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Maximal frequency, amplitude, kinetic energy and elbow joint stiffness in cyclic movements

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The present work deals with those properties of the human motor system that characterize cyclic movements of maximum (expected to be maximum) intensity. It presents the results of an experiment carried out on 11 subjects and aimed at measuring the kinematic characteristics of cyclic movements of the elbow joint executed at maximal frequency under unloaded conditions. The movements of five amplitude levels ranging from 0.1 rad to approximately 1.2 rad were considered. An observable and unequivocal relation was found between the amplitude and the maximal movement frequency. The said relation is one of an inverse relationship types described by the equation of a shifted hyperbola intersecting the frequency axis at a point marking the value of maximal movement frequency f_{max} whose mean value was 8.42 Hz in the group investigated. It was also established that elbow joint movements executed at maximal intensity show significant similarity to harmonic movement, which points to the "stiff" characteristic of useful driving torque. Relationships between maximal amplitudes of angular velocity, angular acceleration, kinetic energy and movement amplitude were also determined. The nature of the said relationships derives from the superposition of the two abovementioned features – the amplitude–frequency relation and the formal relationships between the values describing harmonic movement. The elbow joint stiffness manifested during cyclic movements appears to be related to both movement frequency and amplitude. Its value increases with frequency and decreases with amplitude growth ranging from approximately 15 to 130 Nm/rad. The source of the said stiffness is to be found in the properties of the tendon–muscle complex and its changes depend on the changes of muscle tension. This feature has been illustrated by the measurement of the relation between elbow joint stiffness and the static torque generated by elbow joint flexors and extensors. It has been established that the stiffness increases with muscle tension squared.

Key words: periodic movements, eigenfrequency, elbow stiffness

1. Introduction

In many types of human motor activity, we can find elements of repeatable nature consisting in reproducing cyclically some action performed either in individual joints or as a complex motion involving kinematic chains. This feature can be noticed in all types of locomotor movement, from walking to, for example, rowing, and in numerous manipulative movements, in particular those involving a necessary energetic cost greater than the energy provided by muscles during a single contraction.

To set the body or a part of the human body in motion the motor system must be supplied with an amount of mechanical energy ΔE. Generally speaking, every modification of the movement parameters requires an adjustment of the system's mechanical energy – so an extra amount of energy has to be supplied or its surplus eliminated. In intentional movements, executed in response to an intention, the source of required energy lies in skeletal muscles which operate as actuators or energy transducers in charge of the motor system's propulsion. In periodic movements, a full cycle of mechanical energy change of the body segments is completed during one full movement cycle. The work supplied to the system, indispensable to execute this cycle, constitutes its net energetic cost and the effects of this work become visible as the kinematical relationships describing the movement course. This

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work is performed by skeletal muscles whose energetic expenditure – of approximately 400 J/kg per contraction – is relatively low [1], [2]. Therefore it must be expected that, for example, the maximal values of kinematical parameters generated during cyclic movements, being dependent on the mechanical energy involved in the movement, will be limited, among other reasons, on account of energetic features of muscle.

The main parameters describing time-repeated phenomena are: frequency, its inverse – period, and amplitude. In cyclic movements of human body segments, the parameters such as the step or crank length respectively in running and cycling or half of the angular distance covered out by the oar in the rowlock, etc., can represent the movement amplitude. The movement course and its kinematics will be determined by the energy involved in its execution (supplied to the system) and since kinematical relationships of cyclic movements can be expressed by relations referring to amplitude and frequency regarded as variables one can expect to observe fairly unequivocal and apparent relations between its amplitude and frequency, especially in the case of movements of maximal intensity. Maximal movement amplitude can thus be expected to decrease with the increase of movement frequency. Such a relationship seems to be quite predictable but only in systems (and conditions) in which potential energy cannot be accumulated. Not all types of human movement belong to this category. While such a possibility does not exist in cycling or rowing (where the amount of potential energy associated with the movement is negligible), the involvement of potential energy (of gravity or tissue elasticity) in, e.g., running, walking or hopping can influence considerably the movement course, and consequently, the nature of the amplitude–frequency relation.

Most of research projects on cyclic motor activity focus on running and walking. They provide ambivalent data on the considered theme. SAITO et al. [3] present the results of their research on the relationship between step length, velocity and step frequency in sprint running. The relationships presented by these authors are rigorous; however, contrary to their hypotheses, the relation was not a decreasing one. Indeed, an increase of the amplitude (step length) was observed for step frequency up to approximately 180 min^{-1}. LAURENT and PAILHOUS [4] observed during their research on walking that step length and frequency both correlate with speed but are relatively independent of each other. MARTIN et al. [5] observed the influence of crank length (amplitude) on pedaling frequency and described the relation as inverse. Other equivocal observations were made in works on cyclic movements of limbs or their segments. POST et al. [6] did not notice any distinct relation between the amplitude and frequency of shoulder joint movements, PEPER and BEEK [7] noticed a slight amplitude decrease with frequency increase of forearm movements.

The works cited above all concern cyclic movements of human body segments. These movements were however executed in different conditions, different to the point that the differences may have had determined the nature (or the absence) of the relations discussed there. Moreover, in the majority of those works the applied ranges of movement amplitudes and frequencies did not cover the entire set of movement possibilities. The question may thus be asked whether cyclic movements performed by the human motor system (bone–joint–muscle) are characterized by an unequivocal and observable relation between movement amplitude and frequency. Moreover, what lies at the basis of such a relationship – what specific features determine its character; do the effects of a cyclic activity of the human motor system such as movement velocity, mechanical energy, external work unmistakably depend on movement amplitude? If this is the case, then what is the nature of this relationship, within what amplitude or frequency ranges and in what way can they manifest themselves in complex movements?

Accordingly, to ascertain whether the biomechanical features of the human motor system determine (as exposed in the above hypothesis) an observable relation between movement frequency and amplitude, appropriate measurements should be carried out in conditions excluding the influence of external forces on the movement course, for the entire practicable frequency band and possible amplitude range. Moreover, the research should focus on one joint having possibly simple motor functions (one degree of freedom) to make sure the observation and the measurement of the movements it executes is as thorough and precise as possible.

The objective of the research undertaken derives from the questions posed above and consists in researching hypothetical relations between the amplitude of the values describing a cyclic joint movement and its frequency, as well as the conditions in which such relations exist and, possibly, the sources (phenomena) lying at the basis of such relationships.

2. Material and method

The research was conducted on 11 volunteers, students of physical education, aged 21 ± 1.4 years, with average body mass of 74.9 ± 4.6 kg and the height of 1.82 ± 0.04 m.

The subjects were instructed to execute cyclic flexion–extension movements with the highest possible frequency f_M within a defined (approximately) joint angle whose central value ($\alpha_0 \cong 1.8$ rad) roughly corresponded to the mid-range of the elbow joint movement. Each subject executed two series of movements successively for each of the five values (defined approximately – no physical limitations were applied) identified within the movement range (in other words, five values of the joint angle amplitude). These approximate values were respectively: ~0.1 rad; ~0.4 rad; ~0.8 rad; ~1.0 rad and 1.2 rad. This is how the amplitude–frequency relationship could be determined for each subject in at least five configurations (out of 10 configurations, on account of amplitude differences between the two series). Every measurement was preceded by a test phase, during which the subjects were supposed to get accustomed to the requirements. The measurement phase lasted around a dozen seconds. The first seconds were assigned for setting the limb in motion and for attenuating transients, and the actual 5-second-long measurement phase was carried out once the limb's movement parameters were settled.

The experimental set-up used in the measurements played a two-fold role. It stabilized the subject's body position and eliminated the influence of gravity on the limb movements. Subjects were seated with the shoulder-joint abducted to 90°. The arm rested on a fixed support (part of the experimental set-up) and was immobilized with an elastic band. The forearm rested on a horizontally movable rotative support – a lever with a vertical rotation axis lined up with the elbow joint axis. The subject's forearm and hand were fixed onto the movable lever with an elastic band. Thus, the lever could rotate along with the forearm following the elbow joint's flexion and extension movements. The movable segment of the experimental set-up was made from a light alloy to limit its inertia's influence on the limb's movements. The moment of inertia of this lever was $I_s = 0.01$ kg m^2. A potentiometer used to measure the lever's angular position and consequently – the joint angle, was placed in the lever's axis. A precise linear potentiometer was used (nonlinearity error $\delta = \pm\ 0.5$ % and 0.1% smoothness). The measurements were carried out using a 12 bit A/D converter with a sampling frequency of $f_p = 128$ Hz. The value directly measured was the relationship between joint angle and time $\alpha_j(t)$. For each movement frequency, numerical differentiation allowed one to establish the time-relationships of angular velocity $\omega(t) = d\alpha(t)/dt$ and acceleration $\varepsilon(t) = d\omega(t)/dt$ and their maximal amplitudes: ω_M and ε_M. Also the values and amplitudes of kinetic energy $E_{kM} = \{0.5 I_c \omega_M^2\}_{\max}$ and external power

$$P_M = \left\{ \frac{d}{dt}[E_k(t)] \right\}_{\max}$$

were calculated.

The moment of inertia $\underline{I_c}$ was defined as the sum of the moment of inertia of the experimental set-up's movable segment $\underline{I_s}$ and the moment of inertia of the limb $\underline{I_k}$ (forearm with hand, about the transversal axis of the elbow joint). The latter was estimated for every subject using the regression equations of ZATSIORSKY and SELUJANOV [8]. For the entire group of participants, the mean value of the limb's moment of inertia was $\underline{I_k} = 0.08 \pm 0.0086$ kg m^2.

3. Results

The maximal value of the amplitude of a cyclic (relatively symmetrical) movement in the elbow joint is limited by the range of joint angle changes and cannot exceed the half of it – approximately 1.4 rad. For safety reasons, the amplitude is usually slightly lower. This means that the joint movement amplitude can be comprised between 0 and approximately 1.4 rad. The maximal limiting value of movement frequency, which constitutes the upper limit of the frequency band at which autonomous movements can be executed (movements initiated solely by an action of the motor system), represents another parameter defining the range of possible realizations of a given movement type. Both ranges (amplitudes from 0 to 1.4 rad and frequency from 0 to $f_{\max}$) delineate the limits of the scope of possible realizations of cyclic movements. The question of whether they constitute the only limits for the said parameters is addressed in figure 1a. It presents the relation averaged for the 11 subjects between the maximal frequency f_M developed in cyclic forearm movements and the maximal movement amplitude α_M.

The pattern described by the measurement results presented in figure 1a exposes another limit in the range of possible realizations of elbow joint movements. It suggests that to increase the frequency of movements of a maximal nature, the movement amplitude must be decreased. Similarly, an increase of the movement amplitude entails the necessity to decrease its frequency. Clues pointing to the existence of such a relationship can be found in the publications of PEPER and BEEK [7] in which a linear amplitude decline was observed along with a frequency increase within the band of 1÷2.8 Hz and in the work of BEEK et al. [9] who occasionally observed a linear or even "hyperbolic" relation between movement amplitude and frequency for the movements with amplitudes below 0.5 rad within the band of 1÷6 Hz. A relationship of a similar character was observed in cyclic combined movements which involved several joints [10].

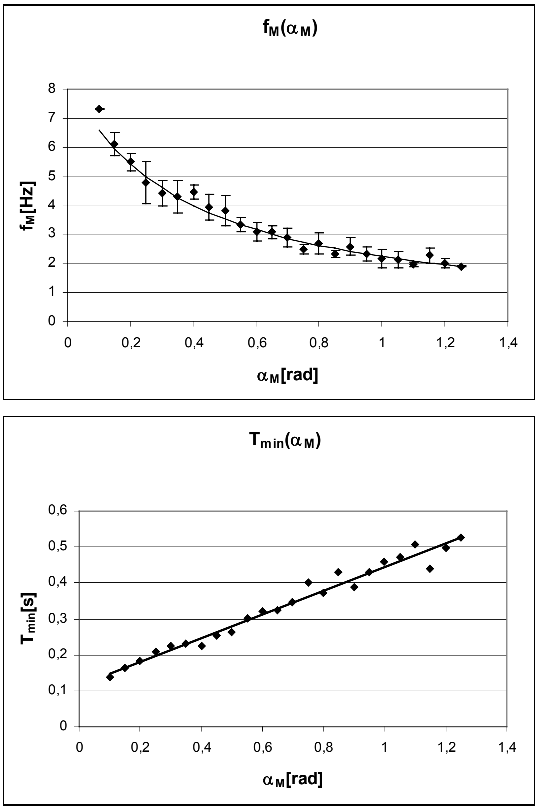


Fig. 1. Maximal frequency f_M (a) and movement period $T_{min} = 1/f_M$ (b) shown as functions of maximal amplitude α_M of cyclic forearm movements. Averaged measurement results for the 11 subjects are marked with points and the characteristic described by equation (1), with solid line

Unlike in those publications, the results discussed in the present work relate to the full movement range of the joint. It can thus be said that the obtained relationships $\alpha_M(f_M)$ provide a complete illustration of the relation between movement amplitude and frequency. This is confirmed by the fact that the type of relationship presented in figure 1a was found in each of the 11 subjects.

An inverse formula – the relation between movement amplitude and movement period T_{min} described in figure 1b – was analyzed in order to obtain more explicit information on the relationship discussed. It is clearly linear so the curve in figure 1a represents a shifted hyperbola, and the relation between maximal amplitude α_M and cyclic movement frequency f_M is an inverse relationship. The empirical relationship from figure 1b was described, using the least squares method, by the equation of a straight line, which following the substitution $T_{min} = 1/f_M$ becomes:

$$\alpha_M = \frac{3.048}{f_M} - 0.362, \quad R^2 = 0.97,$$

or in its general form: (1)

$$\alpha_M = \frac{a}{f_M} - b.$$

The above equation and the nature of the relation presented in figure 1b show that for a certain movement frequency f_{max} the amplitude value is $\alpha_M = 0$. The f_{max} is thus the limiting frequency setting the upper limit of the frequency band at which cyclic elbow joint movements can be executed. The said limiting frequency was $f_{max} = 8.4$ Hz in the group investigated. It is easily demonstrable that the value of the coefficient a in a general form of equation (1) equals the product of f_{max} and the value of amplitude α_M for which the amplitude of angular acceleration reaches its maximal value. The free term b in equation (1) equals the value of movement amplitude for which ε_M reaches its maximal value. This occurs for a movement frequency constituting half of the limiting frequency f_{max}.

Cyclic movements of human body segments constitute a specific type of oscillation, generally constrained, partly by muscle torque. A usually justifiable tendency to apply simplified, linear movement models can be noticed in publications devoted to cyclic movements in biological systems. We can quote works by authors such as ALEXANDER [11], BEEK et al. [9], [12], LACQUANITI et al. [13] and others, in which cyclic movements were regarded as driven harmonic oscillations.

The efficacy of such simplified models does not and cannot imply that every cyclic movement can be automatically considered similar to a harmonic movement and vice-versa; neither should such a possibility be automatically excluded. The nature of the movement type discussed in the present work is illustrated by the relationships presented in figures 2 and 3. They are – in figure 2 normalized to maximal values – a time dependent trajectory $\alpha_j(t)$, velocity $\omega(t)$, angular acceleration $\varepsilon(t)$, and kinetic energy $E_k(t)$ recorded for one subject (figure 2), as well as the relationships $\varepsilon(\alpha)$ showing a relation between the instantaneous value of acceleration ε and the joint angular position α_j for five values of the movement frequency f_M from the entire frequency band (figure 3). The nature of the

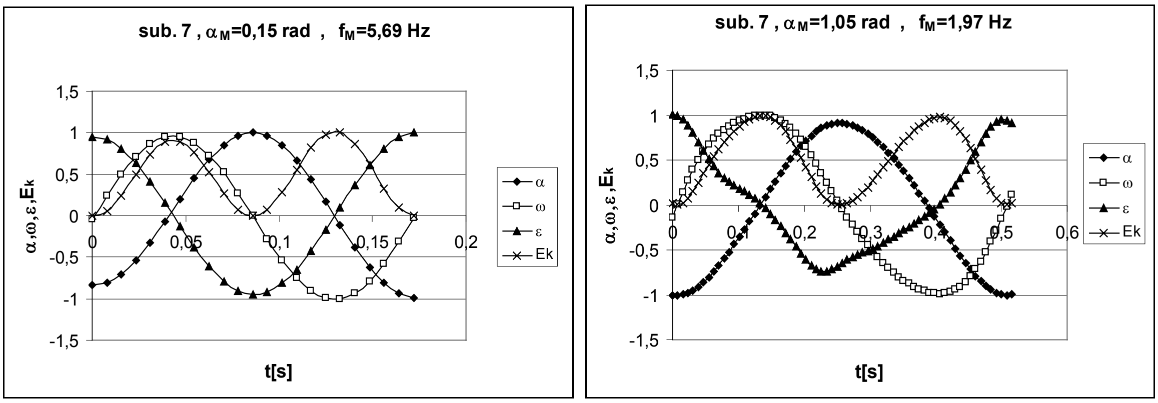
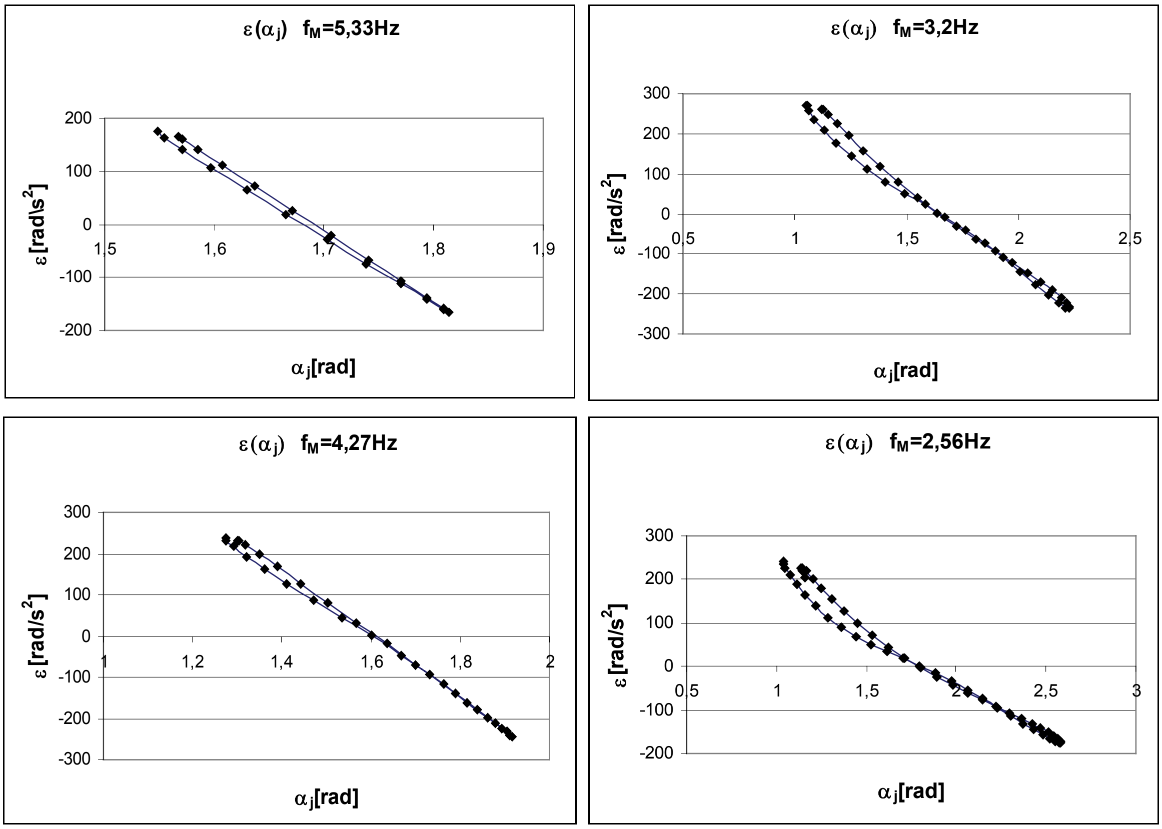


Fig. 2. Example of time versus kinematic parameters and kinetic energy recorded in a cyclic forearm movement for the two extreme values of amplitude α_M and frequency f_M



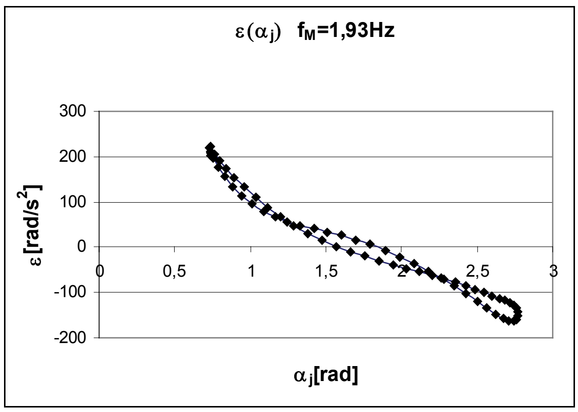


Fig. 3. Relationship between angular acceleration ε and joint angle α_j recorded in cyclic forearm movements at five different amplitudes α_M and frequencies f_M

relationships presented in figure 2 and, even more so, in figure 3 points to a considerable similarity of the studied movements to harmonic movements, which is, for example, confirmed by the linear (for low amplitudes and high frequencies and nearly linear for high amplitudes) relation between angular acceleration and joint angle. The existence of the said relation means that the sense of the functional component of torque generated at the joint (and producing the limb's angular acceleration) is directed opposite to angular displacement $\alpha = \alpha_j - \alpha_0$, and its instantaneous value is proportional to α. Consequently, it acts like a restoring torque produced by the linear joint stiffness K_s – the relation between the instantaneous value of angular acceleration ε and the joint angle α presented in figure 3 can be described as follows: $\varepsilon = -c \cdot \alpha$, while $M_u = I \cdot \varepsilon$, hence:

$$M_u = I \cdot \varepsilon = -I \cdot c \cdot \alpha = -K_s \cdot \alpha \,. \qquad (2)$$

The coefficient $K_s = I \cdot c$ can thus be regarded to be a "substitute stiffness" discernable at the joint, related to the functional torque M_u active in the joint. The results obtained within the framework of the present work indicate that the value of the said functional torque is proportional to the angular deflection of the limb from the central position α_0, and its maximal value shows a relation with movement frequency. It may be asked what causes the nonlinearity of the relation $M_u(\alpha)$ recorded in high amplitude movements (visible in figure 3). Such a nonlinearity appears only in the regions of low joint angle values, in other words when the limb is strongly flexed (α < 1 rad). This pattern is similar for all subjects. NAGASAKI [14] and WANN et al. [15] had observed a similar phenomenon and described it as resulting from the "minimum jerk" model of movement control.

It seems that the cause of this nonlinearity is prosaic and of a purely mechanical nature. As it occurs in all subjects, but only in the end region of joint flexion, it seems to be provoked by an additional distortion of peri-joint tissues (which appears in this interval of joint angle). Such distortions occur in the junction region of the exterior surface of the forearm and arm and both the surface area and the extent of the distortions increase as the joint angle decreases, which results in a locally increased stiffness K_s.

Figures 4, 5, 6 illustrate the experimental relationships between the maximal values of angular velocity amplitude ω_M, angular acceleration ε_M and kinetic energy E_{kM} dependent on the movement amplitude α_M. They delineate the edge of range of possible realizations of forearm situated under the curves thus traced out. In the figures, the measurement values (averaged over the group investigated) were marked with dots, and the solid curves were obtained by inserting the empirical characteristic described by equation (1) into the equations describing the relations between the

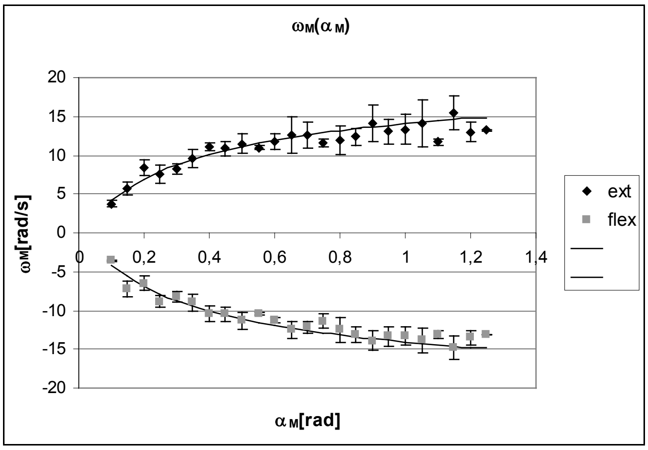


Fig. 4. Relationship between angular velocity ω_M and movement amplitude α_M in flexion and extension phase of cyclic forearm movements at the elbow joint. The solid line illustrates the characteristic described by equation (3)

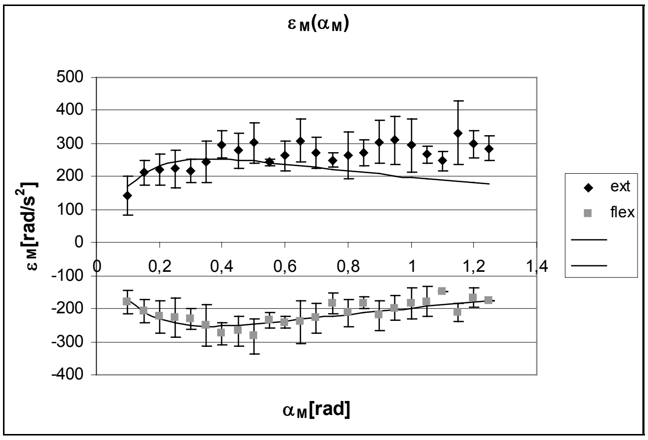


Fig. 5. Relationship between maximal value of angular acceleration ε_M and movement amplitude α_M in flexion and extension phase of cyclic forearm movements. The solid line illustrates the characteristic described by equation (4)

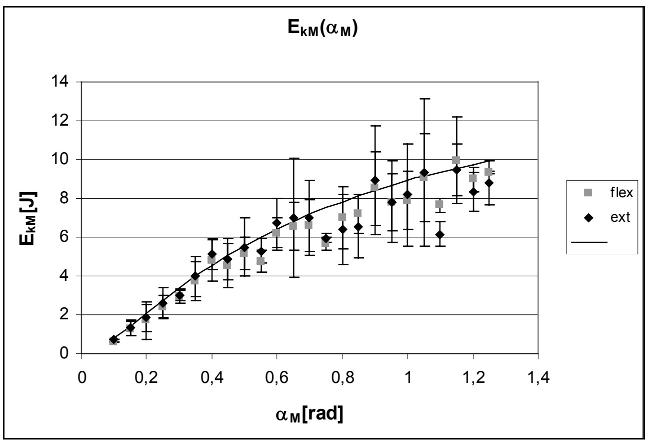


Fig. 6. Maximal value of kinetic energy E_{kM} observed in flexion and extension phase of cyclic forearm movements as a function of movement amplitude α_M. The solid line illustrates the characteristic described by equation (5)

amplitudes of deflection, velocity, and acceleration in harmonic movement:

$$\omega_M = 2\pi f_M \cdot \alpha_M = 2\pi \alpha_M \frac{3.048}{\alpha_M + 0.362}, \quad (3)$$

$$\varepsilon_M = (2\pi f_M)^2 \cdot \alpha_M = 4\pi^2 \alpha_M \left(\frac{3.048}{\alpha_M + 0.362} \right)^2, \quad (4)$$

$$E_{kM} = \frac{1}{2} I \omega_M^2 = \frac{1}{2} \cdot 0.09 \cdot 4\pi^2 \left(\frac{3.048 \cdot \alpha_M}{\alpha_M + 0.362} \right)^2. \quad (5)$$

The characteristics presented in these figures illustrate the degree of similarity of the movements discussed in the present work with a harmonic movement and help assess the influence of the aforementioned nonlinearity of the $\varepsilon_M(\alpha)$ characteristic on movement course, which is illustrated by the asymmetry of the curves with respect to the direction of movement. It has so far been established that such joint movements are actuated by the torque M_u, characterized by a form of stiffness. On account of the equivalent stiffness K_s related to the torque M_u generated at the joint, the limb can be regarded to be a second-order system with an eigenfrequency oscillation ω_0. This frequency depends on the system's stiffness and inertia:

$$\omega_0 = \sqrt{\frac{K_s}{I_c}}. \quad (6)$$

Considering the fact that both the nature of the movements investigated and the relations between the values describing their course appear to be determined and valid within the entire range of the amplitudes and frequencies observed, it can be inferred that the values of the kinematical parameters developed during their execution depend essentially on the value of the mechanical energy involved in the movement. This energy may in the present case amount to the kinetic energy accumulated in the moving segment of the limb and the potential elastic energy resulting from the joint stiffness, with a possible conversion of one form into the other. Total mechanical energy of a limb in motion is made up by the sum of the kinetic and potential energy produced by the activity of muscles – the flexors and extensors of the elbow joint. The energy thus accumulated results from the balance between the energy provided by the muscles and the energy dissipated due to resistance to movement. The present research is focused on the movements of maximal intensity and it can be said that the relationships shown in figures 1, 4, 5 and 6 constitute a particular (but indirect) energetic feature of the muscle-driven cyclic forearm movements. It has previously been implied that cyclic movements provide an example of constrained oscillations having a frequency near to the eigenfrequency of limbs oscillation. The latter can be modified by varying the value of K_s (according to the needs or conditions in which the movement is executed) within a limited range of values. The relationship between the stiffness K_s appearing in cyclic forearm movement and movement amplitude is presented in figure 7.

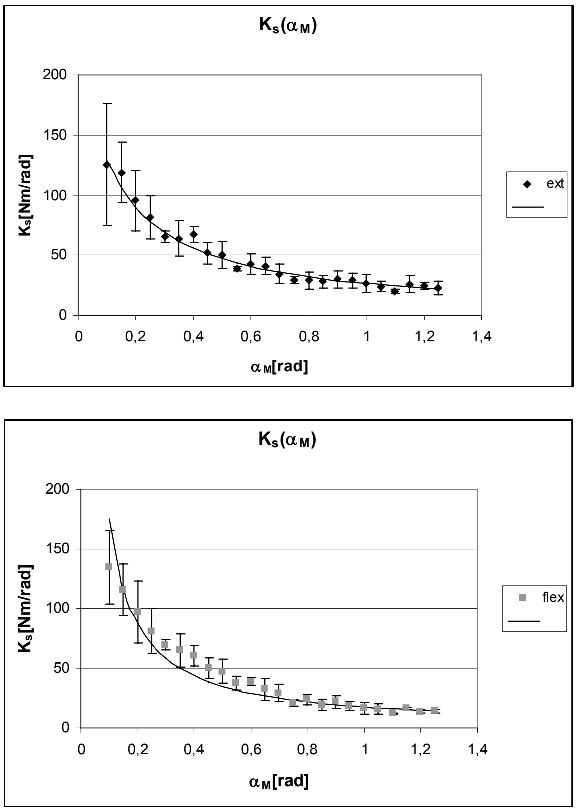


Fig. 7. Relationship between elbow joint stiffness K_s, appearing in cyclic movements, and movement amplitude α_M

This way of executing a movement, consisting in initiating an oscillating joint movement with a frequency near to the eigenfrequency, presents an advantage of a favourable relation between the cost (applied torque) and the result – movement amplitude [13], [15], [16]. Consequently, relationship (1) can, if we take into account relationship (6), be represented as follows:

$$\alpha_M = \alpha_{max} \frac{\dfrac{1}{2\pi \cdot f_M \sqrt{I_c}} - \sqrt{\dfrac{1}{K_{max}}}}{\sqrt{\dfrac{1}{K_{min}}} - \sqrt{\dfrac{1}{K_{max}}}}, \quad (7)$$

where:

α_{max} – the maximal value of elbow joint movement amplitude, equal to half the value of joint movement range,

f_M – the joint movement frequency,

K_{max} – the maximal value of stiffness K_s found in cyclic joint movements,

K_{min} – the minimal stiffness appearing in cyclic joint movements (similar to passive stiffness),

I_c – the moment of inertia of the part of the limb involved in movement.

The above equation testifies to the relation between the range of movement amplitudes, movement frequency and the corresponding range of stiffness changes at the joint. The origin of the said stiffness may be two-fold:

1. It may result from the action of an active component of muscle-torque with the following time dependence: $M_K = M_m \cdot \cos(2\pi \cdot f_m \cdot t)$. To produce this torque, the necessary activation of the flexor and extensor muscles would have to be adequate and in a proper phase with the joint angle time dependence $\alpha = \alpha_m \cdot \cos(2\pi \cdot f_m \cdot t + \phi)$.

2. It may be provoked by a simultaneous static tension of flexors and extensors, and in this case the stiffness occurring at the joint (constituted by the sum of the stiffnesses of extensors, flexors and passive tissues) bears the characteristics of a passive stiffness, which does not require any time-varying control.

In the stable state of forearm cyclic movement, the stiffness-control solution laid out in point 2 seems to be more favourable compared to that of point 1 insofar as:

• The amplitude of the variable component of the torque controlling the movement, and consequently the amount of effort required to execute it, is significantly lower.

• The phase relationships between muscle torque and joint angle time dependencies are simplified since the variable component of torque and the velocity of the joint movement are in the same phase.

• The total amount of muscle mechanical work (supplied by muscle torque) necessary to maintain the movement is fully used to overcome the viscous movement resistance. Therefore the necessity of energy expenditure for acceleration control disappears.

• The control process of the action of flexor and extensor muscles is simplified since in the present case they are activated synchronically (opposite in phase), which means that the process of muscle activation is conformable to Sherrington's law.

• Possible short-lasting disruption of the control signal (muscle torque) does not significantly perturb the movement course.

• The kinetic energy E_{kM} observed in the limb movement (figure 6), regarded as the so-called external work, is fully equivalent to the so-called elastic energy and as such cannot be considered a measure of the energetic cost of the movement.

For a number of reasons, it can be argued that when executing cyclic movements, we resort (at least partly) to the strategy described in point 2. The nature of the relationship between the movement amplitude α_M and the frequency f_M (relationship (1)), in which frequency is raised to the first power, and the relatively low inter-subject scatter of the recorded characteristics suggest that their form results from objective laws underlying movement execution and not simply from the subjects' individual motor qualities. Moreover, research reveals the relationship between muscle tension and stiffness [16], [17], [18], [19], [20], which means that this way of actuating cyclic movements is made possible by objective factors.

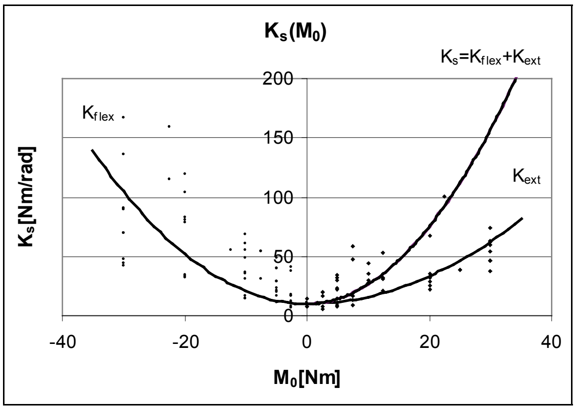


Fig. 8. Example (measured for 3 subjects) of relationship between elbow joint stiffness K_s and static torque M_0 developed by flexor (curve K_{flex}) and extensor (curve K_{ext}) muscles at joint angle $\alpha = 1.8$ rad. Curve K_s shows elbow joint stiffness as a sum of K_{flex} and K_{ext} in the case of flexors and extensors muscle cocontraction [21]

An example (going beyond the scope of the present paper) of the relationship between the stiffness at the elbow joint and the tension of elbow joint flexors and extensors is presented in figure 8. The said relationship (defined using the identification procedure for 3 subjects) illustrates the quality of the relation between muscle stiffness and static tension (measured by the static torque developed by elbow joint flexors and extensors). It is a square relationship – the stiffness of the tendon–muscle complex rises with its tension squared.

The above clues pointing to the possibility of exploiting the influence of static tension of antagonist muscles on the control of joint stiffness do not constitute sufficient evidence proving the control of cyclic joint movements is exercised in this manner. Neither is it proved by the increase in the level of electric activity in muscles involved in movement observed by FELDMAN [17] along with the increase of movement frequency.

It is the author's opinion that convincing evidence should be sought through the analysis of quantitative relations between relationships describing cyclic movements combined with the analysis of phase relations between kinematical parameters and the electric activity of muscles involved in movement.

4. Conclusion

The character of time trajectories of kinematical parameters and the quantitative description of the relationships between them testify to the fact that cyclic flexion/extension elbow joint movements executed at a maximal intensity must be qualified as a constrained oscillation similar to harmonic oscillation. This property is observable for the investigated limb practically in the entire range of possible movement amplitudes and frequencies. It can indeed be said that its behaviour is similar to that of a linear second-order system described by the parameters of inertia I, damping B and stiffness K. The maximal values of kinematical parameters observed in a steady state of such system movement depend on I, B, K parameters and the mechanical energy involved in the movement. This energy may constitute the sum of the kinetic energy accumulated by the moving part of the limb (with the moment of inertia I) and potential elastic energy related to the stiffness K. Because this energy is supplied to the system as a result of the mechanical work executed by muscles actuating the elbow joint and because the investigated movements were of "maximal" intensity, the relationships presented in this work $\omega_M(\alpha_M)$, $\varepsilon_M(\alpha_M)$, and even more so $E_{kM}(\alpha_M)$ indirectly constitute a characteristic of the energetic efficiency of muscle effort in cyclic movements.

The amplitude and frequency of maximally intensive cyclic elbow joint movements show an inverse relationship described by a hyperbola equation. The range of maximal frequencies is comprised between approx. 2 Hz (for movements with maximal amplitude of approx. 1.25 rad) and approx. 8.4 Hz (for amplitudes close to 0). The linear forms of the relationship between angular acceleration ε and joint angle α and the relationship between movement amplitude and movement frequency show the importance of the joint stiffness K_s in the execution of cyclic movements. This stiffness influences decisively the eigenfrequency of the limb oscillation, and the range of its possible changes is considerable (from approx. 14 to 130 Nm/rad). This fact must be considered an important clue pointing to the fact that cyclic movements of a maximal intensity are executed by initiating oscillations at the joint whose frequency is similar or equal to the eigenfrequency of the limb's oscillation. Another argument in favour of this way of cyclic movements drive is the empirical (square) relationship between the joint stiffness K_s and the tension of the muscles by which it is activated (this tension is measured by the value of the static muscle torque operating at the joint). The above property makes it possible to intentionally control the value of joint stiffness by initiating a static simultaneous tension of the two groups of antagonist muscles actuating the joint (through their simultaneous activation). It also implies the possibility of adjusting – according to needs – the eigenfrequency of the limb oscillation.

References

[1] ALEXANDER R. McN., BENNET CLARK H.C., *Storage of elastic strain energy in muscle and other tissues*, Nature, 1977, 265, 114–117.

[2] CAPELLI C., PENDERGAST D.R., TERMIN B., *Energetics of swimming at maximal speed in humans*, European Journal of Applied Physiology and Occupational Physiology, 1998, 78, 385–393.

[3] SAITO M., KOBAYASHI K., MIYASHITA M., HOSHIKAWA T., *Temporal patterns in running*, [in:] Biomechanics IV: Proc. of IV International Seminar on Biomechanics (ed. by R.C. Nelson and Ch.A. Morehouse), University Park Press, Baltimore, London, Tokyo, 1974, 106–111.

[4] LAURENT M., PAILHOUS J., *A note on modulation of gait in man. Effects of constraining stride length and frequency*, Human Movement Science, 1986, 5, 333–343.

[5] MARTIN J.C., BROWN N.A., ANDERSON F.C., SPIRDUSO W.W., *A governing relationship for repetitive muscular contraction*, Journal of Biomechanics, 2000, 33, 969–974.

[6] POST A.A., PEPER C.E., BEEK P.J., *Relative phase dynamics in perturbed interlimb coordination: the effects of frequency and amplitude*, Biological Cybernetics, 2000, 83, 529–542.

[7] PEPER C.E., BEEK P.J., *Are frequency-induced transitions in rhythmic coordination mediated by a drop in amplitude?* Biological Cybernetics, 1998, 79, 291–300.

[8] ZATSIORSKY V., SELUYANOV V., *The mass and inertia characteristics of the main segments of human body*, [in:] Biomechanics VIII-B: Proc. of VIII International Congress of Biomechanics (ed. by M. Matsui and K. Kobayaski) Champaign, Il., Human Kinetics, 1983, 1152–1159.

[9] BEEK P.J., RIKKERT W.E., van WERIGNEN P.C., *Limit cycle properties of rhythmic forearm movements*, Journal of Experimental Psychology: Human Perception and Performance, 1996, Vol. 22, No. 5, 1077–1093.

[10] MARTIN J.C., SPIRDUSO W.W., *Determinants of maximal cycling power: crank length, pedaling rate and pedal speed*, European Journal of Applied Physiology, 2001, 84, 413–418.

[11] ALEXANDER R. McN., *Optimum muscle design for oscillatory movements*, Journal of Theoretical Biology, 1997, 184, 253–259.

[12] BEEK P.J., SCHMIDT R.C., MORRIS A.W., SIM M.Y., TURREY M.T., *Linear and nonlinear stiffness and friction in biological rhythmic movements*, Biological Cybernetics, 1995, 73, 499–507.

[13] LACQUANITI F., LICATA F., SOECHTING J.F., *The mechanical behavior of the human forearm in response to transient perturbations*, Biological Cybernetics, 1982, 44, 35–46.

[14] NAGASAKI H., *Asymmetrical trajectory formation in cyclic forearm movements in man*, Experimental Brain Research, 1991, 87, 653–661.

[15] WANN J., NIMMO-SMITH I., WING A.M., *Relation between velocity and curvature in movement: equivalence and divergence between a power law and minimum–jerk model*, Journal of Experimental Psychology: Human Perception and Performance, 1988, Vol. 14, No. 4, 622–637.

[16] LATASH M.L., *Virtual trajectories, joint stiffness and changes in the limb natural frequency during single-joint oscillatory movements*, Neuroscience, 1992, 49, 1, 209–220.

[17] FELDMAN A.G., *Superposition of motor programs – I. Rhythmic forearm movements in man*, Neuroscience, 1980, Vol. 5, 81–90.

[18] GOTTLIEB G.L., AGARWAL G.C., *Compliance of single joints: elastic and plastic characteristics*, Journal of Neurophysiology, 1988, Vol. 59, No. 3, 937–951.

[19] CALANCIE B., STEIN R.B., *Measurement of rate constants for the contractile cycle of intact mammalian muscle fibers*, Biophysical Journal, 1987, Vol. 51, 149–159.

[20] ZAWADZKI J., KORNECKI S., *Influence of joint angle and static tension of muscle on dynamic parameters of the elbow joint*, [in:] Biomechanics XI-A; Proc. of XI International Congress of Biomechanics (ed. by: G. de Groot, A.P. Hollander, P.A. Huijing, G.J. van Ingen Schenau), Free University Press, Amsterdam, 1988, 94–99.

[21] ZAWADZKI J., *Zależność sztywności w stawie łokciowym od stanu naprężenia mięśni zginaczy i prostowników stawu*, Annales Universitatis Mariae Curie-Skłodowska, Medicina, 2006, Vol. LX, Suppl. XVI, No. 8, 452–455.