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Optimization solutions depend on the choice of coordinate system

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The assumption that optimization results depend on coordinate system selected to describe a biomechanical model is tested by comparing two solutions obtained with generalized and natural coordinate systems. A 5-degrees of freedom planar musculoskeletal model actuated by 9 Hill-type musculotendon units was created to simulate lifting a leg up. Each individual muscle force was discretized into a set of independent design variables, and an inverse dynamic parameter optimization method was used in the computations. The optimal time characteristics of the predicted muscle forces for both solutions are presented. Some remarks concerning the efficiency of natural coordinates for solving optimal control problems are also included.

Key words: *optimization solutions, biomechanical model description, coordinate systems*

1. Introduction

The process of optimization depends upon a number of factors. The most important ones, well-documented in the literature on the subject (e.g. [11]), indicate the proper choice of the merit function, the algorithm applied in the calculations, or the quality of the input data. The results obtained in [6] also emphasize the significant influence of the coordinate system selection on static optimization solutions. It can be then assumed that any kind of optimization, including the parameter one, is affected by the latter factor.

There are two basic kinds of parameter optimization. The first approach is well-known in biomechanics [2], [3], [12]. The basic idea starts from the assumption that any control history can be parameterized by means of a set of nodal points, from which the control function is reconstructed by linear interpolation. The system of differential equations is integrated forwards in time, and the nodal points are the variables optimized. The drawback of this procedure is that it may be too time-consuming for multibody biomechanical systems [2].

The other approach is called the inverse dynamic parameter optimization. Both state or control variables can be discretized and optimized [1], [10], whereas remaining variables are computed through inverse dynamics. A significant advantage of this formulation is that it does not require the system of differential equations to be numerically integrated and thus makes such an approach computationally efficient.

The aim of this work was to examine whether the inverse dynamics parameter optimization used for a biomechanical application depends on the choice of the coordinate system. Two coordinate systems were chosen to describe the biomechanical model. The first one is represented by five independent generalized coordinates, whereas the other one by eight dependent natural coordinates.

The other goal of the work was to check the usefulness of natural coordinate system for solving a parameter optimization problem.

A simple and predictable motor task like lifting a leg up is analysed.

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2. Biomechanical model

The biomechanical model of the human leg is composed of three rigid bodies. The unconstrained model has 5 degrees of freedom. Its configuration is described by five generalized coordinates (x_H, y_H, φ_1, φ_2, φ_3) (figure 1, left).

denotes the torques exerted by the passive joint structures (ligaments), **C** is the 5×8 Jacobian matrix of the constraints imposed on the hip joint trajectory and associated with constant distance conditions between two successive basic points (with non-constant coefficients in m), and **λ** is the 5×1 vector of constraint reactions (in N) in the hip joint and in the Lagrange multipliers (in N/m). The explicit form of

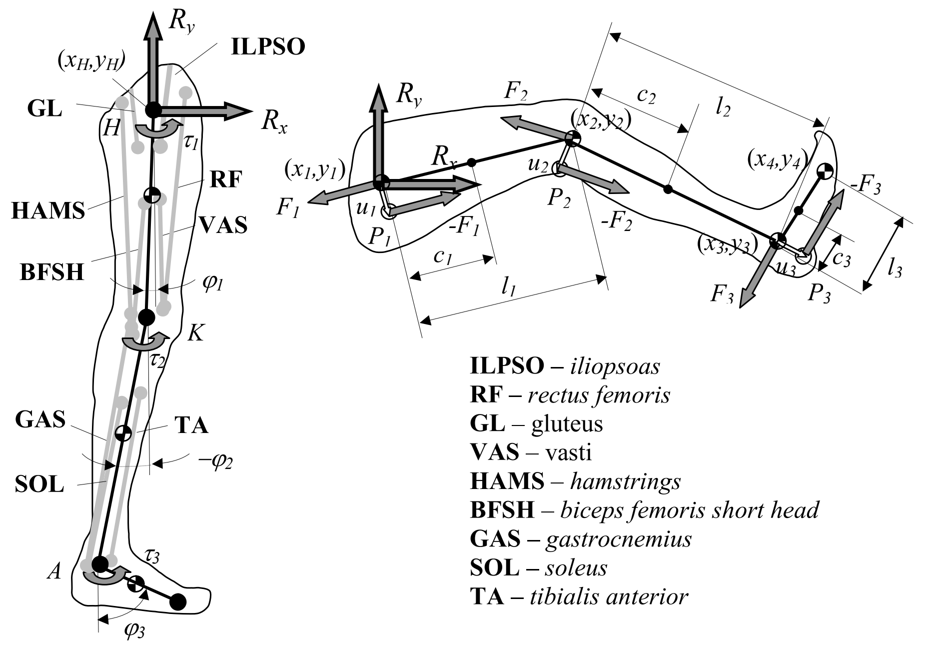

Fig. 1. Biomechanical model defined in generalized and natural coordinate systems

Eight coordinates (x_1, y_1, ..., x_4, y_4), which are the Cartesian coordinates of the basic points located at the joints and at the metatarsal part of the foot, are selected as natural coordinates (figure 1, right). The muscle net torques are represented by the pairs of the forces $\mathbf{F}_i$ and $-\mathbf{F}_i$ acting on the beginnings and ends (P_i) of the unit vectors $\mathbf{u}_i$ oriented perpendicularly to the segment i [8]. The gravitational force exerted on each segment is distributed between its basic points P_i and P_{i+1}. The positions of the leg in figure 1 indicate the range of motion of the analysed motor task. Both models share the same muscle apparatus consisting of nine muscles.

The dynamic equations of the motion of the model described in natural coordinates can be given by

$$\mathbf{M}\ddot{\mathbf{q}} = \mathbf{f} + \mathbf{B}^T(\mathbf{q})(\boldsymbol{\tau} + \boldsymbol{\tau}_{\text{pas}}) - \mathbf{C}^T(\mathbf{q})\boldsymbol{\lambda}, \qquad (1)$$

where **M** is the 8×8 mass matrix of the system (constant coefficients in kg), **q** is the 8×1 vector of natural coordinates, **f** is the vector of external loads containing the gravitational forces, $\mathbf{B}^T$ is the 8×3 matrix of control distribution (nonzero entries in m^{-1}), $\boldsymbol{\tau}$ is the vector of the net muscle torques at the joints, $\boldsymbol{\tau}_{\text{pas}}$ equation (1) is reported elsewhere [6], whereas equation (1) expressed in generalized coordinates can be found in [4].

Throughout the optimization process the net muscle torques $\boldsymbol{\tau}$ in equation (1) are replaced by individual muscle forces

$$\boldsymbol{\tau} = \begin{bmatrix} F_m^1 r_H^1 + F_m^2 r_H^2 - F_m^4 r_H^4 - F_m^5 r_H^5 \\ F_m^2 r_K^2 + F_m^3 r_K^3 - F_m^5 r_K^5 - F_m^6 r_K^6 - F_m^7 r_K^7 \\ -F_m^7 r_A^7 - F_m^8 r_A^8 + F_m^9 r_A^9 \end{bmatrix}, \qquad (2)$$

where F_m^i (in N) (i = 1, ..., 9) are the forces of ILPSO, RF, VAS, GL, HAMS, BFSH, GAS, SOL and TA, and r_H^i, r_K^i, r_A^i are the arms (in meters) of these forces with respect to the hip, knee and ankle joints.

The muscle forces were calculated by means of a 4-element Hill-type muscle model. The physiological and geometrical as well as elastic and damping properties of the muscles were estimated according to [7], [9], [13].

3. Optimization

As a novel approach, the inverse dynamics parameter optimization executed within the natural coordinates' domain is briefly explained in this section.

First, a set of $n = 61$ nodal points of the analysed motor task uniformly distributed over the time is defined. Second, the parameterization of the constraint and muscle forces is performed resulting in the vector of design variables

$$\mathbf{q}_{\text{opt}} = [\mathbf{q}_1^T \;\; \ldots \;\; \mathbf{q}_j^T \;\; \ldots \;\; \mathbf{q}_n^T]^T , \tag{3}$$

where

$$\mathbf{q}_j = [\lambda_{1j} \;\; \ldots \;\; \lambda_{5j} \;\; F_m^{1j} \;\; \ldots \;\; F_m^{9j}]^T \tag{4}$$

is the 14×1 vector containing the values of design variables at the time step t_j ($j = 1, ..., n$). Inserting equation (2) into (1) yields a system of linear equations in the components of $\mathbf{q}_j$

$$\mathbf{A}_j \mathbf{q}_j = (\mathbf{C}_j^T, -\mathbf{B}_j^T)^{-1}(-\mathbf{M}\ddot{\mathbf{q}}_j + \mathbf{f} + \mathbf{B}_j^T \boldsymbol{\tau}_{\text{PAS}_j}), \tag{5}$$

where $\mathbf{A}_j$ is the 8×14 matrix containing the arms of the muscle forces at the time instant t_j. Finally, the inverse dynamics parameter optimization scheme can be written as

$$\begin{cases} \text{minimize} & F_0(\mathbf{q}_{\text{opt}}) \\ \text{with respect to} & \\ \mathbf{A}\mathbf{q}_{\text{opt}} = \mathbf{b} \;\; \Leftrightarrow & \begin{bmatrix} \mathbf{A}_1 & & & & \\ & \ddots & & \mathbf{0} & \\ & & \mathbf{A}_j & & \\ & \mathbf{0} & & \ddots & \\ & & & & \mathbf{A}_n \end{bmatrix} \begin{bmatrix} \mathbf{q}_1 \\ \vdots \\ \mathbf{q}_j \\ \vdots \\ \mathbf{q}_n \end{bmatrix} = \begin{bmatrix} \mathbf{b}_1 \\ \vdots \\ \mathbf{b}_j \\ \vdots \\ \mathbf{b}_n \end{bmatrix} \\ \text{and} & \\ & 0 \le a_{ij} \le 1 \quad \text{or} \quad 0 \le u_{ij} \le 1, \end{cases} \tag{6}$$

where F_0 is a cost function, $\mathbf{A}$ is the 488×854 block diagonal matrix of linear constraints, a_{ij} (i =1, ..., 9) (u_{ij} when the activation dynamics is included) are the lower and upper bounds imposed on the muscle activations/excitations, and

$$\mathbf{b}_j = (\mathbf{C}_j^T, -\mathbf{B}_j^T)^{-1}(-\mathbf{M}\ddot{\mathbf{q}}_j + \mathbf{f} + \mathbf{B}_j^T \boldsymbol{\tau}_{\text{PAS}_j}). \tag{7}$$

The cost function, taken from the work [5], ensures the physiologically relevant distribution of muscle forces, minimizing mechanical energy expenditure. The muscle activations/excitations were estimated using the inversion of the contraction (contraction and activation) dynamics as described in the work [1].

4. Results

A twenty-two-year-old male, with the height of 180 cm and the body mass of 70 kg, lifted up his right leg several times. The recorded activity regarded as being the most appropriate was subjected to the analysis. The data acquisition process as well as handling the raw kinematic data were similar to those described elsewhere [4].

A sequential programming method (SQP) was used to solve optimization problems. The computations were performed with a Pentium IV 3.1 GHz PC. The initial and final conditions were the same for both models. Some technical aspects associated with the computations are specified in the table. The initial guess "0" means that all the components of $\mathbf{q}_{\text{opt}}$ were set at zero, whereas SO denotes the static optimization solution obtained with starting values set at zero as well.

The solution in generalized coordinates was achieved faster and with the lower value of the cost function. When the contraction dynamics is only implemented, the differences in the computation time in both coordinate sets seem to be sensitive to the initial guess. The computation lasts for a drastically longer time when the activation dynamics is included, even if a simple relation between the activation a and the excitation u [14] is presumed.

The patterns of muscle forces, computed from the same initial guess (the table, col. 4) for reasons of comparison, are shown in figure 2. Having similar shapes, these characteristics reveal the significant differences between both solutions. The gluteus, vasti and soleus forces have been predicted in natural

Table. Computation time and selected attributes of optimal solutions

Coordinate systems	No. of optimized variables	Cost function [J]	Comp. time (contr. dynamics) Init. guess "0"	Comp. time (contr. dynamics) Init. guess SO	Comp. time (act. dynamics) Init. guess SO
Generalized	671	514.76	26.79 min	15.95 min	2.03 h
Natural	854	575.96	28.05 min	44.88 min	8.43 h

coordinate system only. It is evident that the solutions obtained depend on the coordinates chosen to describe the biomechanical model. This seemingly confusing result can be explained by the fact that the searching spaces of both models differ in dimension, and a sequential programming method used to solve the optimization problem does not guarantee a global minimum. Thus the differences between solutions may originate from the numerical procedures used for the computations.

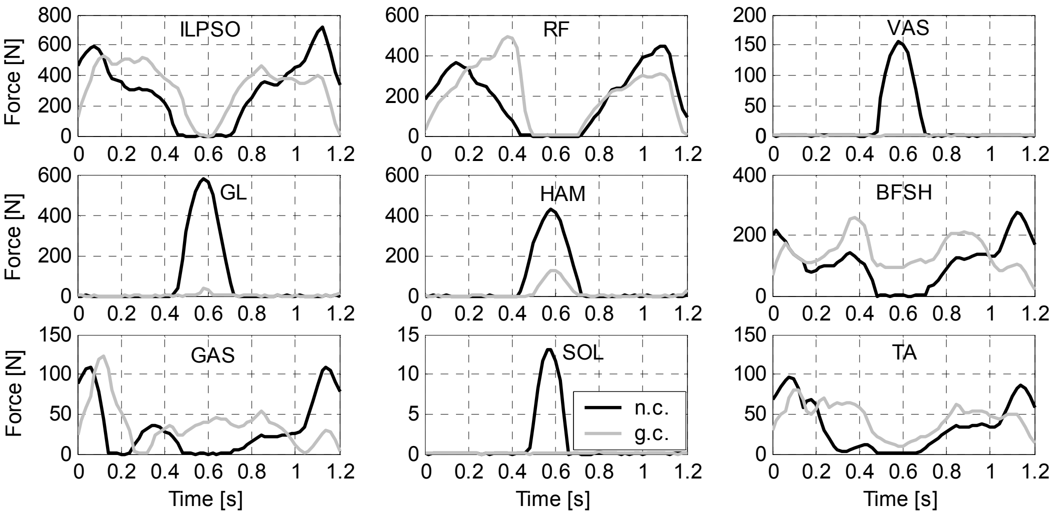

Fig. 2. Optimal muscle forces predicted in natural and generalized coordinate systems

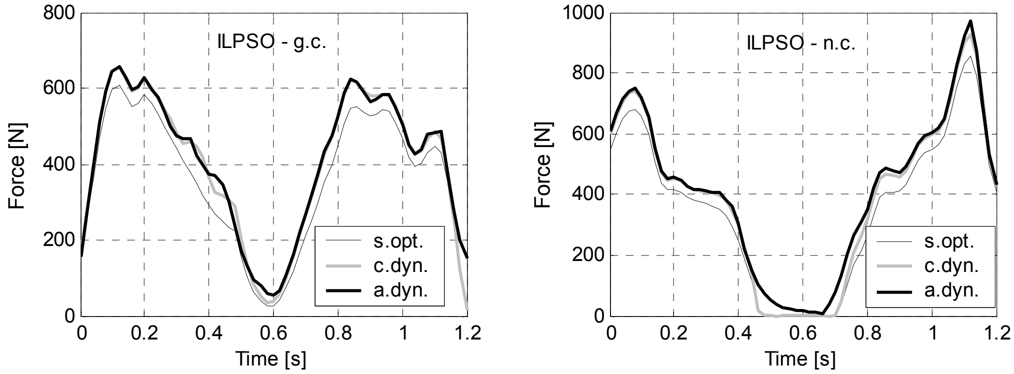

Fig. 3. Iliopsoas forces obtained for different optimization procedures

All the force estimates for the iliopsoas muscle as one of the main contributors to the movement under analysis are presented in figure 3. It is clearly visible that the static (the thin black line) and inverse dynamics parameter optimization (the thick black and grey lines) do not provide the same solutions. A high computational burden associated with the use of the activation dynamics is not justified by the quality of the results obtained because they closely match those achieved just through the contraction dynamics.

5. Conclusions

Inverse dynamic parameter optimization solutions, as static optimization solutions, depend on the coordinate sets used to describe the configuration of the biomechanical model. The differences that occur seem to arise from the numerical background of the simulations. Thus a researcher should take into consideration the fact that the choice of dependent or independent coordinates, usually carried out at the beginning of the modelling process, may influence the final results.

Static and inverse dynamic parameter optimization solutions for the lifting of the leg up are not equivalent. This observation does not match gait analysis results, where static and dynamic optimization solutions are practically equivalent.

A large number of muscle forces predicted in natural coordinate system are important from the physiological standpoint. The computations in such an environment, although time-consuming, provide a useful framework for optimization methods.

In recent years, a growing complexity of musculoskeletal models in biomechanics can be observed. The results obtained, after the activation dynamics was implemented, show that sophisticated models do not necessarily offer better solutions.

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