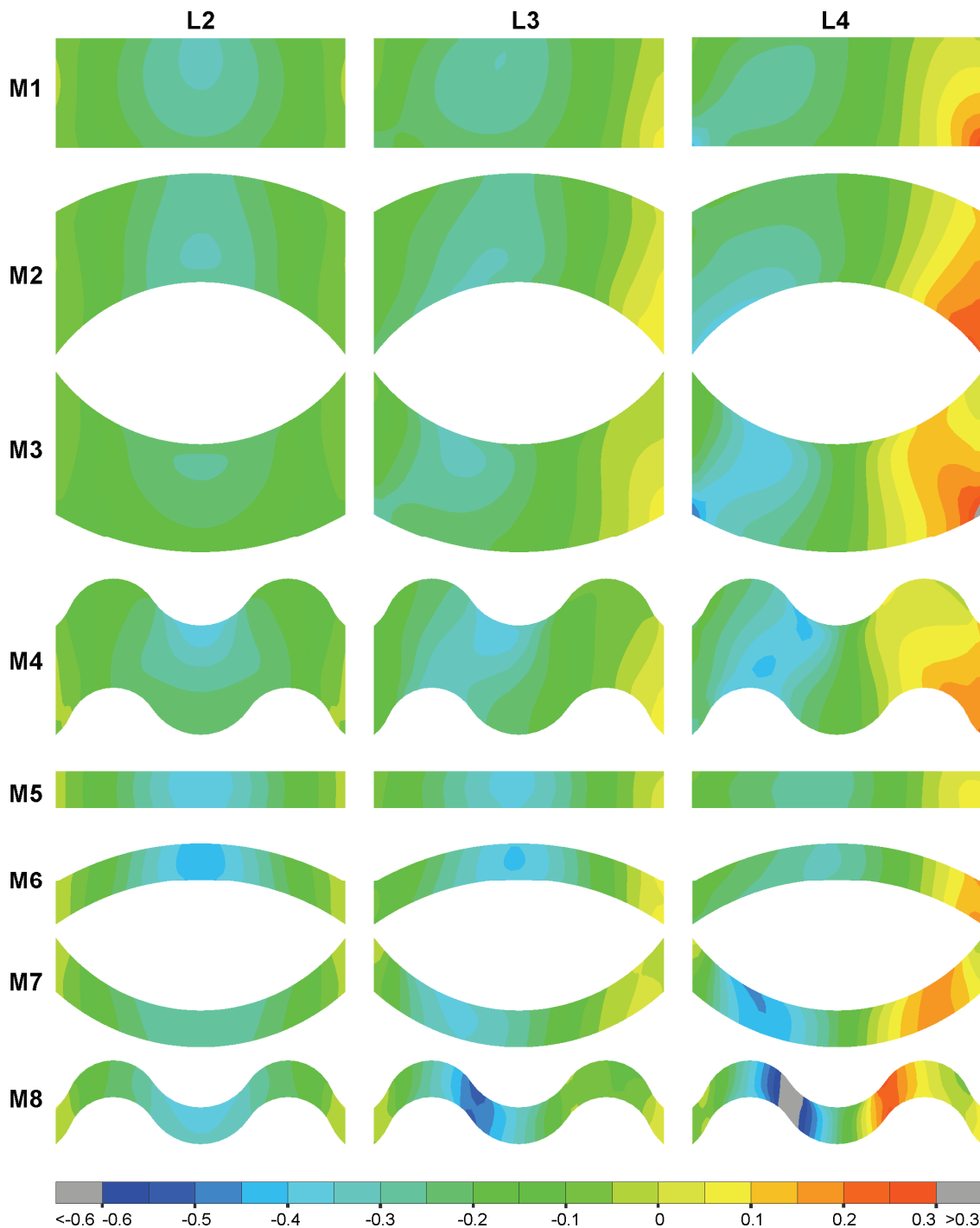


Fig. 6. Patterns of the index GI_2 [MPa] calculated for models M1-M8 under loadings L2 ($\alpha = 0$), L3 ($\alpha = 20^\circ$) and L4 ($\alpha = 45^\circ$). View on the axial cross-section of the growth cartilage model; bone and fibrous parts were intentionally omitted

metrical structure of the model. As a result, Carter's index, being the weighted sum of both these parameters, reflects the interrelation between the loading direction and the shape of cartilage model.

An earlier research [33] has indicated the necessity of increased role of hydrostatic stresses in Carter's formula. Piszczatowski [7], for weighting coefficient $a = 1.7$, obtained a higher correlation between Carter's and Stokes's approaches than for the most frequently

used $a = 0.5$. For these reasons, patterns of GI_1^S index were calculated also for $a = 1.7$ (Fig. 5).

The analysis of these results indicates that greater value of a coefficient relatively decreases the role of shear stresses and, as a result, the model becomes less sensitive on the shape of the cartilage. The domination of compressive stresses is more visible and minimum of GI_1^S index is more correlated with direction of maximal loadings. This effect is well

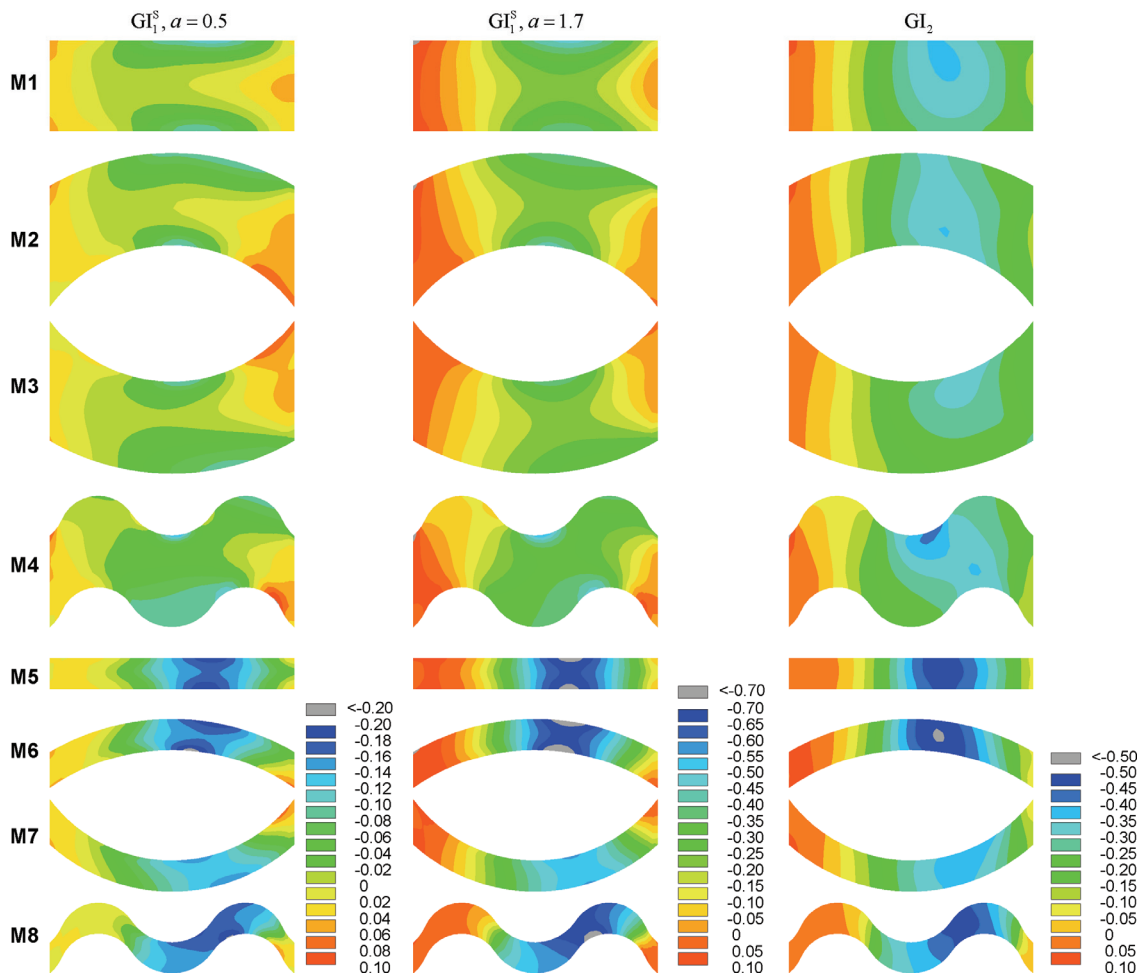


Fig. 7. Patterns of the indexes GI_1^S [MPa] (for $a = 0,5$ and $1,7$) and GI_2 [MPa] calculated for models M1–M8 under loadings L5 (load applied only on the right part of the growth plate model). View on the axial cross-section of the growth cartilage model; bone and fibrous parts were intentionally omitted

visible for flat and convex shapes of the cartilage model.

Patterns of GI_2 index, calculated for various loading directions (L2, L3 and L4) based on the Stokes's concept, were presented in Fig. 6. Similarly to GI_1^S index, patterns of GI_2 index are also non-uniform. For sloped loading direction, patterns are asymmetric in relation to the central plane of the model. The localization of this index minimum, for all models, follows the direction of applied forces and can be found in such regions to which the greatest loadings were directed. For greater angle of loading slope, the non-uniformity of GI_2 patterns for particular models is much more distinct.

The patterns of both GI_1^S and GI_2 indexes, obtained for forces acting only on the right half of the model (L5 variant) were presented in Fig. 7. All patterns are obviously non-uniform with asymmetry in relation to the central plane of the model. Using both Stokes's approach and Carter's formula with higher value of

dilatational parameter $a = 1,7$, the minimum of particular indexes occurs in the right, more compressed part of the growth cartilage model. The same results were also obtained for Carter's formula with lower value of dilatational parameter ($a = 0,5$) and using thinner layer of cartilage (M5–M8 models). Slightly more ambiguous results were obtained using Carter's approach for models with thicker cartilage layer (M1–M4). Patterns of GI_1^S index show, in these cases, a variability in the axial (proximal-distal) direction. The minimum is slightly shifted to the left (into unloaded part of the cartilage) in the central part of the cartilage layer.

Patterns of the GI_1^M index, expressing the application of Carter's theory to multiple loading cases with its variable direction, were presented in Fig. 8. An analysis of these results allows finding the greatest diversification of patterns obtained for particular spatial model configuration. Special attention should be paid to the fact that patterns of GI_1^M index, calculated

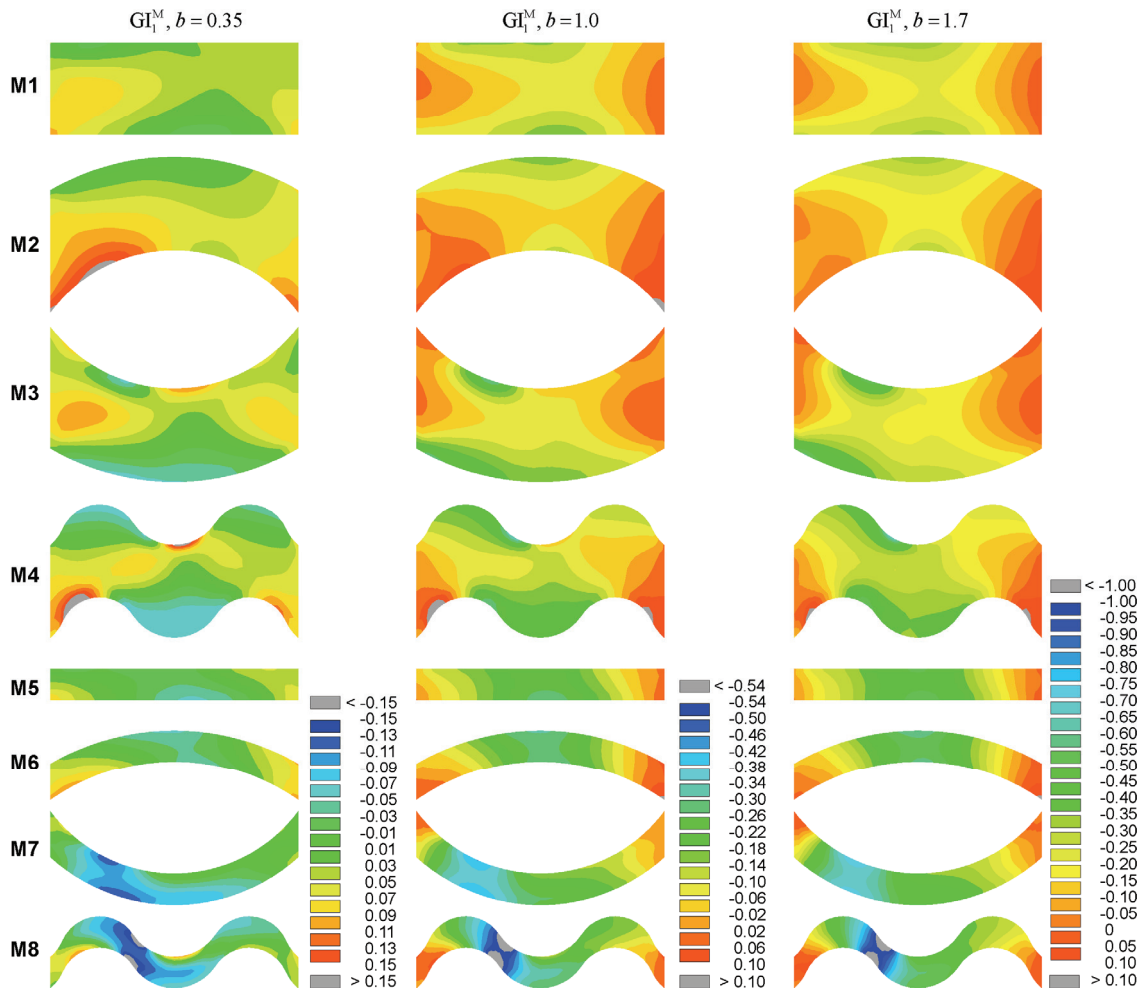


Fig. 8. Patterns of the indexes GI_1^M [MPa] calculated for $b = 0.35, 1.0$ and 1.7 for models M1–M8 under cycle of loading composed of L1, L2 and L3 cases. View on the axial cross-section of the growth cartilage model; bone and fibrous parts were intentionally omitted

for weighting parameter $b = 0.35$, show the region of the most intensive mechanical stimulation of bone growth (greater index value) situated inside the left half of the cartilage model. It means that Carter's formula is able to generate results indicating a more intensive bone growth in regions to which the relatively greater compressive loadings were directed. Such effect could be found, however, only for models with thicker cartilage layer (M1–M4). In such cases, a high variability of the GI_1^M in the axial (proximal-distal) direction occurs. For concave (M3) and wavy (M4) models, the regions of the greatest and the lowest value of GI_1^M index are situated very close to each other. Symptomatic is, however, the fact that results obtained for thinner cartilage layer (M5–M8) are much different to those obtained for thicker models (M1–M4). For concave and wavy shape of the thinner cartilage models (M7–M8) the well visible minimum of GI_1^M is localised in the left, more loaded, part of the cartilage. An increase of the weighting parameter

value ($b = 1; b = 1.7$) does not lead to any important changes in obtained results. Patterns plotted for models with thinner cartilage layer (M5–M8) are almost identical to those obtained for $b = 0.35$. In the case of thicker layer of cartilage (M1–M4), patterns of GI_1^M index calculated for $b = 1$ and $b = 1.7$ become more symmetrical with minimum located close to the central plane of the model.

4. Discussion

In view of the current knowledge, the fact that mechanical loadings can influence endochondral bone growth remains undisputed. The possibility of bone deformity as a consequence of improper biomechanical conditions seems to be very well documented, too. However, the question whether the current level of knowledge allows us to understand,

properly describe and simulate such phenomenon remains open.

One of the main aims of the present research was to analyse the interrelation between loading direction and the growth cartilage shape, in the context of their influence on mechanical stimulation of bone growth. The first general conclusion arising from the results presented above is the fact that loading direction has an important influence on the pattern of mechanical stimuli occurring within the volume of growth cartilage. Such effect is visible both for Carter's and Stokes's approaches. When loading direction is sloped towards the growth plate axis, the patterns of particular indexes describing mechanical stimulation of bone growth become asymmetrical in relation to the central plane of the model. In view of the earlier results, it is well known that Stokes's model is able to quite precisely describe the mechanical influences on the bone development in such situations when axial loadings are of predominant importance [5], [6], [24], [25]. Poor ability of this model to encompass more complex loading conditions was indicated by Lin et al. [33]. The present research has shown that the use of Stokes's model pattern of mechanical stimulation of bone growth depends on loading direction. The sensitivity of Stokes's approach on a spatial configuration of the growth plate – loading condition model is, however, worse compared to Carter's approach. In general, the direction of loadings almost fully determines the localization of the zone where mechanical stimulation of bone growth has a reduced level. The shape of cartilage, in this approach, is of secondary importance. Significant remains, however, the localization of the place where the loading acts on the growth cartilage. This statement can be easily understood by analysing the patterns of GI_2 index obtained for models with convex (M6) and concave (M7) layer of a cartilage (Fig. 6). The localizations of the index GI_2 minimum at the convex model, for both inclined loading variants (L3 and L4), are shifted to the right compared to results obtained for concave model. This effect is probably caused by different localization of the place where the line of resultant forces crosses the growth cartilage. A quite similar result could be observed also for the GI_1^S index. It is visible when comparing patterns obtained for models with thinner cartilage layer (M5–M8) loaded by uniform pressure (L1) and by loading hyperbolic pattern (L2) presented in Fig. 3. The non-uniformity of particular patterns of both GI_1^S and GI_2 indexes obtained for hyperbolic loadings is much higher compared to results obtained for uniform pressure, with the exception of the concave model (M7). Probably, the higher distance from

the proximal surface of the model, where the forces were applied, to the concave layer of the cartilage blurred the more concentrated loading. It should be emphasised that real bone elements have much more complex shapes than the models used in the present research. In such situation, mechanical loadings acting on the bone surface penetrate its tissues and generate the complex stress state within its volume. Loadings acting directly on the growth cartilage, in such situation, strongly depend on the structure of the whole bone element. The very simplified shape of the growth plate models used in the present research was not able to fully encompass all these aspects. The usefulness of the custom-made models in analyses, especially performed for individual patients, is clear visible in such situations.

Results obtained using Carter's methodology (Figs. 4 and 5), in general, are quite similar to those obtained using Stokes's model (Fig 6). The main difference is, however, much higher sensitivity of Carter's approach to interrelation between geometrical structure of the growth plate and loading direction. Growth index GI_1^S , being the weighted sum of octahedral shear stresses and hydrostatic stresses, is able to encompass various stress states resulting not only from loading direction but also from the shape of growth cartilage. Using greater value of the weighting parameter a , index GI_1^S becomes less sensitive to spatial structure of the model. An earlier analysis [7] showed that, for $a = 1.7$, results obtained using Carter's approach are highly correlated to results calculated on the basis of Stokes's model. Presented analyses could confirm the previous statement, but now this conclusion means that GI_1^S index calculated using $a = 1.7$ seems to be less sensitive to the cartilage shape, just like index GI_2 . Symptomatic is that Ribble et al. [34], who were analysing the deformity of femoral bone in the case of cerebral palsy, used only shear stresses acting in the growth cartilage region. All these facts should indicate the urgent necessity of further research to find the rationale of relation between octahedral shear stresses and hydrostatic stresses in Carter's model.

Shefelbine and Carter, in simulation of proximal part of the children femur, used convex shape of growth front model [15]. However, the whole epiphysis placed over growth zone was modelled as a cartilage. Having simulated the cerebral palsy conditions, they used loadings directed more laterally than in normal conditions. As a result, lower stimulation of bone growth in the medial part (less loaded) of the growth plate was obtained. This effect is in contrast to the patterns of GI_1^S index obtained in the present re-

search, where the region of lower mechanical stimulation in general follows the direction of maximum loadings. However, the pattern of GI_1^M obtained for M2 model presents a lower stimulation of bone growth in the region placed out of loading direction. Such effect was not visible, however, for thinner layer of growth cartilage (M6 model), which better reflects the conditions occurring in older children. Further research is necessary to find whether such result means the weakness of the modelling methodology or the response of the growth plate to the same loading conditions really changes with age of the patient. Without any doubt, however, it is possible to state that Carter's approach applied to multidirectional loading analysis (index GI_1^M) is very sensitive to the shape and thickness of a cartilage layer.

Results obtained for loadings acting only on a half of the model (L5) were crucial to find the answer whether any of analysed models are able to explain the more intensive bone growth occurring in a more compressed part of the bone in mild coxa vara or mild genu varum [18], [19]. Unfortunately, obtained results are rather dissatisfying. Both Carter's and Stokes's approaches did not allow to find higher value of mechanical stimulation of bone growth in the compressed part of the cartilage. Obviously, it is possible that another, more sophisticated loading condition or geometrical model of the growth plate, could allow obtaining any new results. An expectation could bring the fact that, for thicker cartilage layer, the zone of minimal stimulation of bone growth is slightly shifted out of loading zone. It can be supposed, however, that a new formula expressing the mechanical stimulation of bone growth must be elaborated to overcome this problem. Usefulness of the multi-phases, poroelastic or viscoelastic models should be checked, too [35], [36], [37], [38]. Both Stokes's and Carter's models do not take into consideration, however, the cyclic loading of the constant direction, whilst such conditions could be present in numerous clinical situations. There are studies indicating that static compression of cartilage decreases biosynthesis within the cartilage [15], [20], [39], [40]. Obtained patterns of indexes GI_1^S and GI_2 are consistent with these results. However, cyclic hydrostatic pressure could increase biosynthetic activity [15], [20], [41]. This effect could be responsible for bone deformity in cerebral palsy and more intensive bone growth occurring in a more compressed part of the bone in mild coxa vara or mild genu varum. At present, both Carter's and Stokes's approach do not allow to simulate such phenomenon.

To sum up, it is possible to state that spatial structure of the growth plate – the loading conditions sys-

tem has an important influence on the mechanical stimulation of endochondral bone growth. The present research indicated the necessity of realistic modelling of the growing bone geometrical structure, including elaboration of the custom-made models. Carter's approach seems to be more sensitive to interrelation between growth plate geometrical structure and loading direction, compared to Stokes's model. Further research is necessary to elaborate the optimal value of dilatational parameter occurring in the Carter's formula. It is clear, however, that for greater values ($a > 0.5$) the sensitivity of this approach to geometrical properties of the model decreases. Taking into account of the cyclic loading of a constant direction should be also the subject of further research in the hope that in this way it will be possible to overcome still existing problems with the explanation of numerous clinical phenomena.

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