# Motion planning for mobile surgery assistant

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The paper presents a method of motion planning for a mobile manipulator acting as a helper providing the necessary tools or a surgery assistant carrying out pre-planned procedures. Mobility of this system makes it possible to reach the position which will give optimal access to the operating field. The path of the end-effector, determined during operation pre-planning, is defined as a curve parameterised by any scaling parameter, the reference trajectory of a mobile platform is not needed. The motion of the mobile manipulator is planned in order to maximise the manipulability measure, thus to avoid manipulator singularities. The method is based on a penalty function approach and a redundancy resolution at the acceleration level. Constraints connected with the existence of mechanical limits for a given manipulator configuration, collision avoidance conditions and control constraints are considered. A computer example involving a mobile manipulator consisting of a nonholonomic platform (2,0) class and a 3 DOF RPR type holonomic manipulator operating in a three-dimensional task space is also presented.

*Key words*: mobile manipulator, path following, trajectory planning, surgery assistant

## 1. Introduction

Robotics is a branch of engineering that deals with designing and controlling mechanisms for imitating manipulation and locomotion features of a human being. One of the most important problems in practical application of robots is to convert high level specification of the task into low level motion description of mechanisms. Motion planning is the area of robotics that provides techniques to solve tasks of this type.

First robots were designed for industrial applications, but currently they are used in many fields, including medicine. Medical robotics has many applications, such as robot assisted surgery [15], [23], prosthesis construction [24], [14] or designing devices used for assisting people with disabilities. The growing impact of robotics on the field of medicine is a consequence of the fact that robot has several abilities superior to human beings, such as precision, geometrical accuracy and repeatability. Robotic surgery seems to be particularly interesting field. Two types of

robotic systems are currently used in surgery: telemanipulators and navigation robots [15]. Telemanipulators allow the motions of surgeon to be transferred into motions of surgical tool ensuring the hand tremor elimination and enabling operator movement scaling. Examples of such systems are Zeus, da Vinci [23] or Polish prototype surgery arm Robin Heart. Navigation robots operate autonomously and they can be used to perform surgery procedures according to operation pre-planning [27], [10]. This type of robotic systems is currently used in radiosurgery (CyberKnife, Accuray Inc.) or orthopedics (Robodoc, Curexo Tech. Corp.).

In this paper, a method of motion planning for a mobile manipulator acting as an assistant surgeon is presented. A mobile manipulator is a robotic system consisting of a mobile platform and a manipulator mounted on top. By combining the mobility of the platform with the dexterous capability of the manipulator, the system gains kinematic redundancy. The redundant degrees of freedom render it possible to accomplish complex tasks in complicated workspaces

with obstacles. This system can be used both as a helper providing the necessary tools or surgery assistant carrying out pre-planned procedures. In this case, the mobile platform enables the position to be reached which will give optimal access to the operating field. Some of the medical procedures require tracing of the pre-planned path, i.e., the robot has to move the end-effector along the curve specified by operator [22]. The task of this type is, for example, hip replacement, in which the robot has to mill implant cavity moving the tool at a particular position along the desired path and then place the implant there. From a technical point of view such tasks are parts assembly (pick and place) and handling and stacking operations. To perform this type of task mobile robot can use, if needed, the wheeled platform or manipulator mounted on its top and the reference path for the platform is not required. The main problem associated with the motion planning of such a robot is to determine the controls (wheel and joint torques) which are necessary to accomplish desired motion. The presented paper is focused on the above mentioned problem.

Introduction of mobile robots has made it difficult to solve the motion planning task. The mobile platform has to roll without slipping, which consequently limits the motion of the mobile manipulator to the allowed instantaneous velocities and positions. Moreover, the platform introduces additional degrees of freedom and the whole robot becomes redundant even if its manipulator is non-redundant. These additional degrees of freedom increase the difficulty of solving the problem due to the ambiguity of the solution. Further, from practical point of view, it is important to ensure high dexterity of the robot during the execution of its task. This can be achieved by ensuring a high manipulability of the robot's manipulator. This property keeps the robot away from its singular configurations, and decreases the probability of reconfiguration of the whole mobile robot during the execution of the task. Most of existing research addresses the problem using only kinematic equations of the mobile manipulator. Due to the method of solving the robot task presented in this paper these approaches will not be discussed. The dynamic interactions between the manipulator and the mobile platform have been studied by Yamamoto and Yun [25]. Desai and Kumar [3] have solved the motion planning problem for multiple mobile manipulators, taking their dynamics and obstacles in the workspace into consideration. In work [13], an input–output feedback linearisation technique is developed to move the mobile platform and the manipulator. The solution at the torque/force level was presented in [8], [9], neverthe-

less control constraints are not taken into consideration.

Some methods require knowledge of the mobile platform and end-effector reference trajectories. Egerstedt and Hu [4] proposed error-feedback control algorithms in which the trajectory for the mobile platform is planned in such a way that the end-effector trajectory is within reach of the manipulator arm. Chung et al. [2] designed two independent controllers for the mobile platform and the manipulator, nevertheless the obtained trajectory is not optimal in any sense. Liu and Lewis [11] developed a decentralised robust controller by considering the mobile platform and the manipulator arm as two subsystems. Mazur [12] presented the solution to the path following control problem decomposed into two separated tasks defined for the end-effector of the manipulator and the nonholonomic platform.

This paper presents a motion planning method for applications where only the knowledge of the end-effector path is needed. The path of the end-effector is a curve that can be parameterised by any scaling parameter. The robot's trajectory is planned in such a manner as to maximise the manipulability measure to avoid manipulator singularities. The proposed method guarantees finding continuous controls which do not exceed their limits and ensures the fulfilment of the constraints imposed by the robot's mechanical limits and collision avoidance conditions.

# 2. Materials and methods

## 2.1. Problem formulation

A mobile manipulator composed of a nonholonomic platform and holonomic manipulator is considered to plan a trajectory. It is described by the vector of generalised coordinates

$$\mathbf{q} = (\mathbf{q}^p \quad \mathbf{q}^r)^T, \tag{1}$$

where $\mathbf{q} \in \mathfrak{R}^n$ – vector of the generalised coordinates of the whole mobile manipulator; $\mathbf{q}^p \in \mathfrak{R}^p$ – vector of the coordinates of the nonholonomic platform; $\mathbf{q}^r \in \mathfrak{R}^r$ – vector of joints coordinates of the holonomic manipulator; $p, r$ – the number of coordinates describing the nonholonomic platform and holonomic manipulator; $n = p + r$.

The platform motion is subject to nonholonomic constraints described in the Pfaffian form

$$\widetilde{\mathbf{A}}(\mathbf{q}^p)\,\dot{\mathbf{q}}^p = \mathbf{0}\,, \qquad (2)$$

where $\widetilde{\mathbf{A}}(\mathbf{q}^p)$ – $(k \times p)$ full rank matrix; $k$ – the number of independent nonholonomic constraints.

To determine the mobile manipulator motion, it is also necessary to know the robot dynamic equations, given in the general form as

$$\mathbf{M}(\mathbf{q})\,\ddot{\mathbf{q}} + \mathbf{F}(\mathbf{q},\dot{\mathbf{q}}) + \mathbf{A}^T\boldsymbol{\lambda} = \mathbf{B}\mathbf{u}\,, \qquad (3)$$

where $\mathbf{M}(\mathbf{q})$ – $(n \times n)$ positive definite inertia matrix, $\mathbf{F} = \mathbf{F}(\mathbf{q},\dot{\mathbf{q}})$ – $n$-dimensional vector representing Coriolis, centrifugal, viscous, Coulomb friction and gravity forces; $\mathbf{A}(\mathbf{q}) = [\widetilde{\mathbf{A}}(\mathbf{q}^p)\quad \mathbf{0}_{k \times r}]$; $\mathbf{0}_{k \times r}$ – $(k \times r)$ zero matrix; $\boldsymbol{\lambda}$ – $k$-dimensional vector of the Lagrange multipliers corresponding to nonholonomic constraints (2); $\mathbf{u}$ – $(n - k)$ -dimensional vector of controls (torques/forces); $\mathbf{B}$ – $(n \times (n-k))$ full rank matrix (by definition) describing which state variables of the mobile manipulator are directly driven by the actuators.

The task of the mobile manipulator is to follow a path given in a parametric form

$$\mathbf{f}(\mathbf{q}) - \boldsymbol{\varphi}(s) = \mathbf{0}\,, \qquad (4)$$

where $\mathbf{f} : \Re^n \to \Re^m$ – $m$-dimensional mapping describing the kinematic equations of the robot; $m$ – dimension of the workspace; $\mathbf{q} = \mathbf{q}(t)$; $t \in [0,T]$; $T$ – unknown time of performing the task; $\boldsymbol{\varphi}(s)$ – the function representing a path of the end-effector; $s$ – the scaling parameter which depends on time $t$, i.e. $s = s(t)$; $s \in [0, s_{\max}]$.

The assumption that the manipulator at the initial moment of motion is in the acceptable (nonsingular, collision free) configuration $\mathbf{q}_0 = \mathbf{q}(0)$ is taken into account. If initial configuration is singular, there are two approaches to solve this problem. The first one requires performing a reconfiguration process, involving movement of the platform without changing position of the end-effector in the workspace. The second solution is to use robust inverse method, which allows the inverse of Jacobian matrix to be determined.

At the initial moment of the motion the end-effector of the manipulator should be at the initial location of the path and at the final moment the entire path should be traced

$$\mathbf{f}(\mathbf{q}_0) - \boldsymbol{\varphi}(0) = \mathbf{0}\,,\ \ s(T) = s_{\max}\,. \qquad (5)$$

From the practical point of view, it is natural to assume that at the initial and final moments of the task

performance, the velocities of the manipulator equal zero

$$\dot{\mathbf{q}}(0) = \mathbf{0}\,,\quad \dot{s}(0) = 0\,,\quad \dot{\mathbf{q}}(T) = \mathbf{0}\,,\quad \dot{s}(T) = 0\,. \quad (6)$$

Constraints connected with the existence of mechanical limits and collision avoidance conditions can be described in the following general form

$$\forall t \in [0,T] \quad \{\beta_i(\mathbf{q}(t)) \geq 0\}\,,\ i = 1:L\,, \qquad (7)$$

where $\{\beta_i(\mathbf{q}(t)) \geq 0\}$ – the set of conditions with scalar functions $\beta_i(\ )$ which involve the fulfilment of the mechanical and collision avoidance constraints (see Numerical example section for exemplary form of these functions); $L$ – the number of inequality constraints.

Additionally, the constant constraints of control are also assumed

$$\mathbf{u}_{\min} \leq \mathbf{u} \leq \mathbf{u}_{\max}\,, \qquad (8)$$

where $\mathbf{u}_{\min}$; $\mathbf{u}_{\max}$ – lower and upper limits on $\mathbf{u}$, respectively.

In practice, it is important for the configuration of manipulator joints to be far away from singular configurations. The assumption corresponds to a search for the trajectory for which the instantaneous performance index is minimized (maximising the manipulability measure [26] generalised in the sense of mobile manipulators) at each time instant $t \in [0,T]$.

$$\widetilde{I}(\mathbf{q}) = -\det(\mathbf{j}\,\mathbf{j}^T)^{1/2}\,, \qquad (9)$$

where $\mathbf{j} = \partial\mathbf{f}(\mathbf{q})/\partial\mathbf{q}$ – $(m \times n)$ dimensional Jacobian matrix of the mobile robot.

The above manipulability measure is only an example, the method presented may be used with other indexes of this type (e.g., measure based on the eccentricity of the ellipsoid [1]).

Dependences (4)–(9) formulate the robotic task as an optimal control problem expressed in rather general terms. The next section will present an approach that renders it possible to solve the above optimisation problem.

## 2.2. Solution

To solve the problem (4)–(9), an approximate implementation of inequality constraints (7) is accepted in the work. The solution proposed is the explication of the trajectory planning method for stationary manipulators described in [16], [18], [21] and it uses

penalty functions approach [5], [7]. This method results in the following function to be minimized

$$I(\mathbf{q}) = \widetilde{I}(\mathbf{q}) + \sum_{i=0}^{L} \kappa_i(\beta_i(\mathbf{q})), \tag{10}$$

where $\kappa_i(\ )$ – the penalty function which associates a penalty with a violation of a constraint.

The solution of the task was divided into two parts: the search for an optimal configuration for a given path point $\varphi(s)$ and trajectory planning from the initial path point to the end point. The task of searching for an optimal configuration for a given path point involves minimisation of the performance index (10) subject to equality constraints (2), (4). Based on the solution presented in [16], [18], [21] and using the necessary condition for the minimum of the performance index $I(\mathbf{q})$ [6], this task can be described as

$$\widetilde{\mathbf{E}}(\mathbf{q},\dot{\mathbf{q}},s) = \begin{pmatrix} \mathbf{f}(\mathbf{q}) - \boldsymbol{\varphi}(s) \\ [((\mathbf{J}^R(\mathbf{q}))^{-1}\mathbf{J}^F(\mathbf{q}))^T \quad -\mathbf{1}_{n-m-k}]\,\mathbf{I}_q(\mathbf{q}) \\ \mathbf{A}(\mathbf{q})\,\dot{\mathbf{q}} \end{pmatrix} = \mathbf{0}, \tag{11}$$

where $\mathbf{J}(\mathbf{q}) = [(\mathbf{j}(\mathbf{q}))^T \quad (\mathbf{A}(\mathbf{q}))^T]^T$ – $((m+k)\times n)$ dimensional full rank matrix, $(m+k)<n$; $\mathbf{J}^R(\mathbf{q})$ – $((m+k)\times(m+k))$ dimensional matrix constructed from $(m+k)$ linear independent columns of $\mathbf{J}$; $\mathbf{J}^F(\mathbf{q})$ – $((m+k)\times(n-m-k))$ dimensional matrix obtained by excluding $\mathbf{J}^R$ from $\mathbf{J}$; $\mathbf{1}_{n-m-k}$ – $(n-m-k)\times(n-m-k)$ identity matrix; $\mathbf{I}_q(\mathbf{q}) = \partial I/\partial \mathbf{q}$.

Mapping $\widetilde{\mathbf{E}}(\mathbf{q},\dot{\mathbf{q}},s)$ may be interpreted as some measure of error between a current configuration $\mathbf{q}$ and acceptable configuration for a given path point $s$. Dependence (11) presents a system of $n$ nonlinear equations which fully specify the configuration $\mathbf{q}$ at a given path point. To find the trajectory of the manipulator $\mathbf{q}(t)$ and the path parameterisation $s(t)$ Equation (11) has been extended by an additional element responsible for tracing the entire path

$$\mathbf{E}(\mathbf{q},\dot{\mathbf{q}},s) = \begin{pmatrix} \widetilde{\mathbf{E}}(\mathbf{q},\dot{\mathbf{q}},s) \\ s - s_{\max} \end{pmatrix} = \begin{pmatrix} \mathbf{E}^I(\mathbf{q},s) \\ \mathbf{E}^{II}(\mathbf{q},\dot{\mathbf{q}}) \end{pmatrix} \tag{12}$$

where

$$\mathbf{E}^I(\mathbf{q},s) = \begin{pmatrix} \mathbf{f}(\mathbf{q}) - \boldsymbol{\varphi}(s) \\ [((\mathbf{J}^R(\mathbf{q}))^{-1}\mathbf{J}^F(\mathbf{q}))^T \quad -\mathbf{1}_{n-m-k}]\,\mathbf{I}_q(\mathbf{q}) \\ s - s_{\max} \end{pmatrix},$$

$$\mathbf{E}^{II}(\mathbf{q},\dot{\mathbf{q}}) = \mathbf{A}(\mathbf{q})\,\dot{\mathbf{q}}.$$

Equation (12) describes the path following error and it is used to obtain the trajectory $\mathbf{q}(t)$ and path parameterisation $s(t)$ parameterised by gain coefficients $\Lambda^I_{V,i}$, $\Lambda^I_{L,i}$ and $\Lambda^{II}_{L,i}$. The trajectory determined using (12) minimises the performance index (9) and fulfils kinematic constraints (7). Then, using dynamic model of the robot, the controls necessary to accomplish trajectory $\mathbf{q}(t)$ will be calculated. At this stage the control constraints will be taken into account and if calculated values of controls are close to limitations imposed the values of gain coefficients $\Lambda^I_{V,i}$ and $\Lambda^I_{L,i}$ will be modified in such a way as to fulfil these constraints.

To solve the trajectory planning problem, the following system of equations is introduced

$$\begin{aligned} \ddot{\mathbf{E}}^I + \boldsymbol{\Lambda}^I_V\,\dot{\mathbf{E}}^I + \boldsymbol{\Lambda}^I_L\,\mathbf{E}^I &= \mathbf{0}, \\ \dot{\mathbf{E}}^{II} + \boldsymbol{\Lambda}^{II}_L\,\mathbf{E}^{II} &= \mathbf{0}, \end{aligned} \tag{13}$$

where $\boldsymbol{\Lambda}^I_V = \mathrm{diag}(\Lambda^I_{V,1},...,\Lambda^I_{V,n-k+1})$, $\boldsymbol{\Lambda}^I_L = \mathrm{diag}(\Lambda^I_{L,1},...,\Lambda^I_{L,n-k+1})$ – $((n-k+1)\times(n-k+1))$ diagonal matrices with positive coefficients $\Lambda^I_{V,i}$, $\Lambda^I_{L,i}$, ensuring the stability of the first equation; $\boldsymbol{\Lambda}^{II}_L = \mathrm{diag}(\Lambda^{II}_{L,1},...,\Lambda^{II}_{L,k})$ – $(k\times k)$ diagonal matrix with positive coefficients $\Lambda^{II}_{L,i}$, ensuring the stability of the second equation.

This is a system of homogeneous differential equations with constant coefficients. In order to solve these equations $(2n-k+2)$ consistent dependencies should be given. These dependencies can be obtained, taking into account initial conditions (6), from (12) for $t = 0$.

Using the Lyapunov stability theory it is possible to show that the above form of the system of differential equations (13) ensures that its solution is asymptotically stable for positive coefficients $\Lambda^I_{V,i}$, $\Lambda^I_{L,i}$ and $\Lambda^{II}_{L,i}$. Lower equation (13) is trivial asymptotically stable for $\Lambda^{II}_{L,i} > 0$. In order to show the stability of the upper equation (13), let us consider the Lyapunov function candidate of the following form

$$V(\mathbf{E}^I,\dot{\mathbf{E}}^I) = \frac{1}{2}\langle \dot{\mathbf{E}}^I, \dot{\mathbf{E}}^I \rangle + \frac{1}{2}\langle \mathbf{E}^I, \boldsymbol{\Lambda}^I_L \mathbf{E}^I \rangle$$

where $<\cdot>$ – the scalar product of vectors.

It is easy to see that $V(\mathbf{E}^I, \dot{\mathbf{E}}^I)$ is non-negative for positive coefficients $\Lambda_{L,i}^I$. After differentiating and substituting $\ddot{\mathbf{E}}^I$ determined from (13), $\dot{V}$ can be written as

$$\dot{V}(\mathbf{E}^I, \dot{\mathbf{E}}^I) = -\langle \dot{\mathbf{E}}^I, \boldsymbol{\Lambda}_V^I \dot{\mathbf{E}}^I \rangle$$

As one can see, for positive coefficients $\Lambda_{V,i}^I$, $\dot{V}(\mathbf{E}^I, \dot{\mathbf{E}}^I) \leq 0$. It can be shown that $\dot{V}(\mathbf{E}^I, \dot{\mathbf{E}}^I) = 0$ for $\dot{\mathbf{E}}^I = \mathbf{0}$. Additionally, if $\dot{\mathbf{E}}^I = \mathbf{0}$ then $\ddot{\mathbf{E}}^I = \mathbf{0}$ and from upper equation (13) it follows that $\mathbf{E}^I = \mathbf{0}$. Based on the La Salle invariant principle it can be concluded that solution of the system of differential equations (13) is asymptotically stable for positive coefficients $\Lambda_{V,i}^I$, $\Lambda_{L,i}^I$ and $\Lambda_{L,i}^{II}$. This property implies fulfilment of conditions (5) and (6) associated with the final moment of the task perform ance, e.g. $\dot{\mathbf{q}}(T) \rightarrow \mathbf{0}$; $s(T) \rightarrow s_{\max}$; $\dot{s}(T) \rightarrow 0$ as $T \rightarrow \infty$.

The above analysis has been carried out on the assumption that $\Lambda_{V,i}^I$, $\Lambda_{L,i}^I$ and $\Lambda_{L,i}^{II}$ are constant coefficients. However, these coefficients are used for scaling the trajectory of the mobile manipulator ensuring fulfilment of control constraints, so $\Lambda_{V,i}^I$, $\Lambda_{L,i}^I$, $\Lambda_{L,i}^{II}$ can vary with time. Due to the fact that rapid changes of gain coefficients lead to large values of controls it is practically reasonable to assume that $\Lambda_{V,i}^I$, $\Lambda_{L,i}^I$, $\Lambda_{L,i}^{II}$ are slowly varying functions of time ($\dot{\Lambda}_{V,i}^I \cong 0$, $\dot{\Lambda}_{L,i}^I \cong 0$, $\dot{\Lambda}_{L,i}^{II} \cong 0$), and in such a case the above analysis holds true.

In addition, if the gain coefficients $\Lambda_{V,i}^I$ and $\Lambda_{L,i}^I$ satisfy the following inequalities

$$\Lambda_{V,i}^I > 2\sqrt{\Lambda_{L,i}^I}, \qquad (14)$$

the function $\mathbf{E}^I(\mathbf{q}, s)$ becomes also a strictly monotonic function. For the initial configuration $\mathbf{q}_0$ satisfying equation (11) this property forces fulfilment of mechanical constraints, collision avoidance conditions and singularity-free robotic motions along a given path (4).

The key moment of the aforementioned analysis was the assumption of the full rank of matrix $\mathbf{J}^R$. It is the consequence of the following reasoning. Let us assume that the mobile manipulator consists of a platform of (2,0) class (as in Numerical example section). In this case matrix $\mathbf{J}$ can be written in the following form

$$\mathbf{J} = \begin{bmatrix} \mathbf{j}^p(\mathbf{q}) & \mathbf{j}^r(\mathbf{q}) \\ \widetilde{\mathbf{A}}(\mathbf{q}) & \mathbf{0}_{k \times r} \end{bmatrix},$$

where $\mathbf{j}^p(\mathbf{q}) = \partial \mathbf{f}(\mathbf{q}) / \partial \mathbf{q}^p$, $\mathbf{j}^r(\mathbf{q}) = \partial \mathbf{f}(\mathbf{q}) / \partial \mathbf{q}^r$, $\mathbf{0}_{k \times r}$ – $(k \times r)$ zero matrix.

The matrix $\mathbf{J}$ will be full rank matrix provided that $\mathbf{j}^r(\mathbf{q})$ and $\widetilde{\mathbf{A}}(\mathbf{q})$ are full rank matrices. $\widetilde{\mathbf{A}}(\mathbf{q})$ is a full rank matrix by definition. The matrix $\mathbf{j}^r(\mathbf{q})$ may be determined as

$$\mathbf{j}^r(\mathbf{q}) = \mathbf{R}(\theta) \frac{\partial \mathbf{f}^r(\mathbf{q})}{\partial \mathbf{q}^r},$$

where $\theta$ – the platform orientation, $\mathbf{R}(\theta)$ – the rotation matrix which specifies orientation of the mobile platform in the base system, $\mathbf{f}^r(\mathbf{q}^r)$ – kinematic model of holonomic manipulator.

Determinant of matrix $\mathbf{j}^r(\mathbf{q})$ is equal to determinant of matrix $\partial \mathbf{f}^r(\mathbf{q}) / \partial \mathbf{q}^r$. In order to ensure the full rank of the matrix $\partial \mathbf{f}^r(\mathbf{q}) / \partial \mathbf{q}^r$ the manipulability measure is maximised.

In order to obtain the trajectory of the manipulator $\mathbf{q}(t)$ and the path parameterisation $s(t)$, after simple calculations equation (13) can be rewritten as

$$\ddot{s} = -(\widehat{\Lambda}_V^I \dot{s} + \widehat{\Lambda}_L^I (s - s_{\max})), \qquad (15)$$

$$\begin{bmatrix} \breve{\mathbf{E}}_q^I \\ \mathbf{E}_{\dot{q}}^{II} \end{bmatrix} \ddot{\mathbf{q}} = -\begin{bmatrix} \dfrac{d}{dt}(\breve{\mathbf{E}}_q^I)\dot{\mathbf{q}} + \breve{\mathbf{E}}_s^I \ddot{s} + \dfrac{d}{dt}(\breve{\mathbf{E}}_s^I)\dot{s} + \breve{\boldsymbol{\Lambda}}_V^I(\breve{\mathbf{E}}_q^I \dot{\mathbf{q}} + \breve{\mathbf{E}}_s^I \dot{s}) + \breve{\boldsymbol{\Lambda}}_L^I \breve{\mathbf{E}}^I \\ \mathbf{E}_q^{II} \dot{\mathbf{q}} + \boldsymbol{\Lambda}_L^{II} \mathbf{E}^{II} \end{bmatrix}, \qquad (16)$$

where $\breve{\mathbf{E}}^I$ – the dependence obtained by excluding $(s - s_{\max})$ from $\mathbf{E}^I$; $\breve{\mathbf{E}}_q^I = \dfrac{\partial \breve{\mathbf{E}}^I(\mathbf{q}, s)}{\partial \mathbf{q}}$; $\breve{\mathbf{E}}_s^I = \dfrac{\partial \breve{\mathbf{E}}^I(\mathbf{q}, s)}{\partial s}$; $\breve{\boldsymbol{\Lambda}}_V^I = \text{diag}(\Lambda_{V,1}^I, ..., \Lambda_{V,n-k}^I)$; $\breve{\boldsymbol{\Lambda}}_L^I = \text{diag}(\Lambda_{L,1}^I, ..., \Lambda_{L,n-k}^I)$;

$$\mathbf{E}_q^{II} = \frac{\partial \mathbf{E}^{II}(\mathbf{q}, \dot{\mathbf{q}})}{\partial \mathbf{q}}; \quad \mathbf{E}_{\dot{q}}^{II} = \frac{\partial \mathbf{E}^{II}(\mathbf{q}, \dot{\mathbf{q}})}{\partial \dot{\mathbf{q}}}; \quad \widehat{\Lambda}_V^I = \Lambda_{V, n-k+1}^I;$$

$$\widehat{\Lambda}_L^I = \Lambda_{L, n-k+1}^I.$$

The mobile manipulator trajectory is the solution of the system of second order differential equations (15)–(16). To determine values of controls which are required to realise the trajectory it is necessary to transform the dynamic equation (3). For this purpose the nonholonomic constraints are expressed by an analytic driftless dynamic system

$$\dot{\mathbf{q}}^p = \widetilde{\mathbf{N}}(\mathbf{q}^p)\, v, \tag{17}$$

where $\widetilde{\mathbf{N}}(\mathbf{q}^p)$ – $(p \times (p-k))$ dimensional matrix; $v$ – $(p-k)$ dimensional vector denotes scaled angular velocities of the platform driving wheels.

Introducing the full rank matrix

$$\mathbf{N}(\mathbf{q}) = \begin{bmatrix} \widetilde{\mathbf{N}}(\mathbf{q}^p) & \mathbf{0}_{p \times r} \\ \mathbf{0}_{r \times (p-k)} & \mathbf{1}_{r \times r} \end{bmatrix} \tag{18}$$

and multiplying the dynamic equation (3) by $\mathbf{N}^T(\mathbf{q})$, noting that $\widetilde{\mathbf{A}}(\mathbf{q}^p)\, \widetilde{\mathbf{N}}(\mathbf{q}^p) = \mathbf{0}$, we obtain equation that allows determination of controls for a current trajectory

$$\mathbf{N}^T(\mathbf{q})\mathbf{M}(\mathbf{q})\, \ddot{\mathbf{q}} + \mathbf{N}^T(\mathbf{q})\mathbf{F}(\mathbf{q}, \dot{\mathbf{q}}) = \mathbf{N}^T(\mathbf{q})\mathbf{B}\,\mathbf{u}. \tag{19}$$

In order to ensure fulfilment of constraints (8) an idea of trajectory scaling, presented in [17], [20], [21], is used. In this case it is suggested to modify values of coefficients $\widehat{\Lambda}_V^I$, $\widehat{\Lambda}_L^I$, $\check{\Lambda}_V^I$, $\check{\Lambda}_L^I$. It can be shown [19] that decreasing these parameters involves reducing control values which are necessary for the execution of the manipulators task. Finally, the solution of equations (15)–(16) with suitable gain coefficients gives an optimal trajectory satisfying path constraints (4), inequality constraints (7), control constraints (8) and boundary conditions (6).

# 3. Results

In the numerical example, a mobile manipulator consisting of a nonholonomic platform of (2, 0) class and a 3DOF RPR type holonomic manipulator working in a three-dimensional task space is considered. A kinematic scheme of the mobile manipulator is shown in Fig. 1.
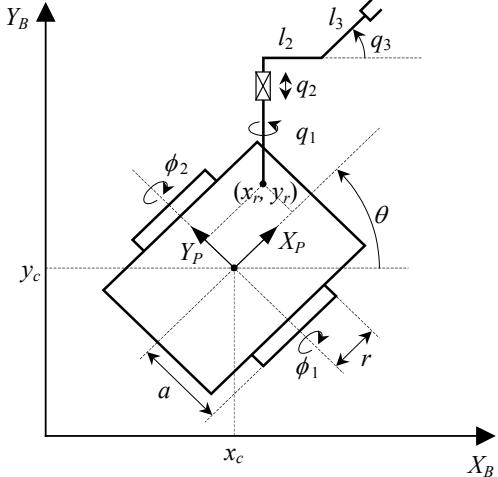


Fig. 1. Kinematic scheme of the mobile manipulator

The mobile manipulator is described by the vector of generalised coordinates

$$\mathbf{q} = (x_c, y_c, \theta, \phi_1, \phi_2, q_1, q_2, q_3)^T,$$

where $(x_c, y_c)$ – the platform centre location; $\theta$ – the platform orientation; $\phi_1, \phi_2$ – angles of driving wheels; $q_1, q_2, q_3$ – angles and offset of the manipulator joints.

The kinematic parameters of the mobile manipulator are given as (all physical values are expressed in the SI system): $l_2 = 0.3$, $l_3 = 0.2$, $a = 0.3$, $r = 0.075$. Respectively, the masses of the mobile manipulator elements amount to: $m_p = 94$, $m_2 = 20$, $m_3 = 20$ ($m_p$ is the total mass of the platform, $m_2, m_3$ are the masses of the manipulator arm).

The motion of the platform is subject to one holonomic and two nonholonomic constraints, so constraints (2) in this case take the following form

$$\dot{\theta} - \frac{r}{2a}\dot{\phi}_1 + \frac{r}{2a}\dot{\phi}_2 = 0,$$

$$\dot{x}_c - \frac{r}{2}\cos(\theta)\dot{\phi}_1 - \frac{r}{2}\cos(\theta)\dot{\phi}_2 = 0,$$

$$\dot{y}_c - \frac{r}{2}\sin(\theta)\dot{\phi}_1 - \frac{r}{2}\sin(\theta)\dot{\phi}_2 = 0,$$

where $r$ – the radius of driving wheels; $a$ – half-distance between the wheels.

The task of the manipulator is to trace a path described by the following equation

$$\boldsymbol{\varphi}(s) = (2s + 0.64, \quad 2s, \quad 0.5s + 0.86)^T, \quad s \in [0, 1].$$

Constraints (5), (7), (8) amount to

$$\mathbf{q}_0 = (0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 1.0, \pi/4)^T,$$

$$\mathbf{q}_{\min}^r = (-\pi, \ 0.5, \ -\pi/2)^T, \qquad \mathbf{q}_{\max}^r = (\pi, \ 2.0, \ 3\pi/2)^T,$$

$$\mathbf{u}_{\min} = (-10.0, -10.0, -2.0, 0.0, 0.0)^T, \qquad \mathbf{u}_{\max} = (10.0, 10.0, 2.0, 400.0, 15.0)^T.$$

The penalty function introduced in order to take into account mechanical and collision avoidance constraints (7) is taken as follows

$$\kappa_i(\beta_i(\mathbf{q})) = \begin{cases} \rho(\beta_i(\mathbf{q}) - \varepsilon_i)^2 & \text{for } \beta_i(\mathbf{q}) \le \varepsilon_i, \\ 0 & \text{otherwise,} \end{cases}$$

where $\rho$ – the positive coefficient determining the strength of penalty; $\varepsilon_i$ – the positive coefficient determining the threshold value which activates the $i$-th constraint.

To preserve mechanical limits scalar functions $\beta_i(\cdot)$ (7) are expressed as

$$\begin{aligned} \beta_i(\mathbf{q}) &= q_j - q_{\min j}, \\ \beta_i(\mathbf{q}) &= q_{\max j} - q_j, \end{aligned} \qquad j = 1 : r.$$

For collision avoidance purposes functions $\beta_i(\cdot)$ specify distances between the manipulator and obstacles. Due to the on-line trajectory planning, in this paper, a method of obstacle enlargement with reduction of the manipulator size is used. In this case, the collision test leads to checking a finite number of inequalities, so functions $\beta_i(\cdot)$ are expressed as

$$\beta_i(\mathbf{q}) = S_j(\mathbf{P}) - \delta,$$

where $S_j(\cdot)$ – equation of the $j$-th obstacle surface; $\mathbf{P} = (x, y, z)^T$ – point from the set of points which approximate the mobile manipulator; $\delta$ – small positive scalar, safety margin.

For sphere with radius $r_j$ and the centre point $(a_j, b_j, c_j)^T$, $S_j(\cdot)$ takes the following form

$$S_j(\mathbf{P}) = \{ \mathbf{P} = (x, y, z)^T : (x - a_j)^2 + (y - b_j)^2$$
$$+ (z - c_j)^2 - r_j^2 = 0 \},$$

and for cylinder with radius $r_j$, height $h_j$ and the centre point of its base placed at $(a_j, b_j, 0)^T$

$$S_j(\mathbf{P}) = \{ \mathbf{P} = (x, y, z)^T : (x - a_j)^2$$
$$+ (y - b_j)^2 - r_j^2 = 0, \ 0 \le z \le h_j \}.$$

Three cases of performing this task are considered. The first one is an end-effector motion along the path without control constraints (8) and collision avoidance constraints. In the second case the same task as in the first experiment is solved, but control constraints (8) are taken into account. In the third simulation, there is an addition of obstacles in the workspace.

The values of gain coefficients have been chosen in such a way as to show their influence on the fulfilment of constraints imposed on controls. The values of coefficients $\Lambda_{L,i}^I$, $\Lambda_{L,i}^{II}$ for the first simulations have been selected experimentally and set to $\Lambda_{L,i}^I = 1$, $\Lambda_{L,i}^{II} = 1$, the values of coefficients $\Lambda_{V,i}^I$ depend on inequality (14) and they have been set to $\Lambda_{V,i}^I = 2.1$. The mobile manipulator controls ($u_1, u_2$ – wheel torques, $u_3, u_5$ – joint torques and $u_4$ – joint force) obtained in the numerical simulations are shown in Fig. 2. For this solution the final time $T$ is 9.78 [$s$], and it can be seen that the controls exceed the assumed constraints.

The second simulation presents the solution of the same task as the first one but control constraints (8) are considered. For simplicity of numerical calculations, in second and third simulations, the values of gain coefficients have been assumed to be constant. $\Lambda_{L,6}^I = 0.1$ has been chosen in such a way as to ensure fulfilling control constraints both in the second and third simulation, $\Lambda_{V,6}^I = 0.66$ results from dependence (14). Such a reduction of gain coefficients results in significantly greater time of the task accomplishment and in this case it equals 23.66 [$s$], but the controls determined do not exceed the assumed constraints. Figure 3 presents the controls obtained in this simulation.

In the third simulation, there are two obstacles which may collide with the mobile manipulator. The first one is represented by a cylinder with radius 0.25 and height 2.0. The centre point of its base is placed at $(1.0, -0.05, 0.0)^T$. The second obstacle is a sphere with radius 0.25 and the centre point placed at $(1.5, 1.0, 0.7)^T$.

If collision avoidance conditions are not taken into account, the platform collides with the cylinder (the

sphere is above the motion plane) and the holonomic manipulator collides with sphere. To show these potential collisions, distances between the mobile manipulator from the second simulation and the centres of obstacles introduced in the third simulation are shown on the left side in Fig. 4. As can be seen, the robot collides with the first obstacle for $t \in [2.1, 5.0]$ and with the second obstacle for $t \in [5.1, 7.9]$.

Parameters for the third simulation are the same as in the previous one. For this experiment, the final time

$T$ is equal to 23.68 [s] and both control constraints and collision avoidance constraints are satisfied. Due to the obstacles acting on the mobile manipulator the controls are slightly sharp. Figures 5 and 6 present the manipulator motion, and controls obtained in this simulation. Distances between the robot and the centres of the obstacles are shown on the right side in Fig. 4. As can be seen, the mobile manipulator penetrates the safety zone of the first obstacle for $t \in [1.2, 5.9]$. At the time instant 4.4 robot enters the safety zone of
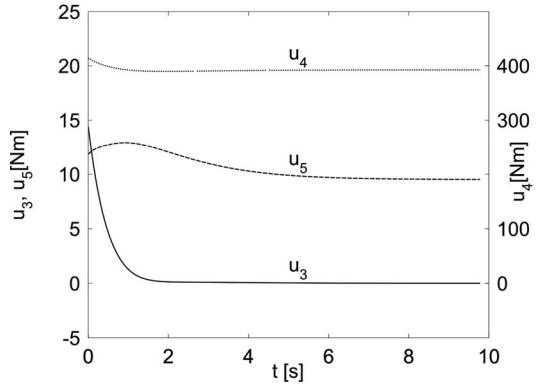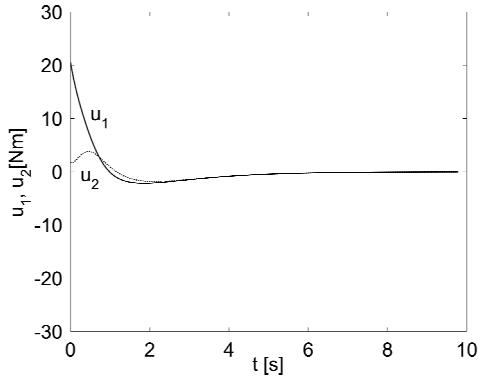


Fig. 2. Controls corresponding to the motion for the first case
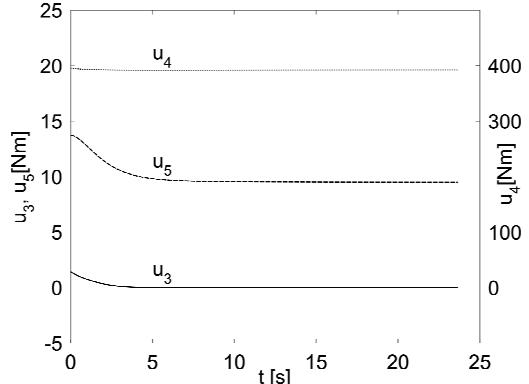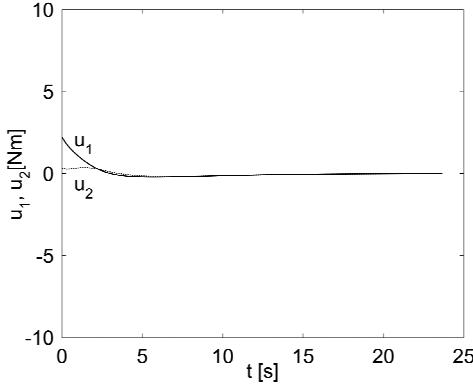


Fig. 3. Controls corresponding to the motion for the second case
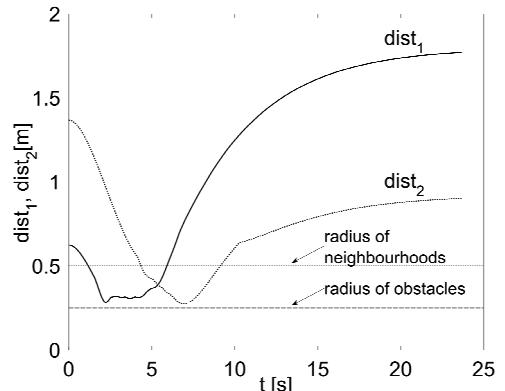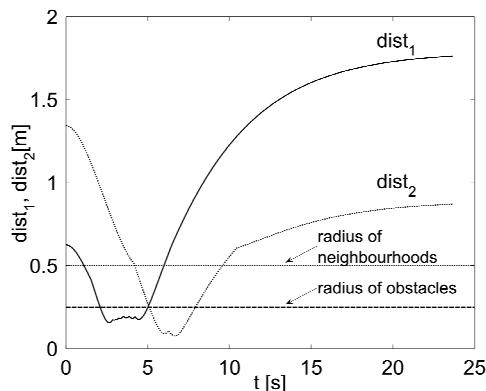


Fig. 4. Distances between the mobile manipulator and the centres of the obstacles:
for the second case (left) and for the third case (right)

the second obstacle and remains there to the time equal to 9.2.
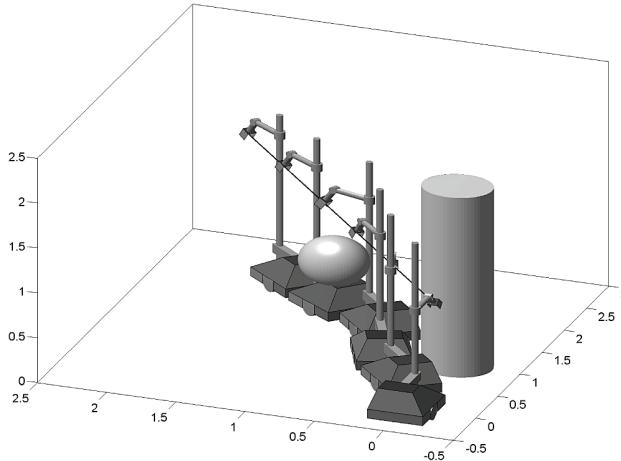


Fig. 5. Collision-free mobile manipulator motion for the third case



Fig. 6. Controls corresponding to the motion for the third case

of reconfiguration of the whole mobile robot during the execution of the task. Additionally, constraints connected with the existence of mechanical limitations, collision avoidance conditions and control constraints have been considered.

To the best of the authors' knowledge, no research has considered the problem formulated in the above manner. Many of the existing research addresses the problem using only kinematic equations of the mobile manipulator, and the dynamics of the robot is not considered at all. There are some references to kinematic and dynamic model-based control methods, nevertheless in these solutions kinematics is represented as a driftless control system and control constraints are not taken into consideration ([8], [9]). Opposite to the above approaches, in our method constraints resulting from movement without slipping are incorporated explicitly to the control algorithm. The solution pre-

## 4. Discussion

In the paper, a concept of the mobile manipulator acting as an assistant of surgeon has been presented. The task of such a system is to move the end-effector along the curve specified by operator during pre-planning of the operation. This path is a curve that can be parameterised by any scaling parameter, the knowledge of the reference trajectory of a mobile platform is not needed. Mounting the arm on top of the mobile wheeled platform extends the performance capabilities of the manipulator and makes it possible to reach the position which will give optimal access to the operating field. Moreover, the manipulator motion is planned in a manner that maximises the manipulability measure in order to avoid manipulator singularities decreasing the probability

sented allows application of scaling techniques [17], [20], [21] for the mobile manipulator trajectory in such away as to fulfill control constraints. In many works authors are mainly concerned with redundancy resolution and their solutions are not optimal in any sense [2]. Our solution maximizes the manipulability measure what results in avoiding a time consuming reconfiguration process of the whole mobile robot. Opposite to similar research (e.g., [12]) in the solution proposed the reference path for the platform is not required, which makes it possible to apply our method in complicated work-spaces including obstacles. The method presented in the paper ensures finding continuous controls. The proposed approach to trajectory planning is a computationally efficient method. The effectiveness of the solution is confirmed by the results of computer simulations.

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