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Numerical estimation of burn degree of skin tissue using the sensitivity analysis methods

E. MAJCHRZAK, M. JASIŃSKI

Silesian University of Technology, Gliwice, Poland

The numerical analysis of thermal processes proceeding in the skin tissue due to external heat flux is presented. Heat transfer in the skin tissue was assumed to be transient and one-dimensional. Thermophysical parameters of successive skin layers are different, at the same time in subdomains of dermis and subcutaneous region the internal heating resulting from blood perfusion and metabolism is taken into account. The degree of the skin burn can be predicted on the basis of the so-called Henriques integrals. The paper deals with the sensitivity analysis of these integrals with respect to the thermophysical parameters. In numerical computations, the boundary element method has been used. In the final part of the paper, the results of computations are presented.

Key words: burn of skin tissue, numerical estimation, thermophysical parameters

1. Introduction

The skin is treated as a multi-layer domain in which one can distinguish the epidermis, dermis and subcutaneous region, with blood perfusion and metabolic heat source in the latter two regions. The skin surface is subjected to an external heat flux which is a function of time. Heat transfer in the skin tissue is assumed to be transient and one-dimensional. In the paper, the mathematical model and also the numerical algorithm based on the boundary-element method are presented. This model allows us to predict the skin temperature and time that passes till the appearance of the symptoms of the first-, second- and third-degree burns due to external heat flux effect. Because thermophysical parameters of the skin vary widely from person to person, the analysis of the sensitivity of temperature field and burn predictions to these variations is also carried out. In the final part of the paper, the results of computations are shown.

2. Governing equations

The skin is divided into three layers, i.e. epidermis Ω_1 of thickness L_1 [m], dermis Ω_2 of thickness $L_2 - L_1$ and subcutaneous region Ω_3 of thickness $L_3 - L_2$. Thermophysical parameters of these subdomains are equal to λ_e [W/(mK)] (thermal conductivity) and c_e [J/(m^3 K)] (specific heat per unit of volume), $e = 1, 2, 3$.

The transient bioheat transfer in the domain of skin is described by the following system of equations [6], [10], [11]:

$$x \in \Omega_e : c_e \frac{\partial T_e}{\partial t} = \lambda_e \frac{\partial^2 T_e}{\partial x^2} + k_e (T_B - T_e) + Q_{me}\,, \tag{1}$$

where G_e [(m^3 of blood/s)/m^3 of tissue)] is the blood perfusion rate, c_B [J/(m^3 K)] is the volumetric specific heat of blood, T_B is the arterial blood temperature and Q_{me} [W/m^3] is the metabolic heat source. It should be stressed that for the epidermis subdomain ($e = 1$) $G_1 = 0$ and $Q_{m1} = 0$.

On the contact surfaces between subdomains considered the continuity conditions in the following form

$$x \in \Gamma_{e,e+1} : \begin{cases} -\lambda_e \dfrac{\partial T_e}{\partial x} = -\lambda_{e+1} \dfrac{\partial T_{e+1}}{\partial x}, \\ T_e = T_{e+1} \end{cases} \qquad e = 1, 2 \tag{2}$$

are given. Additionally

$$x \in \Gamma_0 : \begin{cases} q = q_0, & t \le t_0, \\ q = -\alpha (T_1 - T^\infty), & t > t_0, \end{cases} \tag{3}$$

where $q_1 = -\lambda_1 \partial T_1 / \partial x$, q_0 is the given boundary heat flux, t_0 is the exposure time, α is the heat transfer coefficient, T^∞ is the ambient temperature. For conventionally assumed internal boundary limiting the system, no flux condition can be taken into account. A quadratic initial temperature distribution between 32.5 °C at the surface and 37 °C at the base of the subcutaneous region was used [11].

Thermal damage of skin begins when the temperature at the basal layer (the interface between epidermis and dermis) rises above 44 °C (317 K). HENRIQUES [4] found that the degree of skin damage could be predicted on the basis of the integrals

$$I_b = \int_0^\tau P_b(T_b) \exp\left[-\frac{\Delta E}{R T_b(t)}\right] dt \tag{4}$$

and

$$I_d = \int_0^\tau P_d(T_d)\exp\left[-\frac{\Delta E}{RT_d(t)}\right]dt\,, \tag{5}$$

where $\Delta E/R$ [K] is the ratio of activation energy to universal gas constant, P_b, P_d [1/s] are the preexponential factors, while T_b [K] is the temperature of basal layer (the surface between epidermis and dermis) and T_d [K] is the temperature of dermal base (the surface between dermis and subcutaneous region).

The first-degree burns are said to occur when the value of the burn integral (4) is within the interval $0.53 < I_b \le 1$, while the second-degree burns when $I_b > 1$ [4], [11]. The third-degree burns are said to occur when $I_d > 1$. So in order to determine the values of integrals I_b, I_d the heating curves and next the cooling curves representing the basal layer and dermal base must be known.

3. Sensitivity analysis

The sensitivity analysis of bioheat transfer has been carried out with respect to the parameters λ_e, c_e, G_e, Q_{me}. These parameters we denote by p_s, s = 1, 2, ..., 10, this means $p_1 = \lambda_1$, $p_2 = \lambda_2$, $p_3 = \lambda_3$, $p_4 = c_1$, $p_5 = c_2$, $p_6 = c_3$, $p_7 = G_2$, $p_8 = G_3$, $p_9 = Q_{m2}$, $p_{10} = Q_{m3}$. If the direct method of sensitivity analysis is used [2], [3], [9], then we should consider ten additional boundary-initial problems [8]

$$\begin{cases} x \in \Omega_e : & c_e \dfrac{\partial U_{es}}{\partial t} = \lambda_e \dfrac{\partial^2 U_{es}}{\partial x^2} - G_e c_B U_{es} + \left(\dfrac{c_e}{\lambda_e}\dfrac{\partial \lambda_e}{\partial p_s} - \dfrac{\partial c_e}{\partial p_s}\right)\dfrac{\partial T_e}{\partial t} \\ & + \dfrac{1}{\lambda_e}\dfrac{\partial \lambda_e}{\partial p_s}[G_e c_B T_e - (G_e c_B T_B + Q_{me})] + \dfrac{\partial G_e}{\partial p_s} c_B (T_B - T_e) + \dfrac{\partial Q_{me}}{\partial p_s}, \\ x = 0 : & \begin{cases} Q_{1s} = -\dfrac{1}{\lambda_1}\dfrac{\partial \lambda_1}{\partial p_s} q_0, & t \le t_0, \\ Q_{1s} = -\alpha U_{1s} - \dfrac{1}{\lambda_1}\dfrac{\partial \lambda_1}{\partial p_s} q_1, & t > t_0, \end{cases} \\ x = L_e : & \begin{cases} U_{es} = U_{e+1,s}, \\ \dfrac{1}{\lambda_e}\dfrac{\partial \lambda_e}{\partial p_s} q_e + Q_{es} = \dfrac{1}{\lambda_{e+1}}\dfrac{\partial \lambda_{e+1}}{\partial p_s} q_{e+1} + Q_{e+1,s}, \quad e = 1, 2, \end{cases} \\ x = L : & Q_{3s} = 0, \\ t = 0 : & U_{es} = 0, \end{cases} \tag{6}$$

where

$$U_{es} = \frac{\partial T_e}{\partial p_s} \tag{7}$$

and

$$Q_{es} = -\lambda_e \frac{\partial U_{es}}{\partial x} . \tag{8}$$

For example, the sensitivity with respect to the thermal conductivity of epidermis ($p_1 = \lambda_1$) is determined by the following boundary-initial problem

$$\begin{cases} x \in \Omega_e : & c_e \frac{\partial U_{e1}}{\partial t} = \lambda_e \frac{\partial^2 U_{e1}}{\partial x^2} - G_e c_B U_{e1} + \frac{c_e}{\lambda_e} \frac{\partial \lambda_e}{\partial \lambda_1} \frac{\partial T_e}{\partial t} \\ & + \frac{1}{\lambda_e} \frac{\partial \lambda_e}{\partial \lambda_1} [G_e c_B T_e - (G_e c_B T_B + Q_{me})], \\ x = 0 : & \begin{cases} Q_{11} = -\frac{1}{\lambda_1} q_0, & t \le t_0, \\ Q_{11} = -\alpha U_{11} - \frac{1}{\lambda_1} q_1, & t > t_0, \end{cases} \\ x = L_e : & \begin{cases} U_{e1} = U_{e+1,1}, \\ \frac{1}{\lambda_e} \frac{\partial \lambda_e}{\partial \lambda_1} q_e + Q_{e1} = \frac{1}{\lambda_{e+1}} \frac{\partial \lambda_{e+1}}{\partial \lambda_1} q_{e+1} + Q_{e+1,1}, \end{cases} \\ x = L : & Q_{31} = 0, \\ t = 0 : & U_{e1} = 0, \end{cases} \tag{9}$$

where $\partial\lambda_1/\partial\lambda_1 = 1$, $\partial\lambda_2/\partial\lambda_1 = 0$, $\partial\lambda_3/\partial\lambda_1 = 0$, while the sensitivity with respect to the volumetric specific heat of dermis ($p_5 = c_2$) is connected with the boundary initial problem

$$\begin{cases} x \in \Omega_e : & c_e \frac{\partial U_{e5}}{\partial t} = \lambda_e \frac{\partial^2 U_{e5}}{\partial x^2} - G_e c_B U_{e5} - \frac{\partial c_e}{\partial c_2} \frac{\partial T_e}{\partial t} , \\ x = 0 : & \begin{cases} Q_{15} = 0, & t \le t_0, \\ Q_{15} = -\alpha U_{15}, & t > t_0, \end{cases} \\ x = L_e : & \begin{cases} U_{e5} = U_{e+1,5}, \\ Q_{e5} = +Q_{e+1,5}, & e = 1, 2, \end{cases} \\ x = L : & Q_{35} = 0, \\ t = 0 : & U_{e5} = 0, \end{cases} \tag{10}$$

where $\partial c_1/\partial c_2 = 0$, $\partial c_2/\partial c_2 = 1$, $\partial c_3/\partial c_2 = 0$.

It should be pointed out that the analysis of the temperature changes due to the changes of p_s requires the knowledge of primary solution, because in the mathematical model of the sensitivity, the values resulting from this solution appear. Taking into account forms (4), (5) of the functionals I_b , I_d , the sensitivity of these integrals with respect to the parameters p_s is calculated using the formulas

$$\frac{\partial I_b}{\partial p_s} = \int_0^{\tau} P_b \frac{\Delta E}{R T_b^2} \exp\left[-\frac{\Delta E}{R T_b} \right] U_{bs} \, d\,t \tag{11}$$

and

$$\frac{\partial I_d}{\partial p_s} = \int_0^{\tau} P_d \frac{\Delta E}{R T_d^2} \exp\left[-\frac{\Delta E}{R T_d} \right] U_{ds} \, d\,t \,, \tag{12}$$

where $T_b = T_1 (L_1, t) = T_2 (L_1, t)$, $T_d = T_2 (L_2, t) = T_3 (L_2, t)$ (c.f. equations (2)) and $U_{bs} = U_{1s} (L_1, t) = U_{2s} (L_1, t)$, $U_{ds} = U_{2s} (L_2, t) = U_{3s} (L_2, t)$ (c.f. equations (6)).

The change of burn integrals connected with the change of parameter p_s results from the Taylor formula limited to the first-order sensitivity, this means

$$I_b(p_s \pm \Delta p_s) = I_b(p_s) \pm \frac{\partial I_b}{\partial p_s} \Delta p_s \tag{13}$$

and

$$I_d(p_s \pm \Delta p_s) = I_d(p_s) \pm \frac{\partial I_d}{\partial p_s} \Delta p_s \,. \tag{14}$$

4. Boundary element method

The primary and also the additional problems resulting from the sensitivity analysis have been solved using the 1st scheme of the BEM for 1D transient heat diffusion [1], [7]. So, the following equations for successive layers of skin are considered

$$c_e \frac{\partial F_e}{\partial t} = \lambda_e \frac{\partial^2 F_e}{\partial x^2} + S_e \,, \tag{15}$$

where $F_e = F_e (x, t)$ denotes the temperature or functions resulting from the sensitivity analysis, $S_e = S_e (x, t)$ are the source functions. The functions S_e take a form

- for the primary problem

$$x \in \Omega_e: \; S_e = G_e c_B (T_B - T_e) + Q_{me}, \tag{16}$$

- for the problems of the sensitivity with respect to p_s

$$S_e = -G_e c_B U_{es} + \left(\frac{c_e}{\lambda_e} \frac{\partial \lambda_e}{\partial p_s} - \frac{\partial c_e}{\partial p_s} \right) \frac{\partial T_e}{\partial t} + \frac{1}{\lambda_e} \frac{\partial \lambda_e}{\partial p_s} [G_e c_B T_e - (G_e c_B T_B + Q_{me})]$$
$$+ \frac{\partial G_e}{\partial p_s} c_B (T_B - T_e) + \frac{\partial Q_{me}}{\partial p_s}. \tag{17}$$

At first, we introduce the time grid

$$0 = t^0 < t^1 < t^2 < \ldots < t^{f-1} < t^f < \ldots < t^F < \infty, \quad \Delta t = t^f - t^{f-1}. \tag{18}$$

If the 1st scheme of the BEM is taken into account, then the boundary integral equations (for successive layers of skin – $e = 1, 2, 3$) corresponding to the transition $t^{f-1} \to t^f$ are of the form [1], [7]

$$F_e(\xi, t^f) + \left[\frac{1}{c_e} \int_{t^{f-1}}^{t^f} F_e^*(\xi, x, t^f, t) J_e(x,t) dt \right]_{x=L_{e-1}}^{x=L_e}$$
$$= \left[\frac{1}{c_e} \int_{t^{f-1}}^{t^f} J_e^*(\xi, x, t^f, t) F_e(x,t) dt \right]_{x=L_{e-1}}^{x=L_e} + \int_{L_{e-1}}^{L_e} F_e^*(\xi, x, t^f, t^{f-1}) F_e(x, t^{f-1}) dx$$
$$+ \frac{1}{c_e} \int_{L_{e-1}}^{L_e} S_e(x, t^{f-1}) \int_{t^{f-1}}^{t^f} F_e^*(\xi, x, t^f, t) dt \, dx, \tag{19}$$

where F_e^* are the fundamental solutions given by formula

$$F_e^*(\xi, x, t^f, t) = \frac{1}{2\sqrt{\pi a_e (t^f - t)}} \exp\left[-\frac{(x-\xi)^2}{4a_e(t^f - t)} \right], \tag{20}$$

where ξ is the point at which the concentrated heat source is applied and $a_e = \lambda_e / c_e$.

The heat fluxes resulting from the fundamental solutions are equal to

$$J_e^*(\xi, x, t^f, t) = -\lambda_e \frac{\partial F_e^*(\xi, x, t^f, t)}{\partial x} = \frac{\lambda_e (x-\xi)}{4\sqrt{\pi}[a_e(t^f - t)]^{3/2}} \exp\left[-\frac{(x-\xi)^2}{4a_e(t^f - t)} \right], \tag{21}$$

while $J_e(x, t) = -\lambda_e \, \partial F_e / \partial x$. Assuming that for $t \in [t^{f-1}, t^f]$: $F_e(x, t) = F_e(x, t^f)$ and $J_e(x, t) = J_e(x, t^f)$ one obtains the following form of equations (19)

$$F_e(\xi, t^f) + g_e(\xi, L_e) J_e(L_e, t^f) - g_e(\xi, L_{e-1}) J_e(L_{e-1}, t^f)$$

$$= h_e(\xi, L_e) F_e(L_e, t^f) - h_e(\xi, L_{e-1}) F_e(L_{e-1}, t^f) + p_e(\xi) + z_e(\xi), \tag{22}$$

where

$$h_e(\xi, x) = \frac{1}{c_e} \int_{t^{f-1}}^{t^f} J_e^*(\xi, x, t^f, t)\, d\, t = \frac{\operatorname{sgn}(x-\xi)}{2} \operatorname{erf} c\left[\frac{|x-\xi|}{2\sqrt{a_e}\Delta t}\right] \tag{23}$$

and

$$g_e(\xi, x) = \frac{1}{c_e} \int_{t^{f-1}}^{t^f} F_e^*(\xi, x, t^f, t)\, d\, t$$

$$= \frac{\sqrt{\Delta t}}{\sqrt{\pi \lambda_e c_e}} \exp\left[-\frac{(x-\xi)^2}{4 a_e \Delta t}\right] - \frac{|x-\xi|}{2\lambda_e} \operatorname{erf} c\left[\frac{|x-\xi|}{2\sqrt{a_e \Delta t}}\right], \tag{24}$$

while

$$P_e(\xi) = \int_{L_{e-1}}^{L_e} F_e^*(\xi, x, t^f, t^{f-1})\, F_e(x, t^{f-1})\, dx$$

$$= \frac{1}{2\sqrt{\pi a_e \Delta t}} \int_{L_{e-1}}^{L_e} \exp\left[-\frac{(x-\xi)^2}{4 a_e \Delta t}\right] F_e(x, t^{f-1})\, dx, \tag{25}$$

at the same time

$$Z_e(\xi) = \int_{L_{e-1}}^{L_e} S_e(x, t^{f-1}) g_e(\xi, x)\, dx. \tag{26}$$

For $\xi \to L_{e-1}^+$ and $\xi \to L_{e-1}^-$ for each domain considered one obtains the system of equations

$$\begin{bmatrix} g_{11}^e & g_{12}^e \\ g_{21}^e & g_{22}^e \end{bmatrix} \begin{bmatrix} J_e(L_{e-1}, t^f) \\ J_e(L_e, t^f) \end{bmatrix} = \begin{bmatrix} h_{11}^e & h_{12}^e \\ h_{21}^e & h_{22}^e \end{bmatrix} \begin{bmatrix} F_e(L_{e-1}, t^f) \\ F_e(L_e, t^f) \end{bmatrix} + \begin{bmatrix} P_e(L_{e-1}) \\ P_e(L_e) \end{bmatrix} + \begin{bmatrix} Z_e(L_{e-1}) \\ Z_e(L_e) \end{bmatrix}. \tag{27}$$

The final form of resolving system results from the continuity conditions for $x = L_1$, $x = L_2$ and conditions given for $x = 0$ and $x = L$. So, for the primary problem and the time $t < t_0$ one has

$$\begin{bmatrix} -h_{11}^1 & -h_{12}^1 & g_{12}^1 & 0 & 0 & 0 \\ -h_{21}^1 & -h_{22}^1 & h_{22}^1 & 0 & 0 & 0 \\ 0 & -h_{11}^2 & g_{11}^2 & -h_{12}^2 & g_{12}^2 & 0 \\ 0 & -h_{21}^2 & g_{21}^2 & -h_{22}^2 & g_{22}^2 & 0 \\ 0 & 0 & 0 & -h_{11}^3 & g_{11}^3 & -h_{12}^3 \\ 0 & 0 & 0 & -h_{21}^3 & g_{21}^3 & -h_{22}^3 \end{bmatrix} \begin{bmatrix} T_1(0,t^f) \\ T_b(t^f) \\ q_b(t^f) \\ T_d(t^f) \\ q_d(t^f) \\ T_3(L,t^f) \end{bmatrix} = \begin{bmatrix} -g_{11}^1 q_0 + P_1(0) + Z_1(0) \\ -g_{21}^1 q_0 + P_1(L_1) + Z_1(L_1) \\ P_2(L_1) + Z_2(L_1) \\ P_2(L_2) + Z_2(L_2) \\ P_3(L_2) + Z_3(L_2) \\ P_3(L) + Z_3(L) \end{bmatrix}, \quad (28)$$

at the same time

$$q_1(0,t^f) = -\alpha[T_1(0,t^f) - T^\infty]\,. \quad (29)$$

This system of equations for $t > t_0$ is somewhat different (c.f. condition (3)) [5]. The solution of (28) determines the boundary temperature and heat fluxes at a time t^f for $x = 0$, L_1, L_2, L and next the temperature at the internal points can be found using the following formula

$$\xi \in \Omega_e: \begin{array}{l} T_e(\xi,t^f) = g_e(\xi,L_{e-1})\,q_e(L_{e-1},t^f) - g_e(\xi,L_e)\,q_e(L_e,t^f) \\ + h_e(\xi,L_e)T_e(L_e,t^f) - h_e(\xi,L_{e-1})T_e(L_{e-1},t^f) + P_e(\xi) + Z_e(\xi)\,. \end{array} \quad (30)$$

In the case of additional boundary initial problems (6) connected with the sensitivity analysis, for transition $t^{f-1} \to t^f$ one should solve the following system of equations

$$\begin{bmatrix} -h_{11}^1 & -h_{12}^1 & g_{12}^1 & 0 & 0 & 0 \\ -h_{21}^1 & -h_{22}^1 & h_{22}^1 & 0 & 0 & 0 \\ 0 & -h_{11}^2 & g_{11}^2 & -h_{12}^2 & g_{12}^2 & 0 \\ 0 & -h_{21}^2 & g_{21}^2 & -h_{22}^2 & g_{22}^2 & 0 \\ 0 & 0 & 0 & -h_{11}^3 & g_{11}^3 & -h_{12}^3 \\ 0 & 0 & 0 & -h_{21}^3 & g_{21}^3 & -h_{22}^3 \end{bmatrix} \begin{bmatrix} U_{1s}(0,t^f) \\ U_{bs}(t^f) \\ Q_{1s}(t^f) \\ U_{ds}(t^f) \\ Q_{2s}(t^f) \\ U_{3s}(L,t^f) \end{bmatrix}$$

$$
= \begin{bmatrix}
\frac{1}{\lambda_1} g_{11}^1 \frac{\partial \lambda_1}{\partial p_s} q_1(0, t^f) + P_1(0) + Z_1(0) \\
\frac{1}{\lambda_1} g_{21}^1 \frac{\partial \lambda_1}{\partial p_s} q_1(0, t^f) + P_1(L_1) + Z_1(L_1) \\
g_{11}^2 \left[\frac{1}{\lambda_2} \frac{\partial \lambda_2}{\partial p_s} - \frac{1}{\lambda_1} \frac{\partial \lambda_1}{\partial p_s} \right] q_b(t^f) + P_2(L_1) + Z_2(L_1) \\
g_{21}^2 \left[\frac{1}{\lambda_2} \frac{\partial \lambda_2}{\partial p_s} - \frac{1}{\lambda_1} \frac{\partial \lambda_1}{\partial p_s} \right] q_b(t^f) + P_2(L_2) + Z_2(L_2) \\
g_{11}^3 \left[\frac{1}{\lambda_3} \frac{\partial \lambda_3}{\partial p_s} - \frac{1}{\lambda_2} \frac{\partial \lambda_2}{\partial p_s} \right] q_d(t^f) + P_3(L_2) + Z_3(L_2) \\
g_{21}^3 \left[\frac{1}{\lambda_3} \frac{\partial \lambda_3}{\partial p_s} - \frac{1}{\lambda_2} \frac{\partial \lambda_2}{\partial p_s} \right] q_d(t^f) + P_3(L) + Z_3(L)
\end{bmatrix}, \tag{31}
$$

while

$$
Q_{2s}(L_1, t^f) = Q_{1s}(L_1, t^f) - \left[\frac{1}{\lambda_2} \frac{\partial \lambda_2}{\partial p_s} - \frac{1}{\lambda_2} \frac{\partial \lambda_1}{\partial p_s} \right] q_b(t^f) \tag{32}
$$

and

$$
Q_{3s}(L_2, t^f) = Q_{2s}(L_2, t^f) - \left[\frac{1}{\lambda_3} \frac{\partial \lambda_3}{\partial p_s} - \frac{1}{\lambda_2} \frac{\partial \lambda_2}{\partial p_s} \right] q_d(t^f) . \tag{33}
$$

As previously, the resolving system for $t > t_0$ is somewhat different [5]. The values of sensitivity function U_{es} at the internal points are calculated on the basis of formula

$$
\xi \in \Omega_e : \begin{array}{l} U_{es}(\xi, t^f) = g_e(\xi, L_{e-1})\, Q_{es}(L_{e-1}, t^f) - g_e(\xi, L_e)\, Q_{es}(L_e, t^f) \\ + h_e(\xi, L_e)\, U_{es}(L_e, t^f) - h_e(\xi, L_{e-1})\, U_{es}(L_{e-1}, t^f) + P_e(\xi) + Z_e(\xi) . \end{array} \tag{34}
$$

5. Examples of computations

In numerical computations, the following mean values of parameters have been assumed [5], [11]: $\lambda_1 = 0.235$ [W/(mK)], $\lambda_2 = 0.445$ [W/(mK)], $\lambda_3 = 0.185$ [W/(mK)], $c_1 = 4.3068 \cdot 10^6$ [J/(m^3 K)], $c_2 = 3.96 \cdot 10^6$ [J/(m^3 K)], $c_3 = 2.674 \cdot 10^6$ [J/(m^3 K)], $c_B = 3.9962 \cdot 10^6$ [J/(m^3 K)], $T_B = 37$ °C, $G_1 = 0$, $G_e = 0.00125$ [(m^3 of blood/s)/ m^3 of tissue] for $e = 2, 3$, $Q_{m1} = 0$, $Q_{me} = 245$ [W/m^3] for $e = 2, 3$ [11]. Preexponential factors: $P_b = 1.43 \cdot 10^{72}$ [1/s] for $T_b \geq 317$ [K] and $P_b = 0$ for $T_b < 317$ [K], while $P_d = 2.86 \cdot 10^{69}$ [1/s] for $T_d \geq 317$ [K] and $P_d = 0$ for $T_d < 317$ [K]. The ratio of activation energy to universal gas constant: $\Delta E/R = 55\,000$ [K]. The thicknesses of successive skin layers: 0.1, 2 and

10 [mm]. These layers have been divided into 10, 40 and 120 internal cells. Time step: $\Delta t = 0.05$ [s].

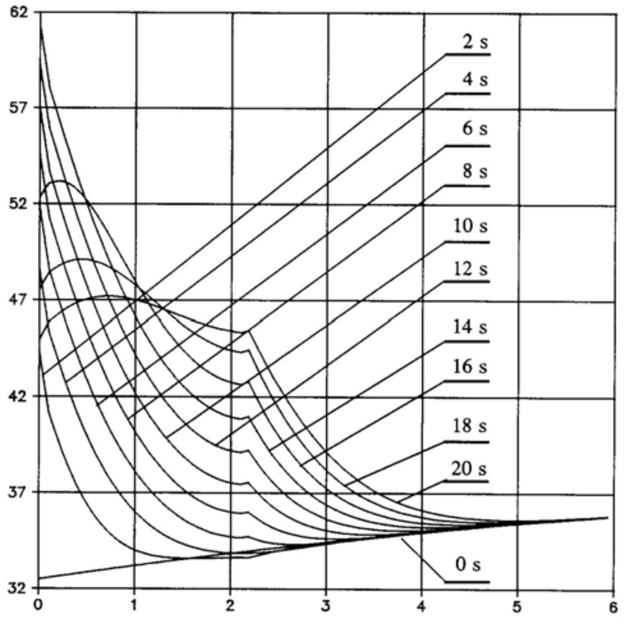

Fig. 1. Temperature distribution ($q_0 = 9000$ [W/m^2], $t_0 = 15$ [s])

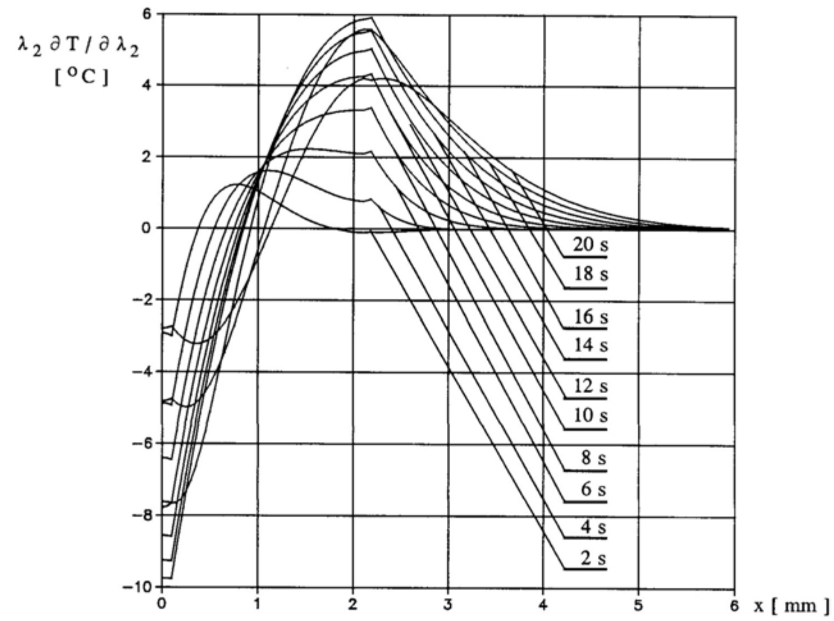

Fig. 2. Distribution of the function $\lambda_2 \cdot \partial T / \partial \lambda_2$ ($q_0 = 9000$ [W/m^2], $t_0 = 15$ [s])

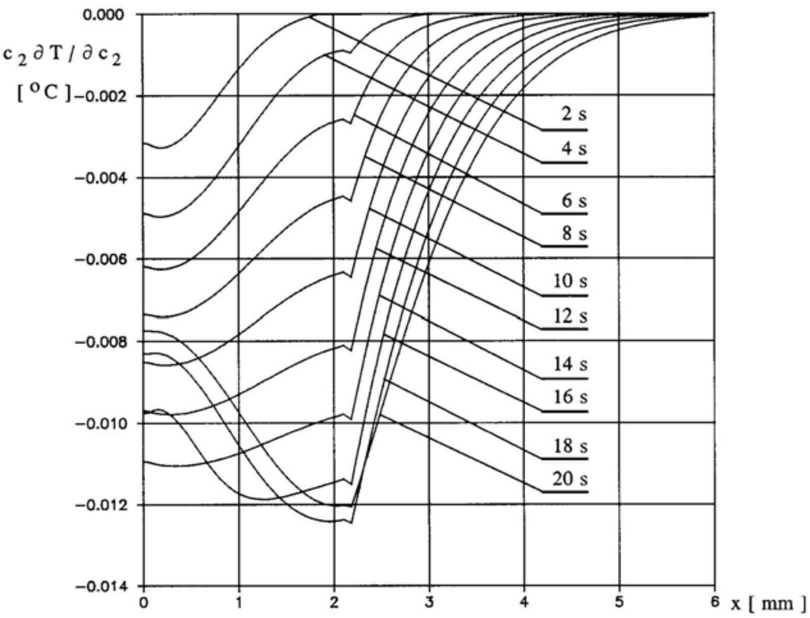

Fig. 3. Distribution of function $c_2 \cdot \partial T/\partial c_2$ (q_0 = 9000 [W/m^2], t_0 = 15 [s])

In the first example, on the skin surface the heat flux $q_0 = 9000$ [W/m^2] is assumed, the exposure time: t_0 = 15 [s]. For $t > t_0$ the Robin condition is accepted (α = 8 [W/m^{2}K], $T^{\infty} = 20$ °C). In figure 1, the temperature distribution in the skin domain is shown. Figures 2 and 3 illustrate the courses of the sensitivity functions $\lambda_2 \cdot \partial T/\partial \lambda_2$ and $c_2 \cdot \partial T/\partial c_2$.

The time necessary for the appearance of the symptoms of the first- and second-degree burn predicted for the mean values of parameters are equal to 11.8 [s] and 12.9 [s], respectively (c.f. figures 4 and 5). The third-degree burn does not appear. The sensitivity analysis of the burn integral I_b has been done with respect to all thermophysical parameters and the changes of I_b connected with the following changes of parameters: $\Delta\lambda_1$ = 0.025 [W/(mK)], $\Delta\lambda_2$ = 0.075, $\Delta\lambda_3$ = 0.025, Δc_1 = $1.32 \cdot 10^4$ [J/(m^3 K)], $\Delta c_2 = 3.3 \cdot 10^5$, $\Delta c_3 = 3.86 \cdot 10^5$, $\Delta G_e = 0.00125$ [(m^3 of blood/s)/m^3 of tissue], ΔQ_{me} = 245 [W/m^3], $e = 2, 3$ [5], [11] have been also calculated. It turned out that the changes of thermal conductivity (figure 4) and volumetric specific heat (figure 5) of the dermis subdomain are especially essential in the case considered. The diminution of these parameters causes that the time necessary for the burn appearance decreases as well. On the other hand, for the top value of λ_2 the second-degree burn does not appear (c.f. figure 4).

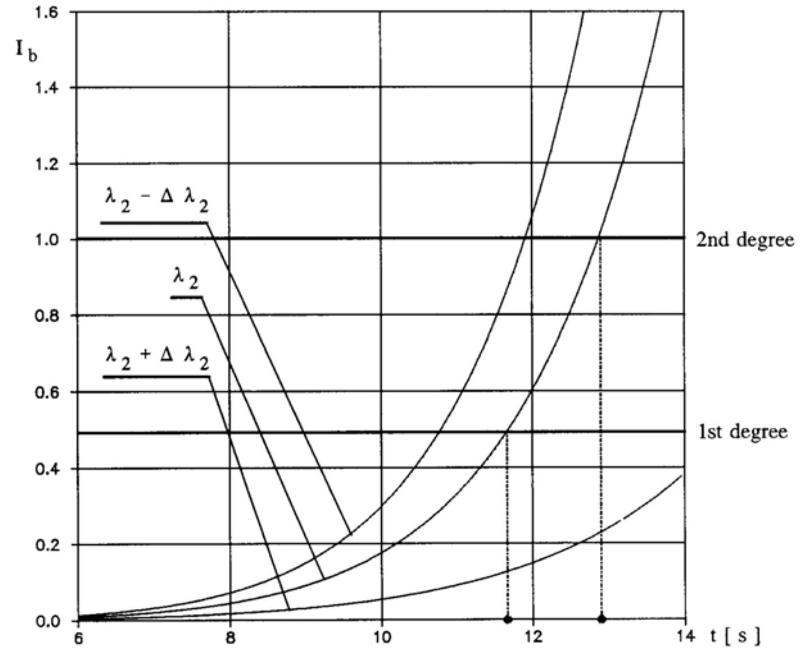

Fig. 4. Burn integral I_b and its sensitivity with respect to λ_2 (q_0 = 9000 [W/m^2], t_0 = 15 [s])

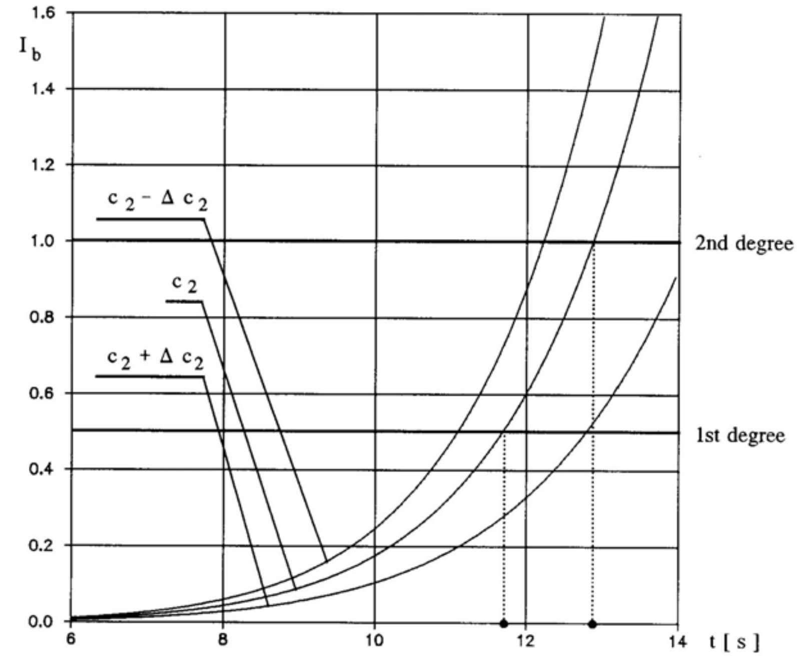

Fig. 5. Burn integral I_b and its sensitivity with respect to c_2 (q_0 = 9000 [W/m^2], t_0 = 15 [s])

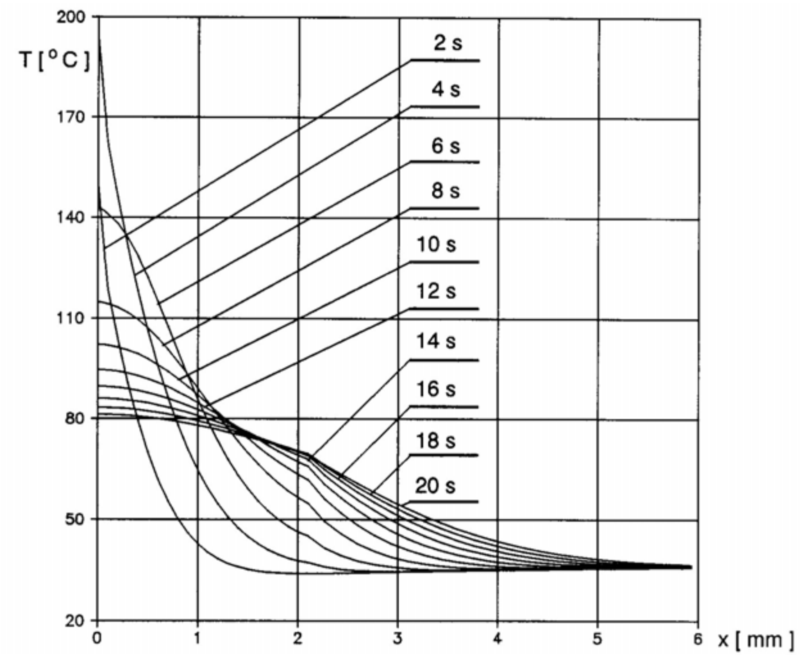

Fig. 6. Temperature profiles (q_0 = 80000 [W/m^2], t_0 = 5 [s])

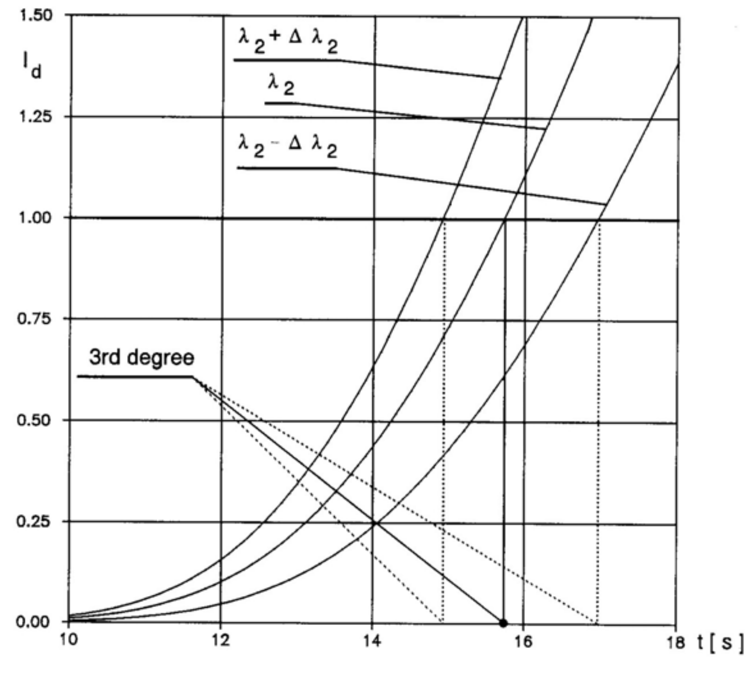

Fig. 7. Burn integral I_d and its sensitivity with respect to λ_2 (q_0 = 80000 [W/m^2], t_0 = 5 [s])

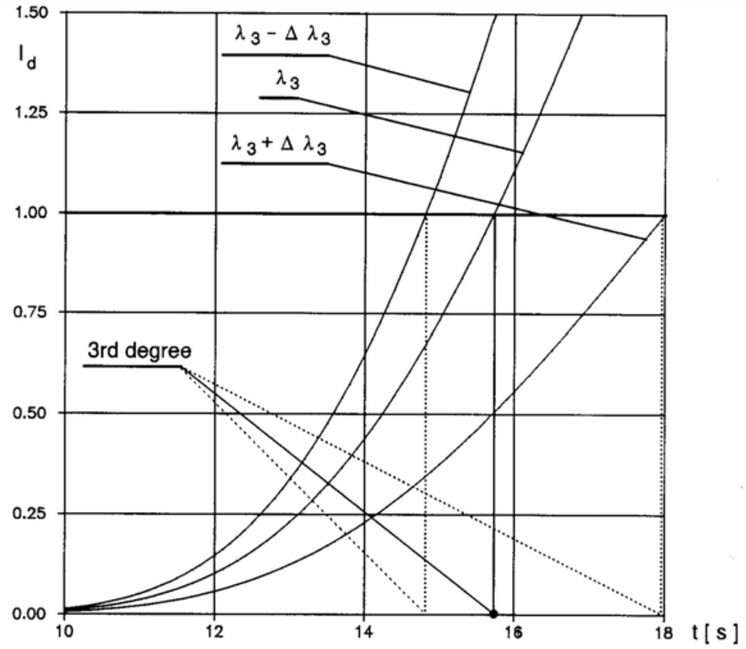

Fig. 8. Burn integral I_d and its sensitivity with respect to λ_3 (q_0 = 80000 [W/m^2], t_0 = 5 [s])

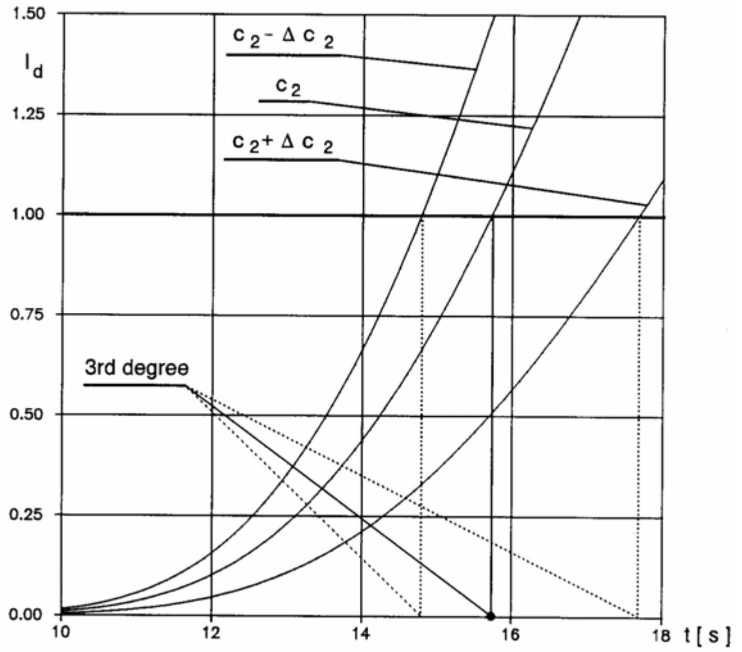

Fig. 9. Burn integral I_d and its sensitivity with respect to c_2 (q_0 = 80000 [W/m^2], t_0 = 5 [s])

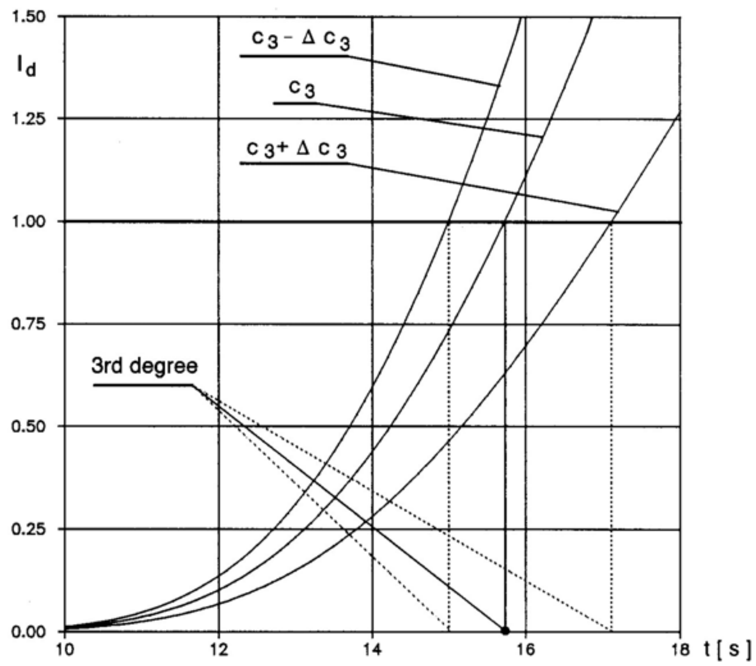

Fig. 10. Burn integral I_d and its sensitivity with respect to c_3 (q_0 = 80000 [W/m^2], t_0 = 5 [s])

In the second example, on the skin surface the heat flux q_0 = 80000 [W/m^2] is assumed, the exposure time t_0 = 5 [s]. In figure 6, the temperature distribution in the skin domain is shown. The time until the appearance of the third-degree burn at mean values of thermophysical parameters is equal to 15.75 [s]. In the case considered, the changes of thermal conductivity and volumetric specific heat in the dermis and subcutaneous region (c.f. figures 7, 8, 9 and 10) are especially essential. The changes of epidermis parameters and also the changes of perfusion rate of blood and metabolic heat source in the dermis and subcutaneous region have minimal influence on the third-degree burn predictions.

6. Conclusions

The algorithm allows us to estimate the effects of variations in thermophysical parameters on skin temperature and burn predictions. It turned out that blood perfusion term and metabolic heat source in the Pennes bioheat transfer equation (1) can be neglected in predicting the degree of burns of the skin tissue subjected to an external heat flux.

Due to wide variations in the thicknesses of skin from person to person, the sensitivity analysis with respect to the geometrical parameters of skin should be also investigated.

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