Acta of Bioengineering and Biomechanics
Vol. 3, No. 2, 2001

The dynamic and timing criteria for assessing the single fin swimming technique

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The aim of the study was to analyse the reaction forces arising on single fin surfaces as a result of swimming movements. The crude data were recorded in the form of time-dependent signals representing the reactive water resistance force exerted on fin surface. They were collected using strain gauges and converted assuming sine harmonics to be a model of stability of propulsion generation parameters. The criteria for assessing the quality of the technique allowed us to explain the problem of both the efficiency and utilisation of the forces exerted during swimming.

Key words: swimming, single fin, reaction forces, timing parameters

1. Introduction

In this study, the problem of swimming efficiency and its economics is considered in the context of effective utilisation of forces, which are the result of fin movements. The large surface of the single fin enables us to explain this problem. Two theories of controlling the mutual dependence of forces that cause dislocation and handicap swimming in the context of its efficiency were considered in this study: the vortex theory [6], [3] and the added mass theory [5]. Earlier studies showed that in order to avoid unprofitable water resistance, stable velocity and suitable rhythm of propulsive movements of single fin (and the adequate values of forces generated on its surfaces) are required [21], [12]. The stability in force initiation is the basic condition for obtaining rhythmical and stable distribution of dynamic parameters of the single fin propulsive motions and also allows one to keep up constant infracycle structure, which is necessary to preserve the constant speed of cyclical locomotion in water [23]. Only Wu [32], [33] and Shuping [24] studied the dynamic motion structure during the single fin swimming. Other studies are mainly based on the results obtained from analogies for fish and dolphin motion structure [28], [29], [1], [31] and refer to the research of kinematics parameters of swimming movements [5],

[18], [25], [17], [27], [26]. Thus, the research aimed at analysing forces generated on single fin surfaces, while performing propulsive movements, which determine swimming velocity.

The dependence of hydrodynamic resistance on the velocity of surfaces moving in water and the structure of propulsive forces generated on single fin surfaces can constitute the basis for the formulation of the following assumption. The resultant reaction force is always perpendicular to the straight line determining the angle of single fin attack. Therefore, it can be assumed that strain gauge measurement of resultant reaction force of single fin surface to water resistance qualifies for the description of dynamic structure of propulsion generation process [21]. The recorded reaction force as a function of time can be graphically depicted as the curved line, which resembles sinusoidal curved line. Thus, it can be assumed that sinusoid is the representative curve describing the stability of dynamic and time parameters of the single fin swimming process [21].

2. Methods

Forty men participated in the study. There were three groups (n = 10 each) of single fin swimmers. All of them were members of Polish National Fin Swimming Team. Every ten participants belonged to different age group (senior – mean age 18.5; junior b – mean age 16.4; junior d – mean age 12.2). The fourth set of ten subjects was the senior group, crawl and dolphin swimmers (mean age 19.3), at the level of Polish Championships Finals. All of the subjects performed one experimental trial, in which they swam with maximal possible speed at 50 m distance under water, using the same standard single fin (figure 1).

Special equipment was used for measuring the force of sag, being the result of its reaction to water resistance. There were strain gauges glued on both sides of single

Fig. 1. The single fin and strain gauges

fin surfaces, in the place where the plate is connected with the boots (figure 1). The upper sensor was glued in parallel to symmetry axis. The second one, glued to the bottom side, was perpendicular to symmetry axis of the fin's plate. The compensatory strain gauges were used to avoid disturbance. Both sensors were connected in the half-bridge configuration. Impulses of direct current from sagging fin and strain gauges were amplified, converted and recorded by PC. Sampling frequency was 20 Hz. A connection cord was used between the parts of the measuring equipment. The cables were shielded. All of the connections were waterproof and the cables were not disturbed during swimming along the whole distance. The following scaling procedure of measuring single fin was applied: five points of scaling on the symmetry axis of single fin were marked (assuming that fin's plate is stiff in its crosswise size). The first point was located as close to the strain gauges as possible. The last one was set on the rear edge of the fin. The points were at the same distance from each other. The same mass (1 kG = 9.81 N) was hanged on each scaling point separately. Next, changes in voltage values resulting from fin's sagging at different points were recorded. Then, mean values of voltage recorded for each sample as well as the coefficient of scaling were calculated.

At the beginning of each experimental trial swimmers were lying on the water surface in special position. The plate of single fin had to be parallel to water (bottom) surface. In this plate position the strain gauges indicated "zero" values of the forces measured. Because of the starting procedures described the propulsion movements as well as the forces recorded were not representative. Therefore the first two cycles were cut off before the main analyses of the signal recorded. The same cutting procedure was used for the last two cycles of the recorded forces. The raw data for further analysis were in the form of time-dependent signal representing the reactive water resistance force, exerted on a single fin surface as a result of upward and downward movements (figure 2).

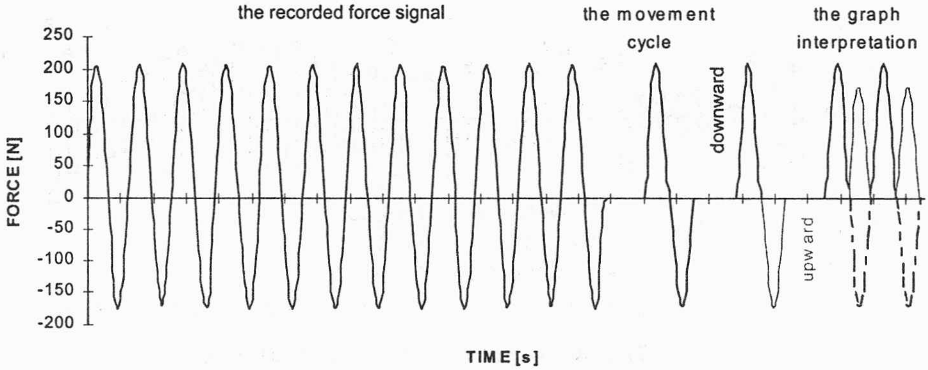

Fig. 2. Example interpretation of the recorded force signals

A time-series analysis was made in order to assess the parameters measured and their level of stability. The recorded force curves were very similar to the sine form. Therefore, another analysis was made assuming sine harmonic to be the model for optimisation of values and stability of parameters in the process of propulsion generation [21]. As a result we obtained mean values of amplitudes of recorded forces, defined Fourier series for these values for all trial courses separately, and approximated the original time function of force by its specific sine harmonic. Two variants of approximation procedure were adopted. In the first one, we approximated forces recorded in the whole series of functions $F(t)$ by their sine models. In the second one, forces recorded in the separate cycles were approximated by their sine (model) curves (figure 8). Finally, it was possible to quantify the areas of harmonic fit between both functions using the integral difference between the approximated curves. A series of the calculated field (impulse) values of the functions being compared constitutes the source of information about the error scale in the stability of the force course in the particular cycles as far as periodicity and amplitude are concerned.

3. Results

The stability level (expressed by standard deviation values of the arithmetic averages of forces examined) (figure 3) clearly differentiates between the competitors under research with respect to the time of covering a distance. The bigger the stability of the forces appearing on a single fin surface, the shorter the time achieved during the attempt. This dependence concerns both the stability of forces in subse-

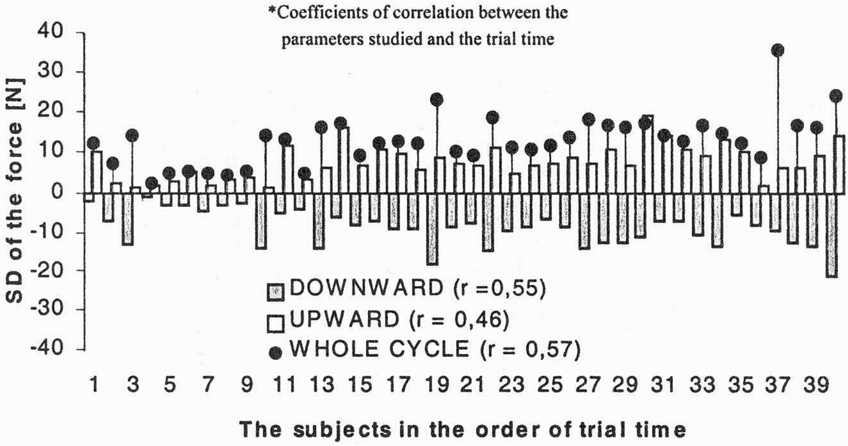

Fig. 3. Values of standard deviation of the force parameters during single fin movements (n = 40)

quent cycle movements and the stability of parameters registered during the upward and downward movement phases of the single fin. Thus, the element on which the maximum speed of swimming depends is the high level of stability of parameters of dynamic movements of a single fin. Maintaining the high level of the stability of forces, generated in full cycles of single fin movement, depends first of all on the ability of stabilising the values of the forces released during the downward movement phases (the correlation ratio between the value of standard deviations in full movement cycles and the values of standard deviations of forces registered during the downward movement phase equals 0.71). Maintaining the high level of the stability of forces of the cycle movements is also influenced by an increase in the value of forces generated during the upward movement phase of the single fin (the correlation ratio between the value of standard deviations in full movement cycles and the values of standard deviations of forces registered during the upward movement phase equals –0.52).

The analysis of the values of standard deviations from the average times of duration of subsequent movements of the single fin (figure 4) implies the existence of differences in the levels of stability of parameters under discussion. Simultaneously, the stability of time intervals describing the downward movements of the single fin is slightly bigger, as compared to the parallel parameters of the upward movement phases. The group under study shows a weak trend in that the stability of the time structure of the cycle movements is generally higher among those who achieved a short time of covering a given distance. The observed regularities allow us to claim that achieving the maximum speed of swimming is not a direct consequence of the level of stability of the time parameters of the single fin movements. No significant differences have appeared within the values of correlation ratios between the time

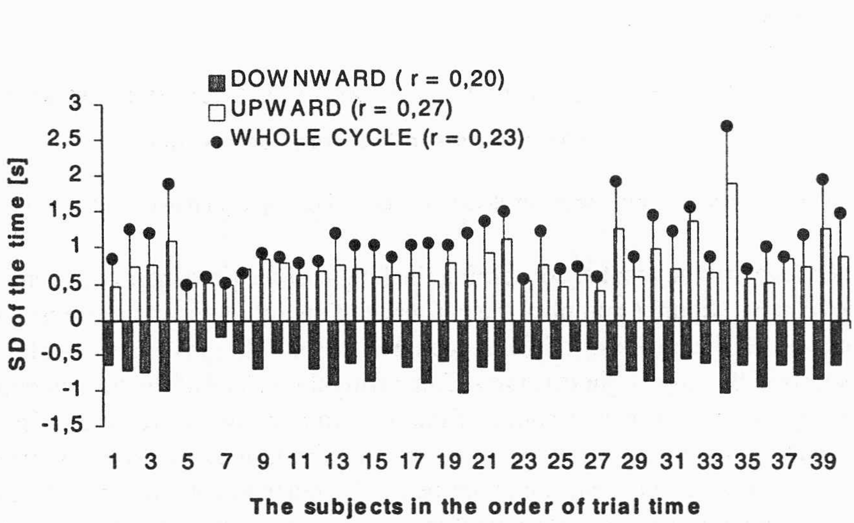

Fig. 4. Values of standard deviation of the timing parameters during single fin movements ($n = 40$)

of covering the distance and the standard deviations of the time of duration of the upward and downward movements of the single fin. However, the results obtained allow us to state that the stability of the time of force generation in upward movement phase potentially affects the speed of swimming.

The research results indicate slight diversities of the forces created on a single fin surface in subsequent movement cycles (figure 5). The persons under research perform movements with greater force in the downward movement phases compared with the force registered during the upward movement phase. The higher value of the forces, characteristic of the single fin movement in subsequent movements, is favourable to the shortening of the time of covering the distance. Simultaneously, those who achieved the shortest times during swimming, generated, as a result of the upward movements of the single fin, higher values of forces than those swimmers who placed themselves as the last competitors in the rating of the results achieved. The achievement of good results was also accompanied by almost equal force values generated in the upward and downward movement phases.

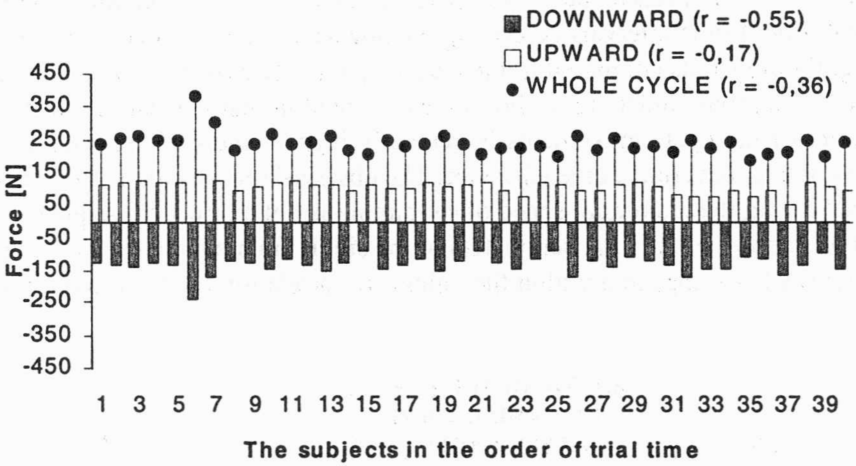

Fig. 5. Mean values of the parameters of forces during single fin movements ($n = 40$)

The survey of the results, concerning the length of the time intervals of the single fin movements (figure 6), implies the existence of the following regularities: the time sequences of the process of force generation in full movement cycles were characterised by only slight diversity. Comparing the values of the time parameters, describing the time of performance of the upward and downward single fin movement phases indicated the existence of very big similarities. However, certain tendency emerged, namely that the swimmers who reached a short time of covering certain distance performed subsequent movements of a single fin in a shorter time than their colleagues who swam slower. A clear difference in the time of covering

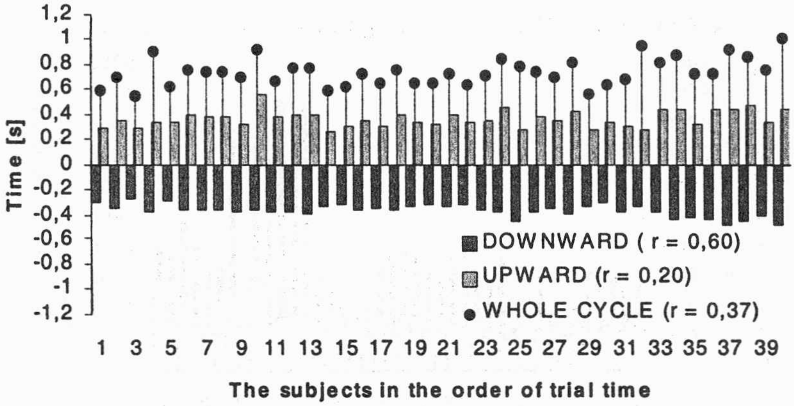

Fig. 6. Mean values of the timing parameters during single fin movements ($n = 40$)

the distance by particular swimmers was related to minimisation of the time of the force generation during downward single fin movement.

In the course of research, the mutual correlation has been examined, that is correlation between the values of forces and the time of their generation and the parameters of the stability of single fin movement techniques. Interpretation of the correlation under discussion has been based on the analysis of error values, resulting from the comparison of the registered courses of the functions $F(t)$ and their model sine equivalents (figure 8). The group of swimmers under investigation showed certain differences in the time of covering the distance, depending on the value of the sums of errors generated while performing the single fin movements. The swimmers, who covered the distance in the shortest time, generated on fin surface forces,

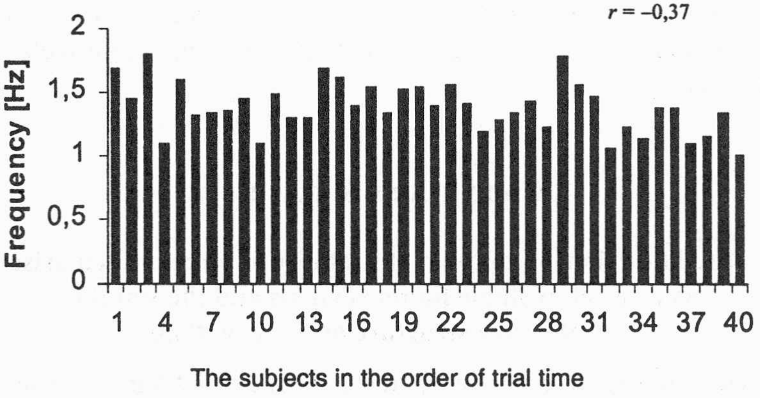

Fig. 7. Mean values of the frequency of single fin movements ($n = 40$)

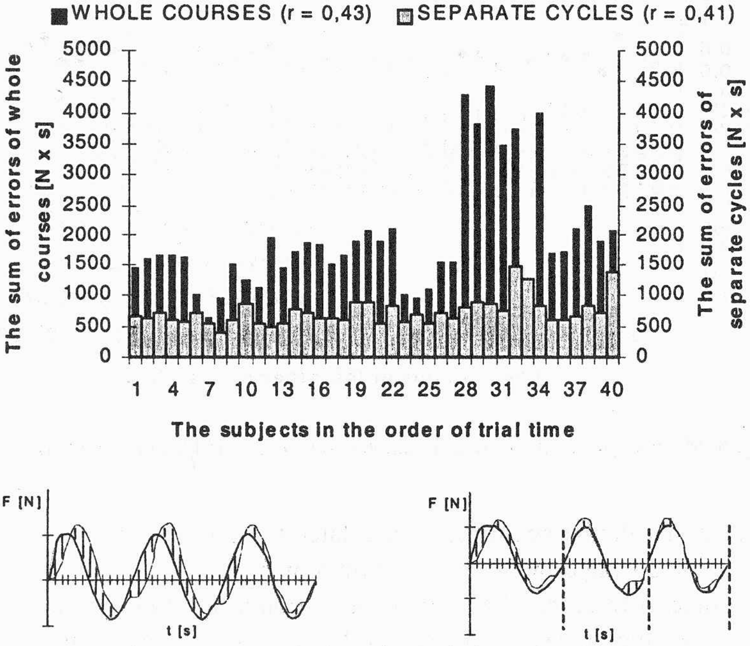

Fig. 8. Values of the level of errors (the areas of errors estimated by approximating the functions of the forces recorded by their sine models) (*n* = 40)

whose courses were similar to the sine curves (taking into consideration both full courses of the functions *F*(*t*) and the records of the particular cycles of these functions). The single fin movements, considered with respect to the whole of the courses of the functions compared, were burdened with bigger error. The errors resulting from the discrepancy between the particular movement cycles and their sine models were of significantly lower value. This fact implies a regularity, manifesting itself in the reduction of the error values in the whole of the function courses under discussion, as a result of decreasing the error rate, appearing in particular single fin movement cycles.

4. Conclusions & discussion

4.1. The criterion of the high stability of forces which arise as a result of single fin movements and the stability of the time structure of these motions

Stabilisation of dynamic and time parameters of single fin movements is a result of complex mechanics aspects related to hydrodynamics of swimming. As a result of movements of a single fin, the turbulent flow of water and vortices are evoked.

The whirling water and the changes in pressure cause the appearance of an additional source of propulsion, as well as a"surplus" component of the lift force [28], [29], [7]. Taking into account the phenomenon of the whirling movement of liquid in terms of the principle of energy conservation, while moving around the surface of a single fin, water "regains" the kinetic energy of its oscillatory movements [15]. The outcome of the evoking of the vortex will be positive for the generated propulsion only in the case its structure is stable. It enables, among others, mono-surface movement of the single fin (without side movements) [3]. The measure of the technique advancement of the "dolphin" movements is also stable rotation of the vortex till the moment the vortex comes off the fin surface [32]. Additionally, the fading of the vortex off one surface need be accompanied by the immediate evoking of the next vortex on the opposite surface of the fin [10]. In order to fully use the additional component of the drag force, stemming from the vortex circulation, it is necessary to avoid sudden changes in the speed of rowing movements. Too sudden and nonstable movements generate accidental vortices, which spread in an uncontrolled manner [4].

The outcome of the study implies that intensification of the forces created during fin movements does not guarantee obtaining the maximal swimming speed. In this situation, the decisive aspect as to the covering of the distance appears to be the ability of adjusting the power engaged in the movement to the value ensuring maintaining the stable dynamic structure of movements of a single fin. Arelano [2] confirmed that the competitors of a high sports level, while swimming the butterfly, performed movements of optimal trajectory (and consequently, power). Lyttle et al.[16] noticed that optimal parameters of oscillatory swimming movements ensure the use of "slide energy" and they compensate for the speed losses, caused by the non-effective phases of propulsion movements. The outcomes presented imply that the short duration of the single fin movements are favourable to the high speed of swimming (figure 6). Simultaneously, it has been observed that too high frequency of movements, reflected by the higher number of movement cycles, results in the reduction of stability of the time structure of propulsion generation (figure 7). The necessity of adjusting the frequency of single fin movements to the level ensuring maintaining high stability of movements is confirmed by the results published by Yi-Ch-Pai and Hay [34]. The ability of maintaining the stable dynamic and time structure of each movement cycle also eliminates the differences in the phase coincidence of movements performed by the fin along the whole distance (figure 8). Under such circumstances, the stable rhythm of cyclic movements of the single fin is preserved [22]. According to Kornecki [12] the stability of the time-dynamic structure (rhythm) of the "dolphin" movements is the measure of using the movement potential of the swimmer to develop and maintain the maximal swimming speed.

The stability of single fin movements is also decisive as to the reduction of the resistance generated on the body surface of the swimmer. The effectiveness of the dolphin movement depends not only on the strength of muscles involved in the movement of rowing surfaces, but also on the stability of the laminar flow of water around the swimmer's body [13]. Thus, the values and time parameters of the forces transferred onto the fin surface need be constant. Stable dynamic and time structure of the fin movements is necessary to avoid unfavourable changes of water resistance, resulting from the effect of inertia forces of the swimmer's body. Therefore, the ability of using the positive aspects of inertia forces in each movement cycle is considered to be an important criterion of mechanic effectiveness of swimming technique, based on the "dolphin" movements [19]. Kornecki [12] formulates the criterion of avoiding the fluctuations in momentary speed, being the factor determining effective solution of the movement task in water.

4.2. The criterion of equal proportion between the forces generated on a single fin surface during its up and down motions

The research into the structure of movements of the tail-fin of dolphin implies that the values of the forces observed during the phase of the downward movement of the fin were slightly higher than the values registered in the phase of the upward movement [28] and this corresponds to the results published in this paper. The research results of the "dolphin" movements of a human being excluded the influence of the power possibilities of the flexors and extensors of lower limbs on the technique of performing both phases of movement cycles, and also determined that the duration of phases of movements up and down is nearly identical [3]. The propulsion in swimming is a consequence of interdependence within the system of using the drag force and lift force. It has been proved [15] that the coefficient of the stability of these forces reaches the maximal value as a result of symmetrical movements of the rowing surfaces in both directions. It happens so, since the size and width of the whirlpool are determined by the scale of amplitude and speed (frequency) of the movements, as well as by the angle of attack of the surface of the fin upon the water surface [15]. Based on the research outcome, it may be anticipated that the stable equal distribution of the parameters mentioned decides as to the smaller phase coincidence of the periods of subsequent cycles, in which the propulsion forces are generated. Equal values of dynamic and time parameters of the single fin movements up and down determine stable, devoid of sudden accelerations, character of the speed changes in the movement cycle phases under discussion. As a result, the stability of dynamic and time parameters of single fin movement are preserved, on which in turn obtaining the maximal speed of swimming depends.

4.3. The criterion for better usage (intensification) of the upward motion forces in the cycle structure during single fin swimming speed

The confirmation of the reality of this criterion need be searched in hydrodynamic conditions of the effective propulsion. The total value of components of the reaction force of the surface of the single fin to the water resistance (resulting from the 3rd Newton's law) is the highest in the case the extremely sagged fin crosses the straight indicating the direction of swimming [30]. However, the highest values of forces resulting from the whirling movement of water appear at the "top" points of phases of downward and upward fin movements. At points in which the fin surface changes the direction of movement, the coming of the vortices off its surface takes place [5]. The same research proved that the direction of the vortex generated during the phase of upward movement is opposite to the direction of swimming. As a consequence, the meaning of vortex is connected not only with the "delivery of additional mass", but also with initiation of the additional force, which acts based on rejection. This additional effect does not accompany the movements up and down and this is the factor justifying the postulate as to the necessity of intensifying the fin movements up and down in order to increase the effectiveness, economic management and single fin swimming speed.

4.4. The technical skills of readjusting fin movements to the criteria resembled by the dynamic sine model of the ratio of forces generated on the single fin surfaces as a result of obtaining the highest swimming speed

A review of related literature does not provide for numerous examples of research on swimming techniques, which are based on measuring of the reaction force to water resistance by use of strain gauges (see, e.g. [8], [9]). The results of the author's own experimental research [21], [22] allow us to state that the extensometric measurement of the reaction forces to water resistance (including forces that sag single fin surfaces) is the source of data for interpretation of processes taking place during swimming. The above statement is confirmed by the results of the research into kinematic parameters of single fin swimming [27], [26]. In the present paper, the sinusoidal character of the curves describing the dynamics of single fin movements became the basis for the evaluation of the quality of swimming technique. The following arguments confirm the legitimacy of acquiring the sinusoidal function as the model of distribution of dynamic and time parameters of single fin movements, which ensures obtaining the maximal swimming speed. The parameters of the description of the technique of single fin movements, constituting the object of the research, were the following: frequency, periodicity and amplitude of forces generated cyclically on the fin surfaces. These parameters are the variables allowing the respective courses examined to be related to their mathematical, sinusoidal equivalents. The research conducted in the field of movements of dolphins led to the conclusion that high speed of swimming of these animals is a result of, among others, harmonic propulsion movements [28]. This view is also confirmed in the case of swimming movements of a human being. This is proved by the fact that the movement amplitude in particular cycles of fin movement determines their duration and thus, affects the frequency of the movement [23]. Furthermore, there is an inversely proportional relation between the force of sagging of the fin surface in reaction to water resistance and the movement frequency [24]. Additionally, in terms of mathematics, "a complex non-harmonic movement is always a resultant of the sinusoidal function of this movement" [20]. Thus harmonic analysis allows us to calculate each function of periodical course if its shape is known [11]. The harmonic function equation also determines the stability of mutual relations between the dynamic parameters of fin movements as a time function of covering the distance [21]. An outstanding example explaining the legitimacy of acquiring the sinusoidal model of the swimming technique quality with the use of a single fin is an interpretation of mechanic aspects describing the phenomenon of "flapping" [10], [25].

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