Acta of Bioengineering and Biomechanics
Vol. 8, No. 2, 2006

Effect of kinematic boundary conditions on optical and biomechanical behaviour of eyeball model

WIESŁAW ŚRÓDKA

Faculty Division of Deformable Body Mechanics, Wrocław University of Technology,
Wrocław, Poland

Any generally accepted standard of modelling an eyeball's mounting system still does not exist. The comparison of the results of two extremely different mounting models, i.e., fixed and simply supported, permits us to estimate the part of the kinematic boundary conditions in the solutions of the model. In numerical calculations of the relationship between the changes in the eyeball's volume and a rise in the *IOP* (intraocular pressure), the result depends greatly on the type of model's mounting. Similarly, the optical system of the eye is dependent on the same boundary condition.

1. Introduction

The biomechanical model of the eyeball as well as models of different organs or biological tissues should enable investigation of the mechanical functions of original organs. Therefore, the structural problems of modelling are limited only to a correct dimensioning and selecting material parameters. At the same time the optical functions of the eyeball model ought to be considered. This makes the efforts to identify this structure more difficult. A numerical model should not only represent a real deformation of the eye caused by typical stimuli, but also (which is often more important) convert realistically its deformations into the image produced by the optical system of the cornea and crystalline lens. For instance, the calculations in order to keep the track of shifts of this optical system's focus caused by an increase in the *IOP* or laser shaping of the cornea. The third component of the eye's model, apart from its geometry and material, is the type of its mounting system, i.e., the kinematic boundary conditions. In some cases, their effect is as important as that of the load or material.

The results of the calculations presented here include three types of loading and the response of the model evoked by them:

- the change of *IOP* caused by an increase in the eyeball's volume,

stimuli

- the shift of the focus relative to the retina caused by the change of *IOP*,

• the flattening of the corneal apex in the aplanat tonometry.

In each case, the effect of kinematic boundary conditions is investigated.

The flattening of the cornea is significant for clinical purposes. This applies to the measurement of an intraocular pressure. The effect of the equalization of the pressures on both sides of a flattened corneal apex which is theoretically justified by the solution of the shell according to the Laplace equation and also by the results of examinations, is reduced to the following equality:

$$IOP_G = IOP_T, \tag{1}$$

where the IOP_T is the true (real) intraocular pressure and the IOP_G is the measured average pressure of the flat probe. This probe is placed against the corneal apex to flatten the cornea to diameter of 3.06 mm. Equation (1), with the conditions of measurement described as above by Godmann, is named the Imbert–Fick law.

2. Computational model of the human eyeball

2.1. Geometrical and material parameters of the model

The parameters of the model are given in the table. There are average values for the healthy, regular human eye observed in clinical practice.

Table. The parameters of the eyeball model

Parameters used in the Finite Element Model	
Parameter	Value
The conic section approximates the anterior and interior corneal profiles	$z(x) = \frac{1}{e^2 - 1} \cdot \left[\sqrt{{R_0}^2 + x^2 \cdot (e^2 - 1)} - R_0 \right]$
Axial radius of anterior corneal curvature	$R = 7.55$ mm
Axial radius of interior corneal curvature	$r = 6.50$ mm
Central corneal thickness	$d_c = 0.52$ mm
Thickness of peripheral cornea adjacent to limbus	$d_p = 0.65$ mm
Diameter of the cornea	11.5 mm
Average refractive index of cornea	$n_r \cong 1.3771$
Average refractive index of aqueous humour and vitreous body	$n \cong 1.336$
Refractive power of the lens	$P_{lens} = 22.07$ D
Poisson's ratio	$\nu = 0.45$
Nominal intraocular pressure	12 mm Hg

Three types of loading that do not cause any injury to the tissues of the eye and admit the possibility of simplifying the model's material are examined. Thus the isotropic and homogeneous material is assumed. The results presented in [1] indicate that it has to be nonlinear. In this publication, the exponential form of the constitutive equation is assumed according to FUNG [2] as follows:

$$\sigma = A(e^{\alpha\varepsilon} - 1), \quad \varepsilon >= 0, \tag{2}$$

where σ is the stress, and ε – the strain.

The material parameters A and α were determined in two tests of the model:

- the test of conformity with the Imbert–Fick's law,
- the test of free displacement of the corneal apex under the influence of *IOP*.

The comparison of the results of the modelling and the experiment permits us to determine a relatively narrow range of the values of the parameters A and α. Additionally it has been established that the model's solutions have to include the dependence of the material characteristics on the sign of the stress/strain. Thus, the stress in compression in the uniaxial state was taken as the linear function of the strain:

$$\sigma = E_0\,\varepsilon, \quad \varepsilon < 0. \tag{3}$$

The longitudinal modulus of elasticity E_0 was determined as the derivative of the function (2) at $\varepsilon = 0$:

$$E_0 = d\sigma/d\varepsilon = A\alpha, \quad \varepsilon = 0. \tag{4}$$

In these tests, the model imitates a real eye as close as possible at $A = 0.0002$ MPa and $\alpha = 130$. These values were used for the calculations in this research.

2.2. The boundary conditions

Until now, there are no reports on the experimental determination of the mechanical parameters of the tissues that surround the eyeball. In this regard, our knowledge limits us to the intuition moulded by the clinical experience of the ophthalmologists, though the demand for this kind of the research becomes firmer [3]. Theoretically, a real mounting of the eye is determined by two opposing, extreme boundary conditions:

- the fixed posterior hemisphere of the sclera,
- the free movement of eyeball.

It is true that for a number of stimuli the above choice is of little importance, but there are some, which lead to the results greatly dependent on the type of model's mounting.

2.3. The self-adjusting effect

The corneal limbus is particularly difficult to model. The longitudinal rigidity of the limbus in the circumferential direction has an influence on the displacement field of the cornea for each kind of force: the variations in *IOP*, the surgical correction of the corneal profile or aplanation of the corneal apex during the tonometric measurement of *IOP*. The stiffness of the corneal limbus is conditioned by the interactions of a number of tissues of the internal structures of the eye such as the pupillary sphincter, the iris, the ciliary body, the crystalline lens, the blood vessels, etc. It is possible to use the hypothesis of the self-adjusting effect of the eye's optical system and to calculate the equivalent limbal ring stiffness in spite of the modelling of all these structures. Thus, it is assumed that the eyeball has the ability to maintain focus on the retina when the eye is subjected to small oscillatory variations in *IOP*. Moreover, the results of the modelling of the elastic moduli of the cornea, sclera and limbal ring reported in [4] enabled us to determine whether there might be an optimum set of rheological values required for the maintenance of ocular image quality. It turned out that well matched longitudinal rigidity of the limbal ring (figure 1) makes the displacement of the secondary principal plane of the cornea–lens system Δl induced by increasing the *IOP* equal to the elongation of the focal length Δf. Due to the above the axial displacement of the focal point relating to the retina, $B = \Delta l - \Delta f$, is equal to zero. All the results presented here were obtained for the self- adjusting model.

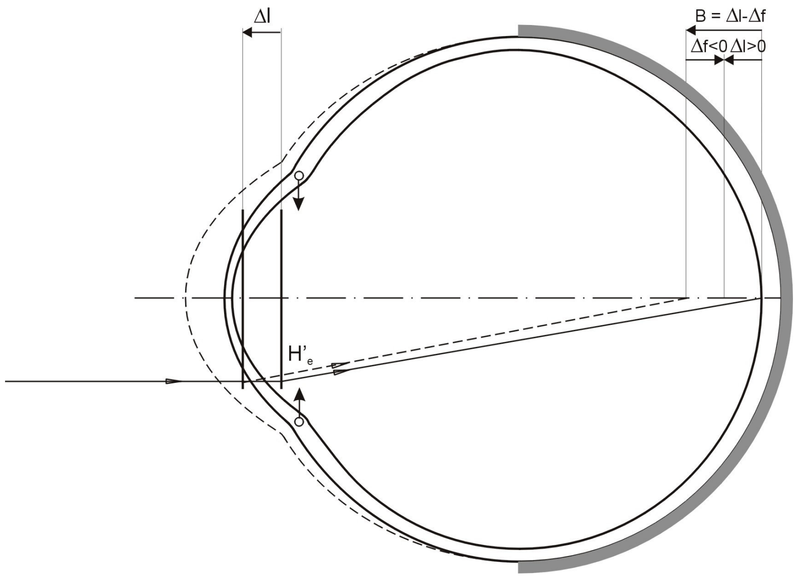

Fig. 1. Two elements (Δl and Δf) of focal point shift. This eyeball is not self-adjusted, after increasing *IOP* we have $\Delta l - \Delta f \neq 0$. Rear hemisphere of the sclera is fixed here

2.4. The computational parameters of the model

In order to make the numerical model symmetrical, it was constructed of the axially symmetric tetragonal elements of the 8-node, plane 2D type (2D continuum). The model is nonlinear both physically (material) and geometrically (parameters of solution). The numerical solutions were obtained using COSMOS/M, and an additional software cooperates with Cosmos for the optical calculations and for determining the IOP_G.

3. The selected solutions of the model and their dependence on the mounting type of the sclera

3.1. The ocular rigidity

The ocular rigidity in the simplest version is defined as the relation of the increase in the *IOP* to the change of the eyeball's volume. It is applied with some techniques of the *IOP* measurement and for that reason is of great clinical importance. FRIEDENWALD

[5] noticed that this *IOP* plotted in the logarithmic scale as a function of the volume increase leads to linear relation and he defined the ocular rigidity as follows:

$$R_{\text{eye}} = \frac{\log(P_2) - \log(P_1)}{\Delta V}\,. \tag{5}$$

P_1 and P_2 are the pressure values at the beginning and at the end of the measurement, $\Delta V = V_2 - V_1$ is the volume increase of the eyeball during the measurement. Fridenwald's definition has survived until now mainly because of a habit and simplicity; in reality this parameter depends on pressure (slightly) as well as on the total initial eyeball's volume V_1 (clearly).

The results obtained directly from the solutions of the two models: fixed and simply supported are compared in figure 2a. In figure 2b, the same numbers were plotted in the logarithmic scale on the *Y*-axis; additionally the regression lines approximate them.

The ocular rigidity measured in the clinical practice reaches 0.01 mmHg/mm^3 (calculated according to (5)), but this value for individuals could be very different.

In the numerical solutions, there are some states of the eyeball's loading (commonly investigated and sometimes significant in clinical meaning), and wrongly assumed boundary conditions lead to false results. The *IOP*–ΔV relation should be rated among these and the differential tonometry introduced by Friedenwald could

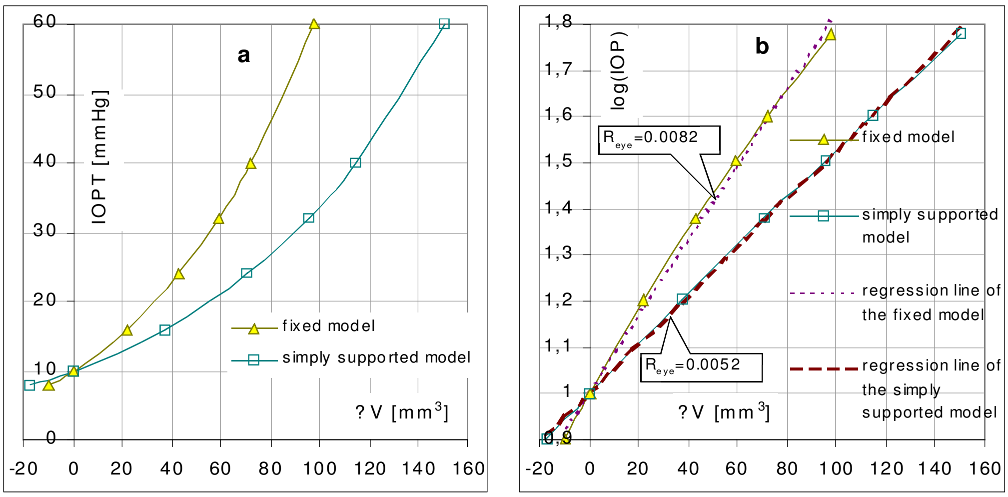

Fig. 2. The relation of the *IOP* (on the left) and the log(*IOP*) (on the right) to the variations of the eyeball's volume; additionally the regression straight lines are plotted (the slope of the straight line corresponds to the ocular stiffness R_{eye})

exemplify the application of this relation. This type of force is also used for identifying material parameters of eye tissues (most often Young's modulus), but as we can see from the results presented here, the result of these calculations greatly depends on the boundary conditions assumed, which are still chosen arbitrarily.

3.2. The displacement of the focus of the eye's optical system

As a result of the increase in *IOP*, the focus of the cornea–lens optical system is displaced. A 0.2 mm shift of the focus relative to the retina requires the optical correction to exceed 0.5 dpt. The corneal apex also distances itself from the retina at the same distance after an increase in the *IOP* ranging from 8 to 16 mm Hg. How does this change in *IOP* influence the place of the focus in the optical self-adjusting model?

Figure 3 shows the series of the solutions for an increase in the *IOP* and for each of the models: fixed and simply supported. The tracing of the displacements of the focus in the initial state of the model loading makes no sense because in the natural state at *IOP* = 12 mm Hg the focus point in the real eye is located on the retina and relative to this point its shifts should be described (maximum on the curve). In order to determine numerically which type of the model mounting influences this aspect of eye's optics, we can compare the difference between intraocular pressures at which each of curves reaches its maximum. The fixed model achieves this maximum at *IOP* = 12.0 mm Hg after the fifth-order polynomial approximation of the curve (figure 3). In the simply supported model, the distance of the focus from the retina reaches maximum at *IOP* = 9.0 mm Hg.

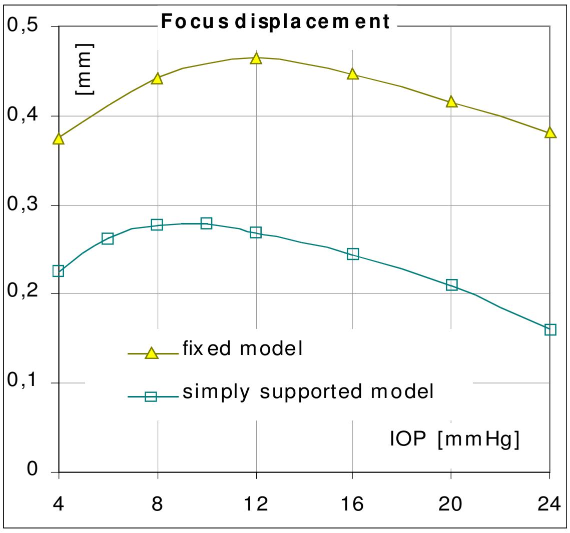

Fig. 3. $B = \Delta l - \Delta f$ as a function of the *IOP* for each type of model mounting

In the case of the fixed model, the maximum occurs at 12 mm Hg because the limbal ring stiffness was specially matched with it. Due to this the focus hardly shifts relative to retina at small variations in *IOP*. Even if the deviation of *IOP* from the nominal value of 12 mm Hg reached ± 4 mm Hg as is shown in figure 3, the displacement of the focus (relative to the maximum, perpendicular to retina) would not exceed 0.02 mm (the simply supported model as well is remarkably tolerant of variations in the *IOP* only around the value of 9 mm Hg). It is converted to the change of the eye's refractive power of 0.05 dpt, being imperceptible (unnoticeable) for a subject. It could be assumed that the fixed model automatically maintains the ocular image quality on the retina when the eye is subjected to variations in *IOP* in the range of 8–16 mm Hg (2/3 of the nominal *IOP*).

In spite of such a wide range of the acceptable variations in *IOP* on that score, the importance of the boundary conditions of the model mounting is clearly noticeable – the optimal value of the *IOP* changes from 12.0 mm Hg for the fixed model to 9.0 mm Hg for the simply supported model. There is a sharp slope of the curve which makes the details of cooperation of the geometrical parameters required for the self-adjusting of the eye dedectable.

3.3. The aplanation of the corneal apex

The simulation of the *IOP* measurement using Goldmann's tonometer is one of theoretical solutions of eye's model most often described in literature. The load consists in the flattening of the corneal apex in the area of 3.06 mm in diameter and the calculation of the average pressure that acts in this area from the outside. A real average eye does not comply exactly with law (1) which is the reason for previous solution and it is necessary to correct the IOP_G obtained due to the instrument. This theoretical solution could provide the scale of these corrections taking into consideration the individual parameters of the eyeball.

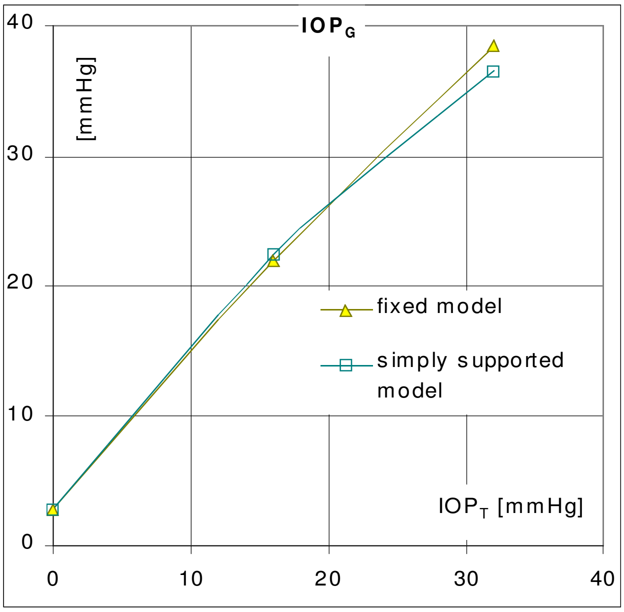

Fig. 4. The calculated contact pressure of the probe IOP_G plotted as a function of real IOP_T for two models: fixed and simply supported

Many theoretical solutions described in the literature are limited to corneal model fixed at the circumference. In the more realistic patterns, the limbus mounted for axial movement is expected (the axis is canted). The most advanced boundary conditions, expected for this solution, include models of the whole simply supported eyeball. Are there any grounds for not taking into consideration even partial fixing of the sclera?

Figure 4 shows the solutions for "measured" pressure (IOP_G) under the probe in two versions of model: fixed and simply supported. It clearly shows that in the range of about 30 mm Hg, the difference in the IOP_G read depending on the type of the model mounting are not large, but it also shows that above this limit this difference increases rapidly and the fixed model gives the large values of the IOP_G.

References

[1] HACHOŁ A., ŚRÓDKA W., *Model biomechaniczny gałki ocznej oparty na zjawisku samonastawności optycznej weryfikowany w tonometrycznym pomiarze IOP*, VIII Sympozjum *Modelowanie i Pomiary w Medycynie*, MPM'2006.

[2] FUNG Y.C., *Biomechanics. Mechanical properties of living tissues*, Springer, 1993.

[3] SCHUTTE S., VAN DEN BEDEM S.P., VAN KEULEN F., VAN DER HELM F.C., SIMONSZ H.J., *A finite-element analysis model of orbital biomechanics*, Vision Research, 2006, 46, 1724–1731.

[4] ASEJCZYK-WIDLICKA M., ŚRÓDKA W., KASPRZAK H., ISKANDER D., *Influence of intraocular pressure on geometrical properties of a linear model of the eyeball. Effect of optical self-adjustment*, Optik, 2004, No. 11, pp. 517–524.

[5] FRIEDENWALD J.S., *Contribution to the theory and practice of tonometry*, Amer. J. Ophthal., 1937, 20, 985–1024.