Acta of Bioengineering and Biomechanics
Vol. 10, No. 1, 2008

Kinematical analysis of mandibular motion in a sagittal plane

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The paper presents a kinematical model enabling the analysis of mandibular motion in a sagittal plane. Based on the recorded trajectories of incisors, the configuration coordinates were identified. The configuration coordinates explicitly identify the position and orientation of the mandible during motion. Such values are basic to the evaluation of alteration in muscle length and to the orientation of forces in particular muscles. This paper also deals with the influence of the coefficients of the weight matrix on the character of the solutions of the configuration coordinates applied in the model study of the kinematical chain. The results of the numerical calculations obtained demonstrated that the trajectory representation was in a considerable concordance with the data recorded.

Key words: *biomechanics, modelling, trajectory, stomatognathic system*

1. Introduction

Mandibular motions against a static maxillary bone and the state of dental arches are responsible for a proper speech and mastication processes. These functions are controlled by the central nervous system, providing guidance according to specific memorized codes of practice. A mandible is attached to the skull with a hinge joint and can move up and down within the range determined by the anatomy of the facial skeleton [1]. The diagnostic records of mandible motions, defined as functional registrations, are one of the most crucial factors determining the choice of treatment method in the dysfunctions of a stomatognathic system [6]. The combination of mandibular motions with the corresponding position of dental arches during prosthetic reconstructions poses a separate problem. The aforesaid problems have considerable importance, therefore the studies on the model analysis of mandible motions have been undertaken. They can provide the results useful both for the estimation of dental arch loads and for new designs of articulators. Based on kinematical relationships describing the nature of mandible motion it is possible to identify motion parameters. These relationships serve this purpose in the cognitive apparatus of mechanics. In order to describe positions, speeds and accelerations, it is necessary to make use of a mathematical apparatus efficient in numerical calculations. In this connection, in order to define the positions and the orientation of the mandible in selected stages of motion, one requires the information on configuration coordinates in these stages. Such information can be gained from clinical trials carried out, for instance, by applying computerized axial tomography. A complete image of mandible behaviour in motion is obtained as a result. However, such a method is very expensive and it is not recommended in numerous cases. Another method providing information on mandibular kinematics is plotting the trajectory of its characteristic points on plates by using a scriber. Such a method is less precise, but non-invasive and inexpensive in use. The precision of the methods with articula-

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Received: May 23, 2007
Accepted for publication: October 16, 2007

tors increases together with the experience of a researcher [13]. The data recorded should be further processed, regardless of the diagnostic devices applied, in order to carry out a kinematical analysis and should be adjusted to a numerical experiment. Only the combination of clinical trial and numerical study can provide us with comprehensive information on a functional behaviour of mandible. Mandibular kinematics can be considered in two ways [3], [4], [7]. The first one is by plotting the trajectory for selected points at given configuration coordinates (direct kinematics task). The second way consists in the plotting of configuration coordinates based on a given trajectory (inverse kinematics task). The configuration of the kinematical chain can be identified by means of several methods, i.e., analytical methods, numerical methods or neural networks. Each of the above-mentioned methods has its advantages and disadvantages. The major disadvantage of neural networks is the need for the preparation of training data. An appropriate preparation of such data is a time-consuming and laborious process, and the efficiency of identification of the kinematical chain depends substantially on the neural network 'training'. Therefore, the application of neural networks in the diagnostics of mandible kinematics is considered to be inefficient. The solutions derived from analytical method provide the most substantial benefits. However, these methods have also some disadvantages. The main one is the lack of the possibility of obtaining unique explicit solutions. Despite a considerably long time required for calculations compared with analytical methods, this drawback does not appear to be highly important in mandibular kinematics. Therefore, the model study of mandibular kinematics demonstrated in this paper applied the aforesaid numerical methods.

2. Biomechanics of mandibular motions

The range of mandibular motions is determined by the surfaces of temoporomandibular joints, ligaments, superficial tooth structure and by the spatial structure of the muscular system. There are two basic types of motion [5], [8], [13]. The first type includes articulation motions which occur in condition of occlusal contacts. The second one includes the so-called free mandibular motions performed when occlusal contact does not occur. These motions result from a synchronised activity of neuromuscular system. Numerical studies of mandibular kinematics are carried out based on the recorded trajectories of its selected points. The characteristic points most frequently chosen are those placed between the two bottom incisors, i.e., the incision point and points located in the surrounding area of the centre of the mandible head. An exact location of the characteristic point enables the definition of mandible orientation in selected time intervals. During the analysis of recorded trajectories the limiting positions, named *border positions*, are observed. The characteristic feature of such positions is their repeatability. The repeatability of limiting trajectories proves to be one of the criteria fundamental to the application of a mathematical model in clinical diagnostics. The mandible, when dislocating within the maximum achieveable range of positions, follows a path defined as a *limiting trajectory*. The limiting trajectory defines a working space within which free and functional motions are performed. The recorded trajectories of free and functional motions, in contrast with the limiting motions, are not required to be repeatable. Generally speaking, there are three basic motions performed in the temporomandibular joints:

- Abduction and elevation.
- Protraction and retraction.
- Rotations in a horizontal plane.

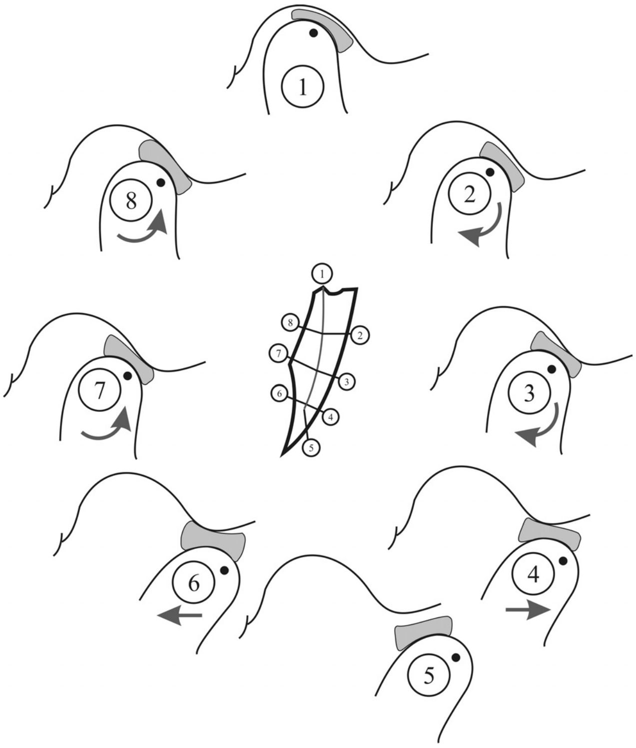

Fig. 1. Model of mandibular head position in a socket during abduction and adduction [5]

Abduction or elevation is performed simultaneously in both joints. The axis intersecting the mandible heads, against which abduction or elevation is performed, is not a fixed axis. The axis of rotation intersects the mandible heads and is located in close proximity to their central points. Concurrently with a gradual increase in mandibular opening, the axis dislocates in anterior direction. Additionally, the motion of mandible heads in temporomandibular joints is considerably influenced by the joint geometry as well as by the susceptibility of intra-articular discs. The model of mandibular head position in a socket during abduction and adduction is demonstrated in figure 1.

During mandible abduction the intra-articular discs slide in a socket in anterior direction. Concurrently, the mandible heads are dislocated towards the inferior surface of the articular tubercle. Such a motion is defined in specialist literature as *rotary sliding*. The range of dislocations of mandible heads is limited by the anatomy of joints, ligaments and joint capsules. The element responsible for mandible abduction is a suprahyoid muscle which, when the hyoid bone is immobilized, causes its abduction. Abduction is enhanced by gravitational force. For the elevation and occlusion of dental arches the temporomandibular muscles are of primary importance. Essential characteristics of mandible heads and the mandible itself are obtained as a result of applying dental diagnostic devices. Such devices allow the mandibular motions to be recorded in three planes. The recording of mandible trajectory in the sagittal plane is performed in order to identify the following basic stages of motion:

- Limiting motion during posterior opening.
- Limiting motion during anterior opening.
- Limiting motion during mandible protraction with occlusal contacts.

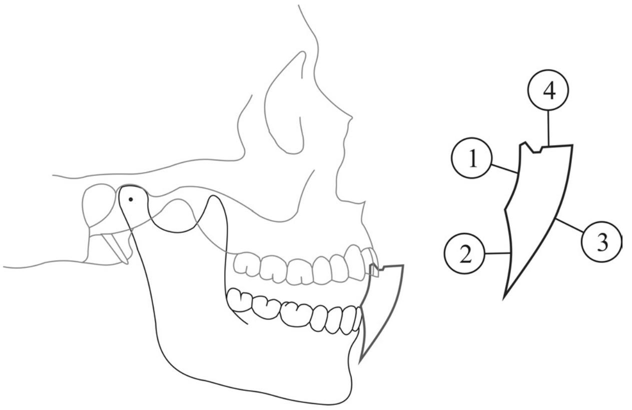

Fig. 2. Mandibular trajectory in the sagittal plane

Limiting motion during posterior opening is performed in two stages. In the first stage, the mandible heads are in fovea articularis. In such a case, the abduction of the mandible can be sufficiently precisely described by rotary motion (figure 2, curve 1), and the rotation axis crosses the central points of the heads. The actual shape of the trajectory plotted by the incision point of the mandible differs from the shape demonstrated in figure 2. The trajectory represents an ideal state of the stomatognathic system.

Theoretically, a purely rotary motion can occur in any position of the mandible heads. A necessary condition for such a motion is a stable constant position of axis about which the mandible rotates. The first stage of limiting motion during posterior opening is performed until the dislocation angle of the mandible attains the value of ca. 10°. Further abduction is connected with the second stage (figure 2, curve 2). At this stage of motion the rotary axis is dislocated in an anterio-inferior direction. The abduction of the mandible during the second stage of motion is performed until the widest opening is reached. The limiting motion during anterior opening represented by curve 3 (figure 2) is clearly determined by two terminal positions representing maximum protraction and opening. Theoretically, when the mandible heads are not subjected to linear dislocations and are stable during the occlusion of dental arches, the transposition from the maximum opening to the position of maximum protraction of the mandible would be performed as a purely rotary motion. However, the maximum protraction of the mandible is affected to some extent by the stylomandibular ligaments which, during occlusion of the dental arches, retract the mandible heads. Accordingly, the limiting motion during anterior opening fails to be performed as purely rotary motion. Thus, it may be inferred that the mandible heads attain the most anterior position during maximum opening, but not during its maximum protraction. The limiting motion due to mandible protraction with occlusal contacts (figure 2, curve 4) is determined by the following factors [13]:

- Range of the difference between the posterior occlusal position and the maximum protraction of the mandible (maximum mandible intercuspation).
- Geometry of the cusps of posterior molar teeth.
- Occlusion of the incisors of the mandible and maxilla in the sagittal and horizontal planes.
- Geometry of the palatal surface of the anterior maxillary teeth.
- State of dentition within dental arches.

The protraction of the mandible in the sagittal plane is a result of the contraction of the lateral pterygoid muscles. During the anterior protraction of the mandible a slight drop in the recorded trajectory is observed. The drop results from the alignment of the mandibular incisors with respect to the maxillary incisors (figure 3). The posterior filaments of the temporal muscles assist in the retraction of mandible.

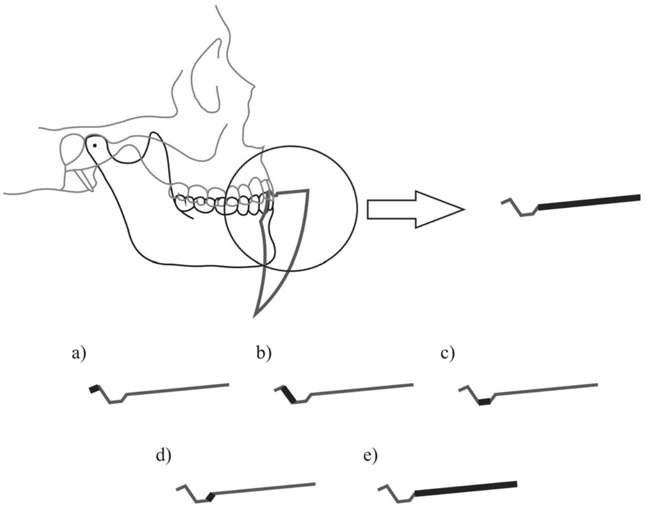

Fig. 3. Limiting motion during mandible protraction with occlusal contacts

Motion connected with the protraction of mandible is affected by numerous factors, and each modification of any of them results in the trajectory disturbance. In the posterior occlusal position, which is the starting point, the contacts between the posterior molar teeth are observed. The first stage of mandible protraction is performed until the mandibular incisors come into contact with maxillary incisors (figure 3a). At the second stage of the motion (figure 3b), the mandibular incisors are dislocated over the incisors of the maxillary bone. Such a motion is inferiorly oriented and persists until the incisors of the mandible and maxillary come into contact with the incisal margins. The contact between the incisal margins is the initial point of the third stage of motion (figure 3c), which is characterised by horizontal motion. This stage persists as long as the incisal margins of the mandibular teeth attain the position in front of the margins of the maxillary incisors. Starting from this moment the mandible begins gradually to move upwards. This motion persists until the occlusion of the premolar teeth is attained. At the last stage of this motion (figure 3e) the shape of the recorded trajectory is considerably influenced by the superficial geometry of the premolar teeth. This stage of motion ends when the mandible heads attain the intercuspal position.

3. Mandibular model in the sagittal plane

In this part of the study, a kinematical analysis of the mandible during its abduction and elevation was carried out. The mandibular motion was modelled by an open kinematical chain with a variable configuration at different time intervals [11], [12]. To formulate a mathematical model the following simplifying assumptions were accepted:

- Mandibular motions were analysed in the sagittal plane.
- The mandible is treated as a perfectly rigid solid.

The point used to define the beginning of a global reference system *XY* (figure 4) with respect to which the motion is being described can be freely accepted. According to the authors of this paper, the best location for the “fixing” of the global coordinates’ system is the point having position near the central points of the mandible heads. Such a place is concurrently the starting position of the mandible against which the trajectory is defined. A characteristic feature of such a location of the global reference system is the fact that a direct access to the initial conditions is obtained. At such a location the initial conditions for linear and angular dislocations of the motion are of zero-value. The initial condition for angular dislocation of the mandible is derived directly from its geometry. The selection of a different point with which the global reference system will be connected results in the need for the identification of all initial conditions.

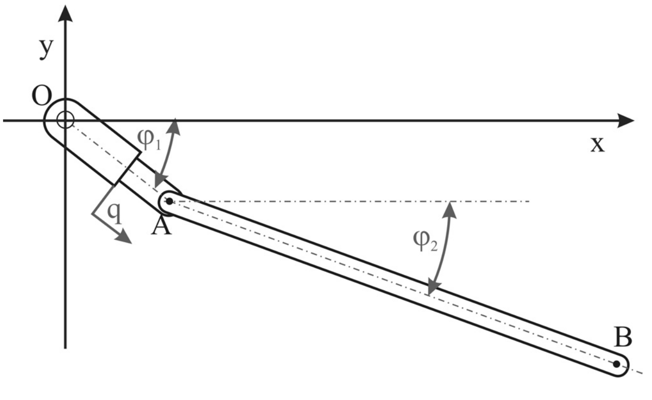

Fig. 4. Kinematical chain modelling mandible motion

A kinematical model of the mandible in the sagittal plane, formulated for numerical study, is constructed of two rotary kinematical pairs and one progressive pair that are responsible for the linear dislocations of the mandible heads. The rotary kinematical pair (the point *A*, figure 4), whose position is explicitly defined by the configuration coordinate φ_2, is responsible for the abduction and elevation of the mandible. The modelling of the elevation motion is determined by the configuration coordinates φ_1 and q. It should be emphasized that at this point the kinematical chain demonstrated in figure 4 and modelling mandible motion in the sagittal plane can be interpreted in two ways. In the first interpretation, the model represents the actual position of mandibular incisors in the sagittal plane (the point *B* in figure 4). Then, the point *A*, representing the motion of mandible head, is the result of the projection onto the plane on which the motion is analysed. In the second interpretation, the situation is inverse, i.e., the point *A* represents the actual motion of the mandible head, whereas the point *B* is the result of projection. The fact that the numerical models represent the ideal mechanical states of the mandible should, however, be taken into consideration. A conscious idealization of motion in temporomandibular joints incorporated in such a way is based on the assumption that the geometry of both joints is the same. The application of matrix notation in the analysis of mandibular kinematics allows the definition of the location of any of its points, with an explicit definition of its orientation with respect to a static reference system. By means of the superposition of elementary relocations of the local reference systems, the following dependence is derived:

$$ {}^{\mathbf{0}}_{\mathbf{B}}\mathbf{T} = \begin{bmatrix} c_{12} & -s_{12} & 0 & c_1 q + c_{12} l \\ s_{12} & c_{12} & 0 & s_1 q + s_{12} l \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}, \tag{1} $$

where:

q – the length of the segment *OA*,

l – the length of the segment *AB*,

$c_{12} = c_1c_2 - s_1s_2$, $s_{12} = c_1s_2 + c_2s_1$.

In dependence (1), the first three rows and columns represent an orientation matrix, and the fourth column is interpreted as a location vector. A mathematical model of mandible motion in the sagittal plane formulated in such a way is a ground for research on inverse kinematics. Generalized coordinates determining the configuration of the mandibular model for quasi-static positions result from kinematical task formulated in such a way. One should not overlook the fact that solving an inverse task they have only the data for the structure of the model, geometrical parameters and trajectory coordinates. From the mathematical point of view the solution of an inverse kinematical task entails specific difficulties which make the explicit configuration of a kinematical chain impossible. Such ambiguities always occur in the case of redundant systems, i.e., systems with more degrees of freedom than necessary to attain a required position. Practically, this means that the system can realize a given trajectory for numerous configurations. The possibility of numerous solutions can also pose the problems lying in the choice of an optimal configuration. The number of configuration solutions depend on the number of kinematical pairs in the mechanism which models a given motion and, additionally, is the function of geometric parameters. Theoretically, there are three situations to be considered which may occur during the solution of an inverse kinematical task [4]:

- The dimension of a one-column matrix of configuration coordinates is smaller than the magnitude of the location vector. The number of the degrees of freedom in such a system are not sufficient. An inverse kinematical task can only be solved in specific cases.
- The dimension of a one-column matrix of configuration coordinates is equal to the magnitude of the location vector. In such a case, the number of the degrees of freedom correspond with a neighbourhood vector. The solution of an inverse task is explicitly possible.
- The dimension of a one-column matrix of configuration coordinates is greater than the magnitude of the location vector. The kinematical chain has a higher number of the degrees of freedom than necessary to realize a given task. The solution of an inverse kinematical task is an infinite set of configurations.

The solutions obtained from numerical methods are based on the Jacobian matrix [3], [4]. This matrix (2) is achieved by the differentiation of the location vector with respect to configuration coordinates. A Jacobian, also named the Jacobian matrix, is one of the most important quantities applied in the analysis and steering of the mechanisms with an open-structure kinematical chain. It is applied, among other things, to the design and realization of smooth trajectories as well as to the identification of singular configurations.

$$ \mathbf{J} = \begin{bmatrix} \frac{\partial x}{\partial \varphi_1} & \frac{\partial x}{\partial \varphi_2} & \cdots & \frac{\partial x}{\partial \varphi_i} \\ \frac{\partial y}{\partial \varphi_1} & \frac{\partial y}{\partial \varphi_2} & \cdots & \frac{\partial y}{\partial \varphi_i} \\ \frac{\partial z}{\partial \varphi_1} & \frac{\partial z}{\partial \varphi_2} & \cdots & \frac{\partial z}{\partial \varphi_i} \end{bmatrix}. \tag{2} $$

The configuration coordinates of an open kinematical chain (figure 4) realizing a given trajectory are determined based on the following iterative equation:

$$ \mathbf{q}_{i+1} = \mathbf{q}_i + \mathbf{J}_P^{-1}(\mathbf{q}_i) \cdot (\mathbf{P}_{i+1} - \mathbf{P}_i), \tag{3} $$

where:

$\mathbf{q}_i$ – the configuration coordinates,

$\mathbf{P}_i$ – the trajectory coordinates,

$\mathbf{J}_P^{-1}$ – the pseudoinverse Jacobian matrix.

The pseudoinverse Jacobian matrix is determined when the model of the system studied is redundant, i.e., when the Jacobian matrix is not a square matrix:

$$\mathbf{J}_P^{-1} = \mathbf{J}^T \cdot (\mathbf{J} \cdot \mathbf{J}^T)^{-1}. \tag{4}$$

Dependence (4) is a specific case of the following equation, assuming that the weight matrix **W** attains the form of an identity matrix:

$$\mathbf{J}_P^{-1} = \mathbf{W}^{-1} \cdot \mathbf{J}^T (\mathbf{J} \cdot \mathbf{W}^{-1} \cdot \mathbf{J}^T)^{-1}. \tag{5}$$

The weight matrix **W** in (5) is a diagonal matrix interpreted as the limits imposed on the motion in particular kinematical pairs. With an increase in weight coefficient the motion in a kinematical pair is subject to limitations. A graphic interpretation of the influence of the weight matrix on the type of motion of the kinematical chain is demonstrated in figure 5.

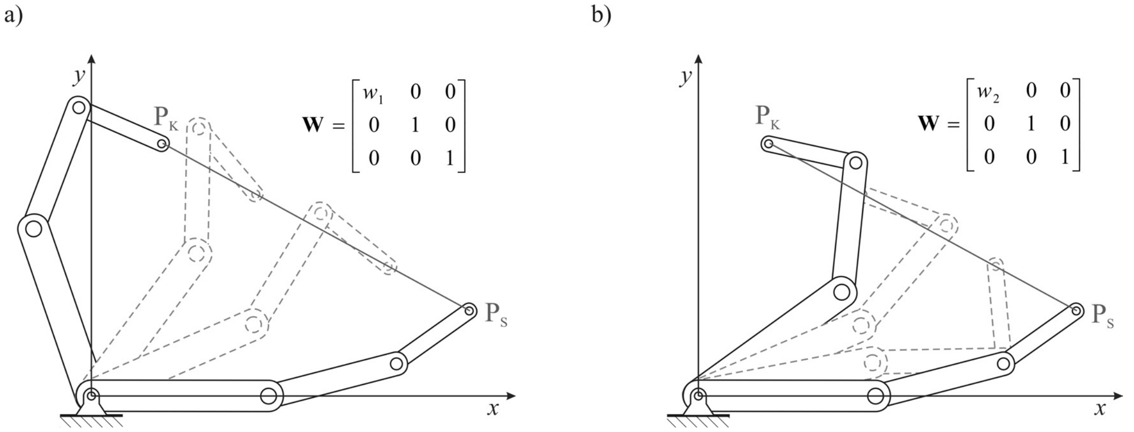

Fig. 5. Graphic interpretation of the influence of a weight matrix: $w_2 > w_1$

Equation (3) is a general recurrent algorithm enabling the numerical solution of an inverse kinematical task. In simulated calculations of this type of task, one should use stable numerical methods, or adequately low frequencies of trajectory sampling, because the errors occurring during its processing have the features of systematic errors which, as a consequence, lead to an imprecise representation of a given trajectory by the kinematical chain.

4. Numerical experiment

The basic action allowing one to carry out a numerical study in inverse kinematics is the processing of the recorded trajectory into a numerical form. Although such an operation does not provide any new information about the trajectory, it may, however, result in the loss or deformation of information. The processing of a trajectory into a numerical form is usually defined as *discretization*. Trajectory discretization consists in the selection of the discrete positions in which the trajectory projections onto particular axes of a global reference system are specified. A discrete series of trajectory values should fulfill a basic criterion of conformity. Such a criterion concerns the possibility of reproducing its actual shape based on discreted samples. If the interval between the discreted samples is incorrectly established, then a precise reproduction of the trajectory is impossible. Such inaccuracies most frequently appear as attenuated oscillations after applying an evolution to the trigonometric series. The oscillations mostly occur near the limiting positions of the mandible. When the period of trajectory sampling is too long, a phenomenon defined in the theory of signals as *frequency masking* or *aliasing* may occur [2], [10]. It should be noted here that although the aliasing does not have to occur, such a phenomenon should always be taken into consideration.

When there is no digital record of a trajectory, the trajectory has to be discreted manually which is a time-consuming process. However, the time required for the discretization of the trajectory can be reduced by interpolation (figure 6a). Interpolation consists in the generation of a full line [9], [14] which crosses the discreted values defined as the interpolation nodes. Interpolation allows the prediction of the values between the interpolation nodes. One of the interpolation methods is the application of the Lagrange interpolation polynomial:

$$L(x)=\sum_{i=0}^{n}\frac{\prod\limits_{\substack{j=0\\ j\neq i}}^{n}(x-x_j)}{\prod\limits_{\substack{j=0\\ j\neq i}}^{n}(x_i-x_j)}\cdot y_i\,. \tag{6}$$

The curve generated in this way is subjected to further numerical discretization at a new and higher sampling frequency. This allows a trajectory with a higher number of discrete values to be obtained (figure 6b). Furthermore, the values of new trajectory are equally distant from each other. Another way enabling us to obtain an increased number of trajectory coordinates is the application of approximation methods. For that purpose, the individual components of a trajectory are approximated by straight lines or parabolas. Owing to the fact that characteristic points of mandible move on closed paths, the best method for trajectory approximation is by evolving it into a trigonometric series (7). In order to apply efficiently the numerical set of values of trajectory coordinates, this set is processed in a way aimed at obtaining modified trajectory components in the form of an explicit function. The use of explicit functions in numerical calculations is more convenient than calculations carried out on a discreted measurement series:

$$\begin{cases} x(t)=\dfrac{a_1}{2}+\sum\limits_{i=1}^{n}[a_{i+1}\cos(i\cdot t)+b_{i+1}\sin(i\cdot t)]\,, \\ y(t)=\dfrac{c_1}{2}+\sum\limits_{i=1}^{n}[c_{i+1}\cos(i\cdot t)+d_{i+1}\sin(i\cdot t)]\,, \\ z(t)=\dfrac{e_1}{2}+\sum\limits_{i=1}^{n}[e_{i+1}\cos(i\cdot t)+f_{i+1}\sin(i\cdot t)]\,, \end{cases} \tag{7}$$

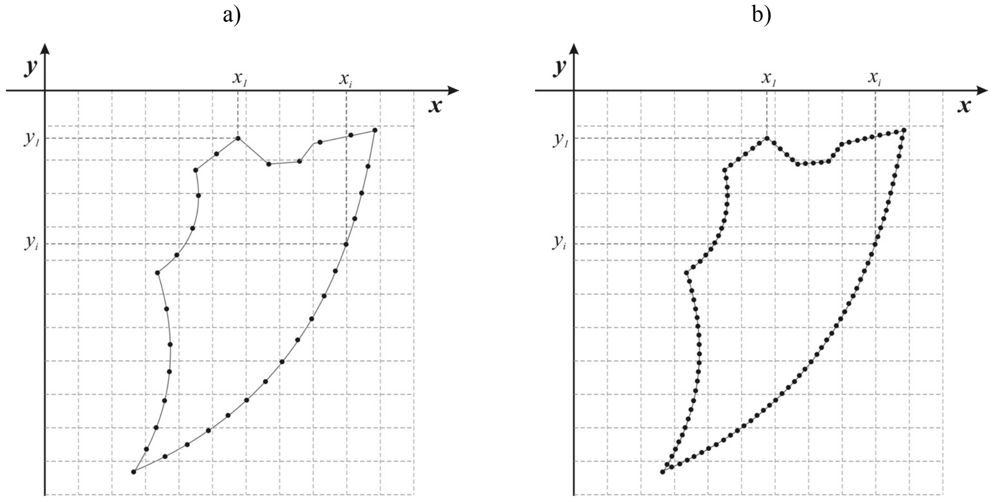
a) b)

Fig. 6. Discretization of a mandibular trajectory: a) before the interpolation of the trajectory, b) after interpolation

where:

a_i, b_i, c_i, d_i, e_i, f_i – the Euler–Fourier coefficients,

n – the number of measurement points,

t – the time or number of a sample.

Figures 7 and 8 demonstrate the influence of the quantity of data for discreted trajectory on the quality of approximation.

The availability of the continuous functions of trajectory coordinates allows for the possibility of defining the values for the configuration of a mandibular model (figure 4). The configuration coordinates are derived from equation (3), where a pseudoinverse Jacobian matrix has been determined from dependence (5). The same results are obtained by the application of dependence (4), assuming that the weight matrix is an identity matrix. It should be emphasized that the iteration algorithm applied in kinematical modelling is efficient, which is proved by the precise representation of a given trajectory (figure 9). A representation error for the trajectory is demonstrated in figure 10.

The influence of the weight matrix coefficients on the type of the solution of the kinematical chain is shown in figures 11–13.

The basic criterion that should be assumed in the evaluation of the results obtained is the periodicity of the solution. This criterion must be fulfilled, because each position of the mandible that performs a limiting trajectory is repeatable. The selection of weight matrix coefficients is crucial for this type of trajectory of the mandibular head, and the determined configuration coordinates are essential for its identification (figure 14).

The results obtained from numerical calculations of inverse kinematics will be incorporated in future computer simulations as the steering values in a dynamic mandibular model.

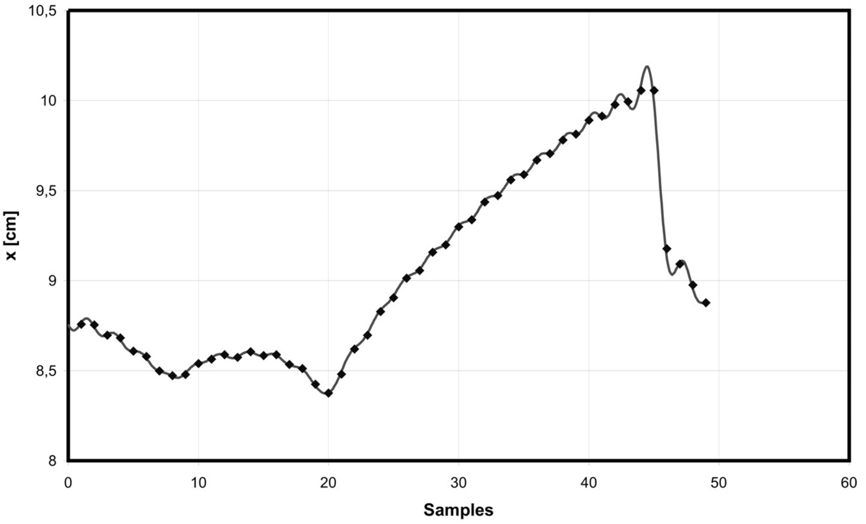

Fig. 7. Approximation of measuring data for an insufficient number of measuring data

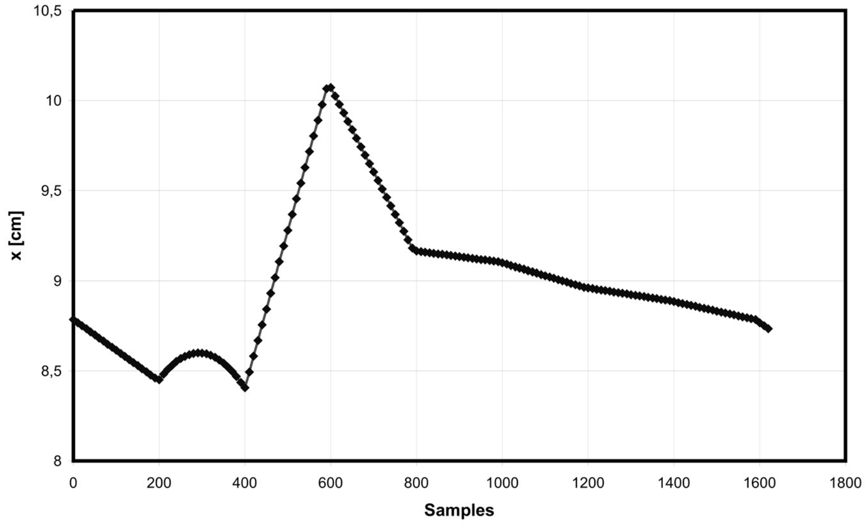

Fig. 8. Approximation of measuring data for a sufficient number of measuring data

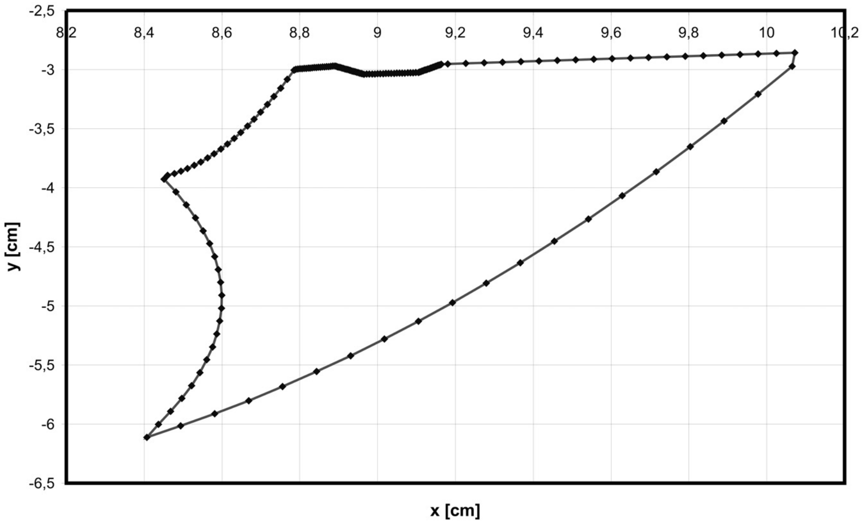

Fig. 9. Trajectory of the mandibular incisors

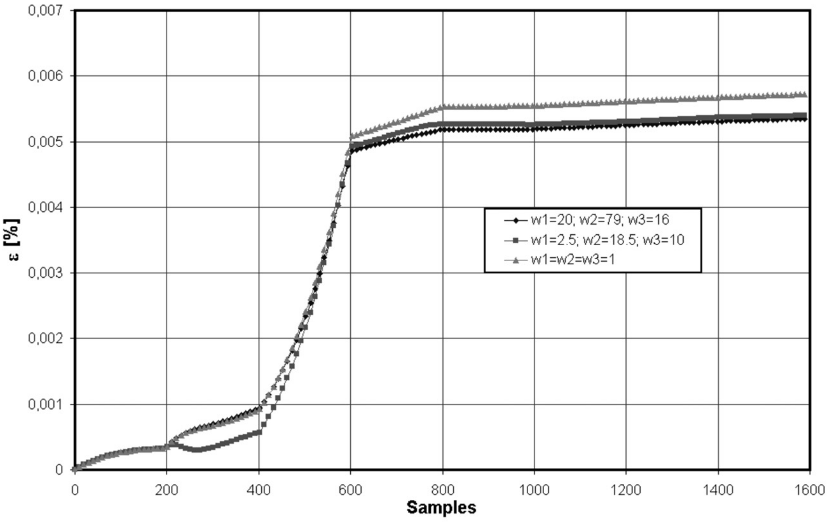

Fig. 10. Representation error of trajectory

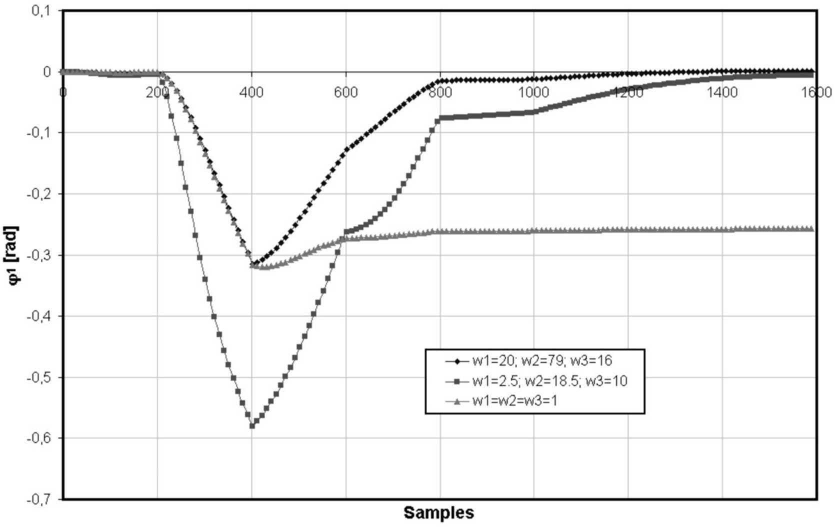

Fig. 11. Results of numerical calculations; configuration coordinate φ_1

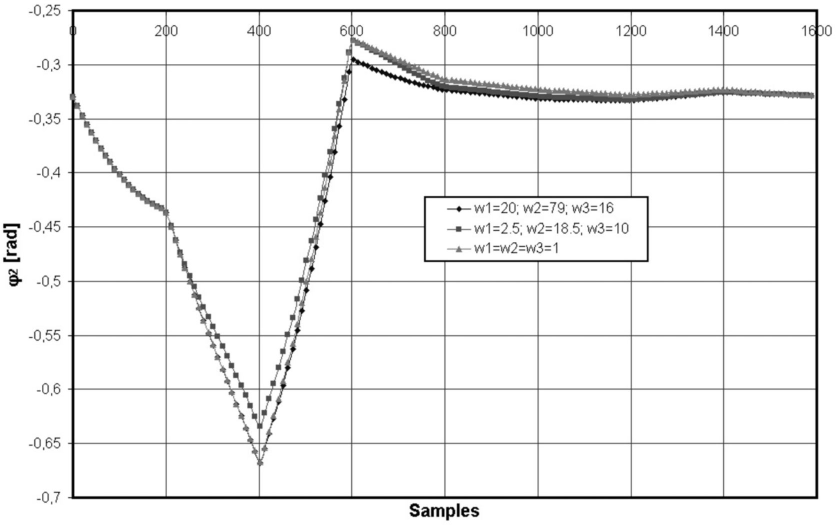

Fig. 12. Results of numerical calculations; configuration coordinate φ_2

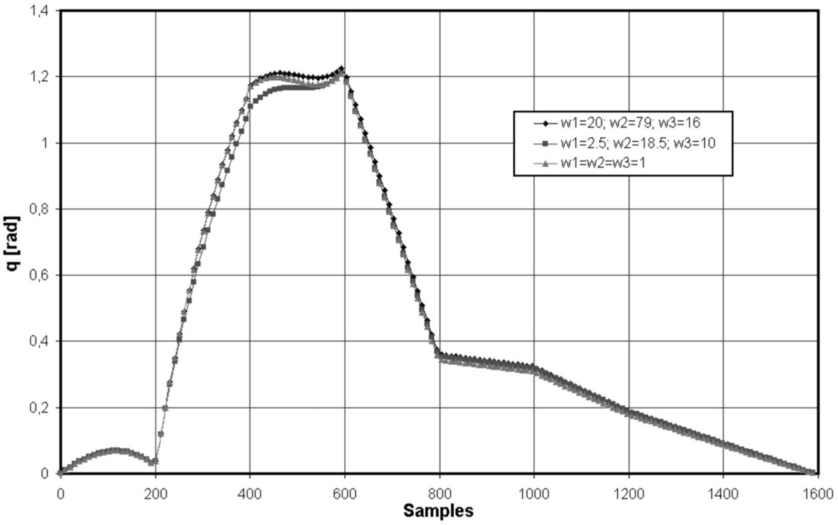

Fig. 13. Results of numerical calculations; configuration coordinate

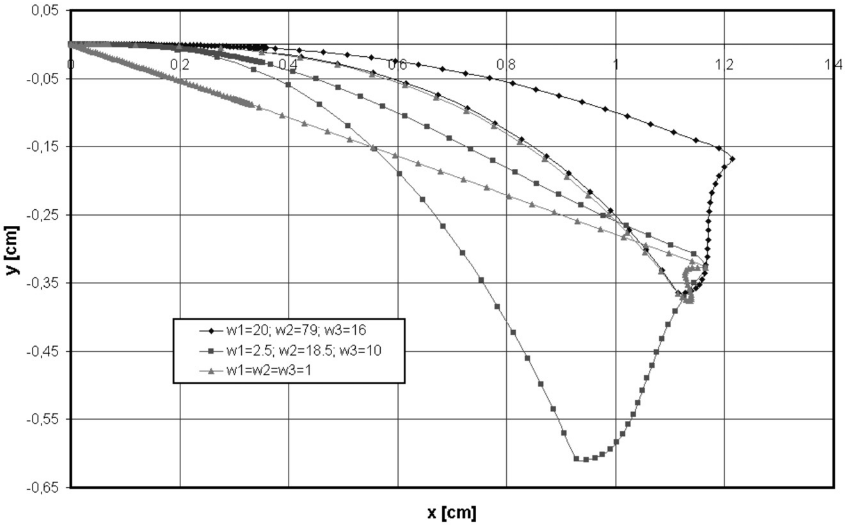

Fig. 14. Results of numerical calculations; trajectories of the central point of the mandibular head

5. Summary

This paper presents a mathematical approach to modelling the kinematics of the limiting mandibular motions in the sagittal plane. The descriptive part applied a method enabling a kinematical analysis based on the coordinates of incisor trajectory as the only available data. The research carried out by the authors demonstrates that the application of the inverse kinematical method proves to be a useful diagnostic instrument enabling the kinematical estimation of mandibular trajectories. An appropriately identified mandibular kinematical model provides a basis for the design of a dynamic model which allows the analysis of the loads occurring in the temporomandibular joint as well as the estimation of muscular forces. An appropriate choice of the coefficients of a pesudoinverse Jacobian matrix enables the path of the mandibular head to be adjusted to the actual trajectory. Moreover, it can be inferred that the choice of coefficients of a weight matrix equal to 1 does not fulfil the criterion of periodicity (figure 12). The numerical study of the inverse kinematics of the mandible failed to prove the existence of a stage connected with a purely rotary motion. Such a motion would occur, provided that the q- and φ_1-coordinates in any given stage of motion attain zero values. A comparison of the numerical results with clinical ones is the final criterion of the positive verification of the model used in this study. It proved to be suitable for application in clinical analysis. The model presented in this paper provides a firm basis for further model studies of mandibular motion in three-dimentional space.

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