Acta of Bioengineering and Biomechanics
Vol. 12, No. 2, 2010

Original paper

On some properties of bone functional adaptation phenomenon useful in mechanical design

MICHAŁ NOWAK*

Division of Machine Design Methods, Poznan University of Technology, Poland.

The paper discusses some unique properties of trabecular bone functional adaptation phenomenon, useful in mechanical design. On the basis of the biological process observations and the principle of constant strain energy density on the surface of the structure, the generic structural optimisation system has been developed. Such approach allows fulfilling mechanical theorem for the stiffest design, comprising the optimisations of size, shape and topology, using the concepts known from biomechanical studies. Also the biomimetic solution of multiple load problems is presented.

Key words: *structural optimisation, trabecular bone remodelling, biomimetics*

1. Introduction

There are many examples of the use of the optimisation method known from the mechanical design in area of biomechanics [1]–[3]. Usually, the mathematical formulas are meant to explain biological processes resulting in structural optimisation. The success of these investigations proves that the living entities are able to solve mechanical engineering problems according to the achievements in mathematical and technical sciences. This paper presents an opposite approach, trying to treat biological processes as a pattern for the formulation of optimisation scenario. Contemporary design methods include optimisation procedures in each of the designing stages. In the case of structural design, the optimisation assists engineers from the earliest design idea all the way to the end of the designing process. In the case of living entities, all kinds of the optimisation must be simultaneous. An example of such a simultaneous adaptation is the phenomenon of the trabecular bone remodelling process.

2. Methods

2.1. Trabecular bone remodelling process

The trabecular bone remodelling process is an excellent example of the structural optimisation problem. There are many models of bone remodelling. Most of them represent the continuation of the Roux's idea of biological regulatory process [4]–[8]. In the "regulatory model" developed by HUISKES [9]–[11], the concept of tissue adaptation is based on the assumption of the existence of homeostasis (perfect balance between bone gain and loss). The remodelling "regulatory model" scheme is depicted in figure 1.

This equilibrium can occur only in the presence of mechanical stimulation. The network of osteocytes plays the role of sensors detecting mechanical energy distribution along trabecular bone tissue. The model postulates strain energy density (SED) on the surface of trabecular bone as a scalar measure of mechanical

---

* Corresponding author: Michał Nowak, Division of Machine Design Methods, Poznań University of Technology, ul. Piotrowo 3, 60-965 Poznań, Poland. Tel.: +48 61 6652041, e-mail: Michal.Nowak@put.poznan.pl

Received: May 21st, 2010
Accepted for publication: June 10th, 2010

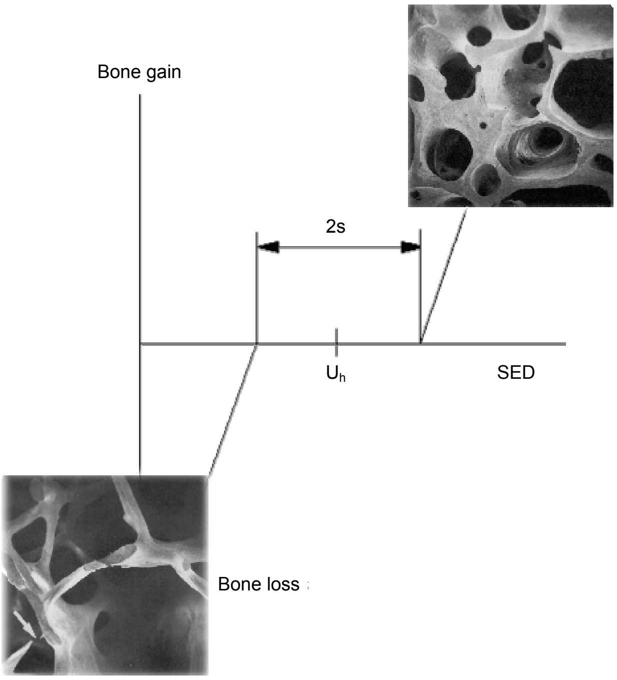

Fig. 1. The Huiskes remodelling “regulatory model” scheme

stimulation and a distinguished value of SED, corresponding to bone remodelling homeostasis. Thus, the regulatory mechanism is responsible for the remodelling process in the trabecular bone on a single cell level. The model assumes also that small deviations from this distinguished energy value do not substantially influence the remodelling phenomenon (Carter’s “lazy zone” [6]). Only significant changes in the mechanical stimulation result in bone loss or gain. The “regulatory model”, presented in figure 1, is described according to [10] by the following equations:

$$\frac{dE}{dt} = C_e(U - (U_h + s)) \quad \text{for} \quad U \rangle (U_h + s), \tag{1}$$

$$\frac{dE}{dt} = 0 \quad \text{for} \quad (U_h - s) \le U \le (U_h + s), \tag{2}$$

$$\frac{dE}{dt} = C_e(U - (U_h - s)) \quad \text{for} \quad U \langle (U_h - s), \tag{3}$$

where:

E denotes Young’s modulus of the tissue,

U_h is the SED value corresponding to homeostasis of bone loss and gain,

$2s$ is the size of the “lazy zone”,

C_e is a constant value.

HUISKES [12] used SED as a remodelling signal as a choice only. He suggested that other mechanical variables could be used as well. The following chapters show that though intuitive, the choice was appropriate.

2.2. The principle of constant strain energy density

The essential assumption of the “regulatory model” is the existence of homeostasis in the remodelling process, described by the distinguished value of SED. But SED (as an energy measure) is also of a prime importance in optimisation research, far from biomechanical applications [13]–[15]. PEDERSEN [15] presented the considerations of the optimisation of structural shape by minimizing the strain energy. Defining the total potential as a sum of elastic energy and work of external forces [15]:

$$\Pi = U_\varepsilon + U_{\text{ext}} \tag{4}$$

the derivative of the total potential Π with respect to an arbitrary parameter h is:

$$\frac{d\Pi}{dh} = \frac{\partial \Pi}{\partial h} + \frac{\partial \Pi}{\partial \varepsilon}\frac{d\varepsilon}{dh}, \tag{5}$$

and with respect to the virtual work principle:

$$\frac{\partial \Pi}{\partial \varepsilon} = 0 \tag{6}$$

for design independent external loads:

$$\frac{\partial U_{\text{ext}}}{\partial \varepsilon} = 0, \tag{7}$$

the derivative of the total potential:

$$\frac{d\Pi}{dh} = \frac{\partial U_\varepsilon}{\partial h}. \tag{8}$$

For a local design parameter h_e that only changes the design in the domain e of the structure:

$$\frac{\partial U_\varepsilon}{\partial h_e} = \frac{\partial(\overline{u}_e V_e)}{\partial h_e}, \tag{9}$$

where:

$\overline{u}_e$ is the mean strain energy density in the domain of e,

V_e is the corresponding volume.

Assuming two parameters h_i, h_j and a constant total volume V of the structure:

$$\Delta V = \frac{dV}{dh_i}\Delta h_i + \frac{dV}{dh_j}\Delta h_j = 0, \tag{10}$$

we arrive at the increment of the elastic energy:

$$\Delta U_\varepsilon = \frac{dU_\varepsilon}{dh_i}\Delta h_i + \frac{dU_\varepsilon}{dh_j}\Delta h_j \tag{11}$$

for design independent loads, and it follows from (9) that only the local energies are involved:

$$\Delta U_\varepsilon = \bar{u}_i \frac{dV_i}{dh_i}\Delta h_i + \bar{u}_j \frac{dV_j}{dh_j}\Delta h_j,$$
$$\Delta U_\varepsilon = -(\bar{u}_i - \bar{u}_j)\frac{dV_i}{dh_i}\Delta h_i. \tag{12}$$

With the assumption of a constant volume the necessary condition for optimality:

$$\Delta U_\varepsilon = 0 \tag{13}$$

leads to the conclusion that the strain energy densities must be equal to:

$$\bar{u}_i = \bar{u}_j. \tag{14}$$

Similarly, with all design parameters:

$$\Delta V = \sum_e \frac{dV_e}{dh_e}\Delta h_e \tag{15}$$

the total energy change equation:

$$\Delta U_e = \sum_e u_e \frac{dV_e}{dh_e}\Delta h_e \tag{16}$$

leads to the conclusion that a necessary condition for the optimality $\Delta U = 0$ with constraint $\Delta V = 0$ is a constant value of the strain energy density. Thus, for the stiffest design the energy density along the shape to be designed must be constant:

$$u_s = \text{const.} \tag{17}$$

Equation (17) proves that SED used as a remodelling signal in the Huiskes's remodelling model is a very good choice. The natural, metabolic trabecular bone remodelling process is, in this context, a practical realization of the structural optimisation.

3. Results

The phenomenon of trabecular bone adaptation has two important features. Firstly, mechanical stimulation is needed to maintain the rebuilding balance. Secondly, the process of resorption and formation occurs on the trabecular bone surface only. These factors together with the "regulatory model" concept were the basis for the generic, three-dimensional system for biomimetic structural optimisation [16], [17]. The method developed mimics the geometry evolution of a real bone, where the mesh of volumetric finite elements and the surface of the trabecular network are controlled during the simulation. By analogy to the bone remodelling model described above, the following assumptions have been made (the variables have the same meaning as in equations (1)–(3)):

Add some amount of material onto the surface of the structure for

$$U > (U_h + s). \tag{18}$$

No action – "lazy zone" for

$$(U_h - s) < U < (U_h + s). \tag{19}$$

Remove some amount of material from the surface of the structure for

$$U < (U_h - \text{s}). \tag{20}$$

In this way, the process of structural optimisation mimics the real biological process of mechanical adaptation of trabecular bone. The material in virtual space is (just like the tissue) added onto or removed from the surface of the structure. The structural form is changed imitating the behaviour of Basic Multicellular Units (BMU), thus the change concerns always the same amount of material (tissue), independently of the current values of SED according to formulas (18)–(20). Such a mechanism preserves the stability of the process and prevents rapid change in the form of the structure. The optimisation process ceases when SED values on the whole surface of the structure are included in the "lazy zone".

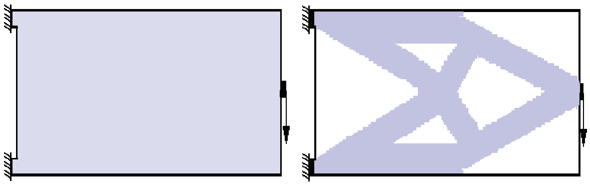

Fig. 2. The optimisation problem of the cantilever beam bending. Left: possible material distribution domain with bending force and two supports. Right: the solution – optimal material distribution within the domain

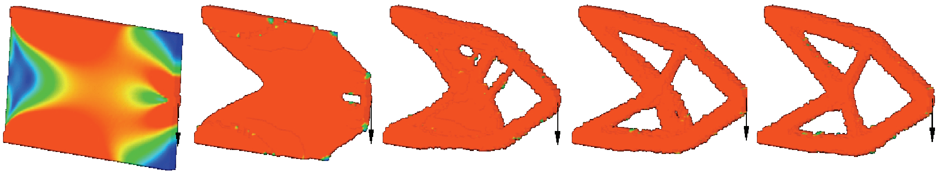

Fig. 3. The optimisation results of the cantilever beam bending – from the left to the right: selected simulation steps

To compare the present approach with other optimisation methods, the standard topology optimization example was chosen [18]. In figure 2, optimisation problem of the cantilever beam bending is depicted. The left picture presents the possible material distribution domain with bending force and two supports. The right picture depicts the solution – optimal material distribution within the domain.

The problem solution with use of the biomimetic system developed is presented in figure 3.

4. Discussion

4.1. Domain-independent structural optimisation

The solution obtained is in accordance with the results of standard topology optimisation method [18]. The solution has another advantage – it comprises the optimisations of size, shape and topology, because every step of the optimisation procedure represents functional structural configuration. This is a unique feature of the biomimetic method – natural in the case of living entities and unseen in the traditional optimisation approach. But in the case of leaving entities, there is no optimisation domain. So, what will happen, if the starting configuration is different? Will the optimisation method lead to the same solution?

Another issue is to define possible supports. In the case of bone, the trabecular tissue modification occurs inside the shell of cortical bone. Also, there are no prescribed supports for the trabecular structure, but rather the area of possible supports only. Solving such a problem with standard approach is not possible, because the change in the definition of boundary conditions leads to the change of the whole problem. To find answers to these questions, the simulation example presented above was modified.

The starting configuration is as simple as possible – the stick connecting the bending force and possible support area. Instead of support definition, there is a clumped wall, as a surface, on which during the optimisation procedure supports are defined. The optimisation result presented in figure 4 is the same, despite different starting configuration.

4.2. Structural optimization under multiple load case

From an engineer's point of view the single load case problem is rather rare. More frequent, but also more valuable for mechanical design, is the problem of structural optimization under multiple loads. The biomimetic optimisation methods should be useful because in real life, multiple independent loads are always present. For the multiple load case simulation the same starting configuration (stick) was studied. Two different load cases were examined. The first, identical with the study presented in figure 4, and the second, with the same definition of boundary conditions and horizontal bending force. The optimisation results for these two configurations treated separately

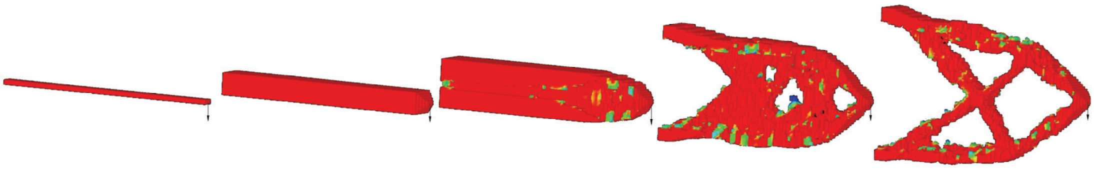

Fig. 4. The optimisation results of the cantilever beam bending without domain definition – from the left to the right: selected simulation steps

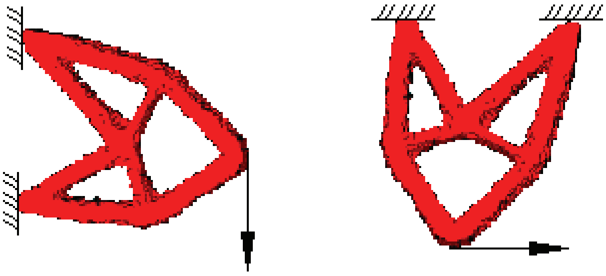

Fig. 5. The optimisation results for the same starting configurations (stick) and different direction of the bending force: left – vertical bending, right – horizontal bending force

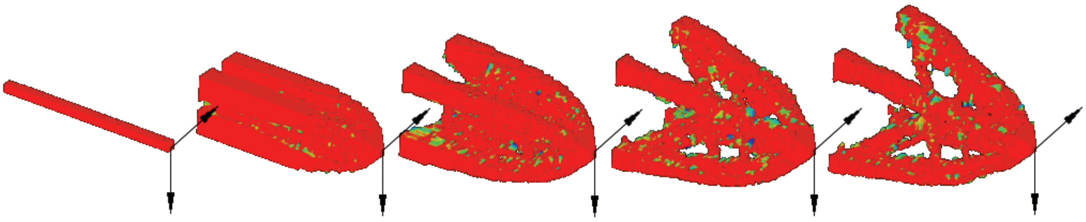

Fig. 6. The result of the multiple load study (altering vertical and horizontal bending forces)

are depicted in figure 5. The solutions have identical form, but rotated according to the direction of the force applied. Figure 6 depicts the result for the same starting configuration but including multiple load cases. The direction of the force applied was switched every two simulation steps from the vertical to horizontal one and vice versa. The result of the multiple load study is presented in figure 6.

The solution obtained is radically different from those obtained for each of the load cases shown in figure 5, nor is their superposition. Due to the unique features of biomimetic structural optimisation process discussed above, the evolution of the structure proceeded smoothly, despite the changes in load definition. The method allows efficient performance of the optimisation process for several cases of loading, when homogenisation of SED on the surface of the structure guarantees the optimality of solution.

5. Conclusions

In the paper, some unique properties of trabecular bone functional adaptation useful in mechanical design were presented. The domain independence, functional configurations during the process of optimisation and possible solution of multiple load problems demonstrate the usefulness of the biomimetic method in mechanical design. Additionally, such an approach allows fulfilling mechanical theorem for the stiffest design, comprising the optimisation of size, shape and topology using the concepts known from biomechanical studies.

The question whether the WOLFF's law [19] is true in terms of a mathematical formula determining a structure form can be now raised otherwise. In fact, there is no mathematical formula nor pattern for tissue form. However, contrary to Huiskes's statements "by wondering about what mathematical rules bone architecture might be the answer to, we do not learn anything useful at all" [12], the principle of constant strain energy density on the surface of the structure leads directly to the stiffest design – minimisation of strain energy in the structure. In this sense, the Wolff's law is true, and the metabolic process developed by Nature can be regarded to be the method of achieving the goal – the stiffest design. Thus, the research of bone remodelling process is interesting not only for biological and medical sciences, but it also may be useful for the development of optimisation methods in the area of mechanical design.

Acknowledgements

This work was supported by the Polish Ministry of Science and Higher Education under the grant no. N N518 328835.

References

[1] KIM H.A., CLEMENT P.J., CUNNINGHAM J.L., *Investigation of cancellous bone architecture using structural optimisation*, Journal of Biomechanics, 2008, 41(3), 629–635.

[2] BAGGE M., *A model of bone adaptation as an optimization process*, Journal of Biomechanics, 2000, 33, 1349–1357.

[3] PENNINGER C.L., PATEL N.M., NIEBUR G.L., TOVAR A., RENAUD J.E., *A fully anisotropic hierarchical hybrid cellular automaton algorithm to simulate bone remodeling*, Mechanics Research Communications, 2008, Vol. 35, Issues 1–2, 32–42.

[4] ROUX W., *Gesammelte Abhandlungen über Entwicklungsmechanik der Organismen II*, Leipzig, 1895.

[5] PAUWELS F., *Biomechanics of the locomotor apparatus*, Springer-Verlag, Berlin, 1963.

[6] CARTER D.R., ORR T.E., FYHRIE D.P., *Relationships between loading history and femoral cancellous bone architecture*, Journal of Biomechanics, 1989, 22(3), 231–244.

[7] FROST H.M., *Laws of Bone Structure*, Springfield, Ill: Charles C. Thomas, 1964.

[8] TELEGA J., GALKA A., TOKARZEWSKI S., *Effective moduli of trabecular bone*, Acta of Bioengineering and Biomechanics, 1999, Vol. 1, No. 1.

[9] HUISKES R. et al., *Effects of mechanical forces on maintenance and adaptation of form in trabecular bone*, Nature, 2000, 404, 704–706.

[10] HUISKES R., *Computational Theories of Bone Modeling and Remodeling, Advanced Course on Modelling in Biomechanics*, MiB03, Institute of Fundamental Technological Research, Polish Academy of Science, Warsaw, 2003.

[11] HUISKES R., WEINANS H., GROOTENBOER J., DALSTRA M., FUDALA M., SLOOFF T.J., *Adaptive bone remodelling theory applied to prosthetic-design analysis*, Journal of Biomechanics, 1987, 20, 1135–1150.

[12] HUISKES R., *If bone is the answer, then what is the question?* Journal of Anatomy, 2000, 197, 145–156.

[13] WASIUTYNSKI Z., *On the congruency of the forming according to the minimum potential energy with that according to equal strength*, Bull. de l'Academie Polonaise des Sciences, Serie des Sciences Techniques, 1960, 8(6), 259–268.

[14] DEMS K., MROZ Z., *Multiparameter structural shape optimization by finite element method*, Int. J. Num. Meth. Eng., 1978, 13, 247–263.

[15] PEDERSEN P., *Optimal Design Structures and Materials Problems and Tools*, 2003.

[16] NOWAK M., *A generic 3-dimensional system to mimic trabecular bone surface adaptation*, Computer Methods in Biomechanics and Biomechanical Engineering, 2006, 9(5), 313–317.

[17] NOWAK M., *Structural optimization system based on trabecular bone surface adaptation*, Struct. Multidisc. Optim., 2006, 32, 241–251.

[18] BENDSOE M.P., SIGMUND O., *Topology Optimization, Theory, Methods and Applications*, Springer-Verlag, Berlin, 2003.

[19] WOLFF J., *Das Gesetz der Transformation der Knochen*, Hirschwald, 1892.