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Identification of muscle forces in human lower limbs during sagittal plane movements
Part II: Computational algorithms

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Using a sagittal model of human body developed in part I, the present paper deals with computational algorithms related to the inverse simulation problem – the determination of driving muscle forces in lower limbs using the measured motion characteristics as input data. The control problem is associated with muscle force redundancy and then affected by the fact that muscles always generate tensile forces. Computational schemes for the determination of reaction forces in the hip, knee and ankle joints are also reported/developed and discussed from the view-point of their effectiveness and applicability.

1. Introduction

In part I of this paper [1], a sagittal n-degrees-of-freedom ($n = 11$) musculoskeletal model of human body was developed, aimed at describing planar movements such as standing long jumps, vertical jumps and jumps down from a height, in which both the lower and upper extremities are moving parallel to each other in the sagittal plane. Since the attention is focused on lower limb control and loadings, a hybrid model of the control $\mathbf{u}$ that creates movements in the $k = 8$ joints of the musculoskeletal system was proposed – the motor control moments in the hip, knee and ankle joints are modelled by a set of $m_F = 15$ muscle forces treated as control inputs, while the actuation in the other $k - 3 = 5$ joints is simplified to $m_\tau = 5$ torques representing the respective muscle action. The dynamic equations of motion for the musculoskeletal model in n independent coordinates $\mathbf{q}$ were derived, initially for the flying phase and then for the support phase where the feet are in contact with the ground.

The present paper deals with computational algorithms related to the inverse simulation problem stated as follows: given the measured motion characteristics $\mathbf{q}_d(t)$, $\dot{\mathbf{q}}_d(t)$ and $\ddot{\mathbf{q}}_d(t)$ in an observed movement, determine the control $\mathbf{u}_d(t)$ that forces the system to complete the motion. The control problem is associated with muscle force redundancy and then affected by the fact that muscles always generate tensile forces. Computational schemes for the determination of reaction forces in the hip, knee and ankle joints are also reported/developed and discussed taking account of their effectiveness and applicability.

2. Determination of inverse simulation control

Two inherently different phases of the sagittal movements considered can be distinguished: the *support phase* when the feet are in contact with the ground with no slip and the *flying phase* when there is no contact between the feet and the ground. During the flying phase the gravity forces are the sole external forces on the system, which are then complemented by the ground reaction forces during the support phase. The dynamic equations that describe the musculoskeletal model motion in the flying and support phases are given respectively by equations (I.13) and (I.15), i.e., equations (13) and (15) of part I. The inverse dynamics problem stated above requires then a slightly different computational scheme in the two phases.

2.1. The flying phase control

The first problem that is concerned with the determination of the musculoskeletal model control during the flying phase stems from the fact that the 'flying' human model is globally underactuated. As is shown in Section I.2, i.e., Section 2 of part I, the n-degrees-of-freedom system is actually controlled by k torques $\mathbf{u}' = [\tau_1 \; ... \; \tau_9]^T$ resulting from the muscle action in the k joints, $k < n$. The torques τ_1, τ_2 and τ_3 in the H, K and A joints are then expressed in terms of muscle forces $F_1, ..., F_{15}$ treated as new controls, and the local control redundancy is achieved in the joints related to the extended control m-vector $\mathbf{u} = [F_1 \; ... \; F_{15} \;\; \tau_4 \; ... \; \tau_8]^T$. Putting the latter problem aside for a while, let us concentrate on removing the global underactuation.

The generalized control force related to $\mathbf{q}$ in equation (I.13), $\overline{\mathbf{f}}_u = \overline{\mathbf{B}}\mathbf{u}$, can alternatively be written as $\overline{\mathbf{f}}_u = \overline{\mathbf{B}}'\mathbf{u}'$. The $n \times k$ matrix $\overline{\mathbf{B}}'$ of distribution of the control torques $\mathbf{u}'$ in $\mathbf{q}$ directions is easy to formulate (see also [2] for illustration), and the same relates to its orthogonal complement, an $r \times n$ matrix $\overline{\mathbf{A}}'$ ($r = n - k$) such that $\overline{\mathbf{A}}'\overline{\mathbf{B}}' = \mathbf{0}$. While the vectors represented as columns of $\overline{\mathbf{B}}'$ span the controlled subspace in the n-space related to $\dot{\mathbf{q}}$, the vectors represented as rows of $\overline{\mathbf{A}}'$ span the uncontrolled subspace (see [3] for more details), and as such $\overline{\mathbf{A}}'\overline{\mathbf{B}} = \mathbf{0}$ as well. For the case at hand we have

$$\overline{\mathbf{B}}' = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \quad \mathbf{A}'^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}. \tag{1}$$

Using a formula similar to equation (I.12), the projection of the dynamic equations (I.13) into the uncontrolled subspace gives

$$\mathbf{a}(\mathbf{q},\dot{\mathbf{q}},\ddot{\mathbf{q}}) = \overline{\mathbf{A}}'(\overline{\mathbf{M}}\,\ddot{\mathbf{q}} + \overline{\mathbf{d}} - \overline{\mathbf{f}}_g) = \mathbf{0}\,, \tag{2}$$

which is equivalent to r = 3 conditions according to which the mass centre of the system moves along a parabola (or vertically) and the total angular momentum remains constant during the flying phase. Based on these conditions we can verify the accuracy of the motion characteristics measured and the correctness of the mathematical model constructed, i.e., $\mathbf{a}(\mathbf{q}_d,\dot{\mathbf{q}}_d,\ddot{\mathbf{q}}_d) \equiv \mathbf{0}$ is demanded.

The projection of the dynamic equations into the controlled subspace gives

$$\overline{\mathbf{B}}'^T\ddot{\mathbf{q}} + \overline{\mathbf{B}}'^T\overline{\mathbf{M}}^{-1}(\overline{\mathbf{d}} - \overline{\mathbf{f}}_g) = \overline{\mathbf{B}}'^T\overline{\mathbf{M}}^{-1}\overline{\mathbf{B}}'\mathbf{u}' \tag{3}$$

or

$$\overline{\mathbf{B}}'^T\ddot{\mathbf{q}} + \overline{\mathbf{B}}'^T\overline{\mathbf{M}}^{-1}(\overline{\mathbf{d}} - \overline{\mathbf{f}}_g) = \overline{\mathbf{B}}'^T\overline{\mathbf{M}}^{-1}\overline{\mathbf{B}}\,\mathbf{u}\,, \tag{4}$$

respectively, and we use $\overline{\mathbf{f}}_u = \overline{\mathbf{B}}'\mathbf{u}'$ or $\overline{\mathbf{f}}_u = \overline{\mathbf{B}}\,\mathbf{u}$ in equation (I.13). The number k of the dynamic equations being projected is now equal to the number of joints (and motor control torques $\mathbf{u}'$) in the system, and the global underactuation of the system is removed. More strictly, $\overline{\mathbf{B}}'^T\overline{\mathbf{M}}^{-1}\overline{\mathbf{B}}'$ in equation (3) is a positive definite $k \times k$ matrix and $\mathbf{u}'_d(t)$ can be calculated as

$$\mathbf{u}'_d(t) = [\mathbf{H}'(\mathbf{q}_d(t))]^{-1}\mathbf{h}(\mathbf{q}_d(t),\dot{\mathbf{q}}_d(t),\ddot{\mathbf{q}}_d(t))\,, \tag{5}$$

where $\mathbf{H}' = \overline{\mathbf{B}}'^T\overline{\mathbf{M}}^{-1}\overline{\mathbf{B}}'$ and $\mathbf{h} = \overline{\mathbf{B}}'^T\ddot{\mathbf{q}} + \overline{\mathbf{B}}'^T\overline{\mathbf{M}}^{-1}(\overline{\mathbf{d}} - \overline{\mathbf{f}}_g)$.

The other question is connected with the distribution of the joint torques in the lower extremity joints H, K and A into the muscle forces $F_1, ..., F_{15}$. Mathematically the problem can be associated with the solution of k algebraic equations, obtained from equation (4) after substituting $\mathbf{q}_d(t)$, $\dot{\mathbf{q}}_d(t)$ and $\ddot{\mathbf{q}}_d(t)$, with respect to m unknowns **u**, $k < m$ (here $k = 8$ and $m = 20$; see Section I.2 for details). The redundant control problem is usually solved using different optimization techniques, see, e.g., [4]–[6]. One possibility of this type is to apply the pseudoinverse method [7], which, for the case at hand, consists in formulating a pseudoinverse to the rectangular $k \times m$ matrix $\mathbf{H} = \overline{\mathbf{B}}'^T \overline{\mathbf{M}}^{-1} \overline{\mathbf{B}}$ introduced into equation (4), i.e., the $m \times k$ matrix $\mathbf{H}^\dagger = \mathbf{H}^T (\mathbf{H}\mathbf{H}^T)^{-1}$ such that $\mathbf{H}\mathbf{H}^\dagger = \mathbf{I}$ (the $m \times m$ identity matrix). Using the pseudoinverse, $\mathbf{u}_d(t)$ can be calculated from equation (4) according to the scheme

$$\mathbf{u}_d(t) = \mathbf{H}^\dagger(\mathbf{q}_d(t))\ \mathbf{h}(\mathbf{q}_d(t), \dot{\mathbf{q}}_d(t), \ddot{\mathbf{q}}_d(t)), \tag{6}$$

where **h** is as defined in equation (5). It can be demonstrated [6] that the pseudoinverse technique automatically computes the solution which minimizes the norm $J = \mathbf{u}^T \mathbf{u} = u_1^2 + \ ...\ u_m^2$.

An inherent feature of the mathematical solution obtained from equation (6) is that both positive and negative muscle forces may be generated. In the case, some of the muscle forces determined are negative, they should be set to zero (or small positive values), and the calculations should be repeated until all the muscle forces are either positive or vanish.

2.2. The support phase control

The musculoskeletal model dynamics during the support phase was given a detailed description in Section I.3. Due to the slipless feet contact with the ground, $r = 3$ constraints are imposed on the system, and the dynamic equations in **q** are projected into the $k = n - r$ dimensional unconstrained subspace and r dimensional constrained subspace, resulted in equations (I.15) and (I.16), respectively. Using alternatively $\bar{\mathbf{f}}_u = \overline{\mathbf{B}}'\mathbf{u}'$ or $\bar{\mathbf{f}}_u = \overline{\mathbf{B}}\,\mathbf{u}$, equation (I.15) can be rearranged to

$$\overline{\mathbf{D}}^T \overline{\mathbf{M}}\ddot{\mathbf{q}} - \overline{\mathbf{D}}^T(\bar{\mathbf{d}} - \bar{\mathbf{f}}_g) = \overline{\mathbf{D}}^T \overline{\mathbf{B}}'\mathbf{u}' \tag{7}$$

or

$$\overline{\mathbf{D}}^T \overline{\mathbf{M}}\ddot{\mathbf{q}} - \overline{\mathbf{D}}^T(\bar{\mathbf{d}} - \bar{\mathbf{f}}_g) = \overline{\mathbf{D}}^T \overline{\mathbf{B}}\,\mathbf{u}, \tag{8}$$

where $\overline{\mathbf{D}}$ is the $n \times k$ matrix defined in equation (I.9). Due to this projection into the unconstrained subspace, the global underactuation of the system dynamics is removed, and equations (7) and (8) correspond closely to equations (3) and (4). Resembling the previous case, $\overline{\mathbf{D}}^T \overline{\mathbf{B}}'$ is an invertible $k \times k$ matrix and $\mathbf{u}'_d(t)$ can be calculated from equation (5) after substituting $\mathbf{H}' = \overline{\mathbf{D}}^T \overline{\mathbf{B}}'$ and $\mathbf{h} = \overline{\mathbf{D}}^T \overline{\mathbf{M}} \ddot{\mathbf{q}} - \overline{\mathbf{D}}^T (\overline{\mathbf{d}} - \overline{\mathbf{f}}_g)$. Then, since both the matrices $\overline{\mathbf{D}}^T \overline{\mathbf{B}}$ in equation (8) and $\overline{\mathbf{B}}'^T \overline{\mathbf{M}}^{-1} \overline{\mathbf{B}}$ in equation (3) are of dimension $k \times m$, $\mathbf{u}_d(t)$ can be calculated according to equation (6) using $\mathbf{H} = \overline{\mathbf{D}}^T \overline{\mathbf{B}}$ and its pseudoinverse.

Having the inverse simulation control $\mathbf{u}_d(t)$ determined, the reactions from the ground can be found as $\overline{\boldsymbol{\lambda}}_d(t) = \overline{\boldsymbol{\lambda}}(\mathbf{q}_d(t), \dot{\mathbf{q}}_d(t), \mathbf{u}_d(t))$ using equation (I.16). The computed values of ground reactions can possibly be compared with the measured ones, which may be another criterion for verification of the accuracy of the motion characteristics measured and correctness of the mathematical models built.

3. Determination of joint reaction forces

The evaluation of internal loads acting on joints, which may be especially high in the upward propulsion and landing phases of jumps, can be essential for a comprehensive inverse dynamic analysis of the sagittal movements under consideration. The information related to the question of which loads cross the joints during these and other activities may be of considerable interest to the clinicians as well. There are at least three possible computational algorithms related to the problem.

A chronologically oldest scheme for the determination of joint reactions is based on D'Alembert's principle. With reference to the formulation of the present paper, the scheme can be based on the initial dynamic equations in absolute coordinates **p**, reported in equation (I.1), and must be proceeded, firstly, by the determination of $\mathbf{u}_d(t)$ and then $\mathbf{p}_d(t)$, $\dot{\mathbf{p}}_d(t)$ and $\ddot{\mathbf{p}}_d(t)$ using the kinematic relationships represented by equations (I.8), (I.9) and (I.10), i.e.,

$$\mathbf{p}_d(t) = \mathbf{g}(\mathbf{q}_d(t)); \quad \dot{\mathbf{p}}_d(t) = \mathbf{D}(\mathbf{q}_d(t))\, \dot{\mathbf{q}}_d(t); \\ \ddot{\mathbf{p}}_d(t) = \mathbf{D}(\mathbf{q}_d(t))\, \ddot{\mathbf{q}}_d(t) + \boldsymbol{\gamma}(\mathbf{q}_d(t), \dot{\mathbf{q}}_d(t)) \tag{9}$$

During the support phase, the right-hand side of equation (I.1) needs also to be supplemented with the appropriately involved (and previously determined/measured) ground reactions to the feet. Fulfilled these preliminary calculations, the joint reaction forces $\boldsymbol{\lambda}_d(t) = [\lambda_{1d}(t) \; \cdots \; \lambda_{ld}(t)]^T$ can be determined recursively using d'Alembert's method and starting from the external segments of the modelled human body (segments 4, 7 and 9) and going inward. The scheme is rather laborious and cumbersome in computer applications.

The second scheme arises from the last l equations of the projection formula introduced into equation (I.12). This projection into the null velocity subspace yields

$$\boldsymbol{\lambda}(\mathbf{q},\dot{\mathbf{q}},\mathbf{u})=(\mathbf{C}\mathbf{M}^{-1}\mathbf{C}^{T})^{-1}\mathbf{C}\,(\mathbf{M}^{1}(\mathbf{f}_{g}+\mathbf{B}\mathbf{u})-\boldsymbol{\gamma}), \quad (10)$$

where $\mathbf{C}$ is the $l\times 3N$ matrix of joint constraints defined in equation (I.6) as $\mathbf{C}(\mathbf{p})$, rearranged then to $\mathbf{C}[\mathbf{g}(\mathbf{q})]$ after using equation (I.8). The determination of joint reaction forces from equation (10) can then be written symbolically as $\boldsymbol{\lambda}_d(t)=\boldsymbol{\lambda}(\mathbf{q}_d(t),\dot{\mathbf{q}}_d(t),\mathbf{u}_d(t))$. The above scheme is decidedly computer-oriented. However, there are three main disadvantages of the scheme (compared to the third scheme presented in the sequel). Firstly, the joint constraint equations in the implicit form of equation (I.5) and the arising $l\times 3N$ matrix $\mathbf{C}$ need to be formulated, which are not needed for the derivation of dynamic equations (I.12). This means some additional modelling effort. Secondly, the $l\times l$ (here 16×16) matrix $\mathbf{C}\mathbf{M}^{-1}\mathbf{C}^{T}$ needs to be formulated and then inverted, which may be a cumbersome task. And finally, all the l constraint reactions $\boldsymbol{\lambda}$ need to be determined, while one may be interested in the constraint reactions only in some joints.

The third scheme for the determination of joint reaction forces uses an augmented formulation of the joint coordinate method [8] reported in Section I.3. In this method, instead of the joint constraint equations given in an explicit form of equation (8), an augmented explicit form of these equations is introduced, i.e.,

$$\mathbf{p}=\mathbf{g}(\mathbf{q},\mathbf{z}), \quad (11)$$

where $\mathbf{z}=[\,z_1\ \ldots\ z_l\,]^T$ are open-constraint coordinates that describe the prohibited relative motions in the joints. Specifically, since $\mathbf{z}=\mathbf{0}$, equation (11) is virtually equivalent to the explicit constraint equation $\mathbf{p}=\mathbf{g}(\mathbf{q})$, and the dependence on $\mathbf{z}$ is needed only to grasp the prohibited motion directions related to $\dot{\mathbf{z}}$ – the directions of constraint reactions in the respective joints. Moreover, the open-constraint coordinates can be introduced only into those joints in which the reaction forces are to be determined. In the case at hand, we open thus only the hip, knee and ankle joints, and thus $\mathbf{z}=[\,z_1\ z_2\ z_3\ z_4\ z_5\ z_6\,]^T$ as can be seen in figure I.4. Only $l^*=6$ reaction forces $\boldsymbol{\lambda}^*=[\,\lambda_1\ \lambda_2\ \lambda_3\ \lambda_4\ \lambda_5\ \lambda_6\,]^T$ in the lower extremity joints will thus be determined, where $\boldsymbol{\lambda}^*$ is a subset of $\boldsymbol{\lambda}$ introduced into equation (I.1). As an example, the relations represented by equation (I.11) that correspond to $\mathbf{p}=\mathbf{g}(\mathbf{q})$ in the augmented form $\mathbf{p}=\mathbf{g}(\mathbf{q},\mathbf{z})$ are:

$$x_{C3} = x_H + l_1 \sin\varphi_1 + z_3 + l_2 \sin\varphi_2 + z_5 + \xi_{C3} \sin\varphi_3 + \eta_{C3} \cos\varphi_3,$$
$$y_{C3} = y_H - l_1 \cos\varphi_1 + z_4 - l_2 \cos\varphi_2 + z_6 - \xi_{C3} \cos\varphi_3 + \eta_{C3} \sin\varphi_3, \tag{12}$$
$$\theta_3 = \varphi_3.$$

As can be seen, the augmentation is a rather trivial task. By differentiating equation (11) with respect to time and then setting $\mathbf{z} = \mathbf{0}$, one arrives at

$$\dot{\mathbf{p}} = \left(\frac{\partial \mathbf{g}}{\partial \mathbf{q}}\right)\Bigg|_{\mathbf{z}=\mathbf{0}} \dot{\mathbf{q}} + \left(\frac{\partial \mathbf{g}}{\partial \mathbf{z}}\right)\Bigg|_{\mathbf{z}=\mathbf{0}} \dot{\mathbf{z}} = \mathbf{D}(\mathbf{q})\dot{\mathbf{q}} + \mathbf{E}(\mathbf{q})\dot{\mathbf{z}}\,, \tag{13}$$

while the explicit formulation of the constraint equation $\mathbf{p} = \mathbf{g}(\mathbf{q})$ yields simply $\dot{\mathbf{p}} = \mathbf{D}(\mathbf{q})\dot{\mathbf{q}}$. Because the joints in the system are all movable, all the entries of **E** are equal either to 0, 1 or –1.

The $3N \times l^*$ (here 27×6) matrix **E** produced in equation (13) has the features of a pseudoinverse matrix to the rectangular $l^* \times 3N$ matrix $\mathbf{C}^*$ which is composed of those rows of matrix **C** which correspond to the open-constraint coordinates **z**, i.e., $\mathbf{C}^*\mathbf{E} = \mathbf{I} \Leftrightarrow \mathbf{E}^T\mathbf{C}^{*T} = \mathbf{I}$, where **I** denotes the $l^* \times l^*$ identity matrix. On the other hand, $\mathbf{E}^T\mathbf{C}^{**T} = \mathbf{0}$, where $\mathbf{C}^{**}$ contains the other rows of **C** (not contained in $\mathbf{C}^*$). By denoting $\mathbf{C} = [\mathbf{C}^{*T} \quad \mathbf{C}^{*T}]^T$, we have finally

$$\mathbf{C}\mathbf{E} = \begin{bmatrix} \mathbf{I} \\ \mathbf{0} \end{bmatrix} \quad \Leftrightarrow \quad \mathbf{E}^T\mathbf{C}^T = [\,\mathbf{I} \quad \mathbf{0}\,]. \tag{14}$$

With the use of **E** defined as above, the projection formula reported in equation (I.12) can be modified to [8]

$$\begin{bmatrix} \mathbf{D}^T \\ \mathbf{E}^T \end{bmatrix} (\mathbf{M}(\mathbf{D}\ddot{\mathbf{q}} + \boldsymbol{\gamma}) - \mathbf{f}_g - \mathbf{B}\mathbf{u} + \mathbf{C}^T\boldsymbol{\lambda}) = \mathbf{0}. \tag{15}$$

The first k = 8 components of equation (15) lead to the dynamic equations in **q** given by equation (I.13), and the last $l^* = 6$ components of equation (15) result in

$$\boldsymbol{\lambda}^*(\mathbf{q},\dot{\mathbf{q}},\ddot{\mathbf{q}},\mathbf{u}) = \mathbf{E}^T[\mathbf{f}_g + \mathbf{B}\mathbf{u} - \mathbf{M}(\mathbf{D}\ddot{\mathbf{q}} + \boldsymbol{\gamma})]\,, \tag{16}$$

which offers the requested effective formula for the determination of joint reaction forces in the lower extremity joints, $\boldsymbol{\lambda}_d^*(t) = \boldsymbol{\lambda}^*(\mathbf{q}_d(t),\dot{\mathbf{q}}_d(t),\ddot{\mathbf{q}}_d(t),\mathbf{u}_d(t))$. As is seen, in the present scheme, the joint reaction forces are obtained directly in a 'resolved' form (no matrix inversion is involved). The scheme does not require the implicit form of the joint constraint equations either, which does not need to be introduced at all. Finally, the constraint reactions only in some chosen joints can be determined.

As a final remark related to the problem of determination of joint reaction forces, let us note that the joint reactions determined by using each of the methods mentioned will be different, depending on $\mathbf{f}_u = \mathbf{B}\mathbf{u}$ or $\mathbf{f}_u = \mathbf{B}'\mathbf{u}'$ involved. In the latter case, the internal tensile forces exerted by muscle on bones (segments of the musculoskeletal model) are neglected which may spoil the reliability of calculation results.

4. Conclusions

The determination of muscle forces during human movements can play an important role in a deeper understanding of the underlying neural control. It can also be essential for the analysis of internal loads acting on bones and joints. While in part I of this paper a background to the mathematical modelling of human sagittal movements and control was presented, in this paper we developed some related computational algorithms. Based on the measured motion characteristics as the input data, computational schemes for the synthesis of driving muscle forces in the lower limbs and motor control torques in the other joints were introduced, being built in slightly different ways for the flying and support phases of the sagittal jump movements analyzed. The discussed pseudoinverse technique of distribution of motor control torques in the lower limb joint into the respective muscle forces is only one of a huge variety of optimization methods. Since many of them give practically equivalent results [5], the proposed automatic method based on pure matrix manipulations seems to be a reasonable choice. Finally, the ranges of computational schemes for the determination of reaction forces in the hip, knee and ankle joints were reported. The novel approach following from the augmented joint coordinate method seems to have the advantage over the other methods, taking account of both the modelling and computations as well as its simplicity and effectiveness.

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