Acta of Bioengineering and Biomechanics
Vol. 12, No. 3, 2010

Original paper

Modelling dynamics and aerodynamic tests of a sport parachute jumper during flight in sitfly position

JUSTYNA MONIUSZKO[1], JERZY MARYNIAK[2],
EDYTA ŁADYŻYŃSKA-KOZDRAŚ[3]*

[1] Warsaw University of Technology, ITLiMS, Warsaw, Polnad.
[2] Air Force Higher Military School, Dęblin, Polnad.
[3] Warsaw University of Technology, IMiF, Warsaw, Polnad.

Based on a model of a parachute jumper, for various body configurations in a sitting position, tests were carried out in an aerodynamic tunnel. Aerodynamic characteristics and dimensionless aerodynamic forces' coefficients were calculated. The tests were carried out for various configurations of the jumper's body. A universal mathematical model of a parachute jumper's body was prepared, thus enabling the analysis of the jumper's movement with a closed parachute in any position. In order to build the model, a digitized model of a jumper allowing changing the body configuration, making appropriate changes of the moment of inertia, distribution of the center of mass and the aerodynamic characteristics was adopted. Dynamic movement equations were derived for a jumper in a relative reference system. The mathematical model was formulated for a jumper with a variable body configuration during the flight, which can be realized through a change of the position and the speed of the parachute jumper's limbs. The model allows analyzing the motion of the jumper with a closed parachute. It is an important jump phase during an assault with delayed parachute opening in sports type jumping, e.g., Skydiving and in emergency jumps from higher altitudes for the parachute's opening to be safe.

Key words: *aerodynamics of a parachute jumper, tunnel aerodynamic testing, mathematical modelling, numerical simulation*

1. Introduction

Parachute jumping has long ago ceased to be only the means of saving the pilot's life or distributing the army in inaccessible places. Skydiving has become one of the so-called extreme sports, which is based on a freefall with a closed parachute as well as its variation – freefly, which is an art of controlled movement in flight. Currently, typical jumps are made from the level of 4000 m, with parachute opening at circa 800 m above the ground. Freefall lasts less than a minute.

One of the basic figures in freefly is the so-called sitting position (figure 1). It is the starting position for further figures in this discipline of jumping. Instructors teaching new adepts base their teaching on their own experience and that of their colleagues, which often makes learning this position not easy, full of trials and mistakes.

Never before have tunnel tests been carried out on a parachute jumper's model in a sitfly position, in order to refute myths and confirm the correct guidelines of parachute instructors allowing the teaching process to be facilitated. The purpose of this thesis is to introduce dynamic movement equations for a parachute jumper in various configurations of the body's positions, with special attention paid to the sitfly sitting position.

* Corresponding author: Edyta Ładyżyńska-Kozdraś, Warsaw University of Technology, IMiF, 02-525 Warsaw, ul. św. Andrzeja Boboli 8, Warsaw, Polnad. E-mail: e.ladyzynska@mchtr.pw.edu.pl

Received: May 26th, 2010
Accepted for publication: August 13rd, 2010

Fig. 1. Sitting position during a freefall with a parachute

2. Materials and methods

2.1. Mass geometry

In order to derive the dynamic movement equations one needs to know the jumper's mass distribution in various configurations. Previous practice in modelling and numerical simulation indicates [1], [2], [4], [8] that it is enough to digitize a jumper by introducing 14 segments of their body (the parachute is adopted as the 15th element) (figure 2). We assume that the body mass is fixed, and the distribution of mass changes, depending on the configuration, which affects the change of the center of mass and the moments of inertia.

Segment	Mass (%)	Location of the center of gravity (%)
Head	7	0.50
Body	43	0.44
Arm	3	0.47
Forearm	2	0.42
Hand	1	0.47
Thigh	12	0.44
Shank	5	0.42
Foot	2	0.44

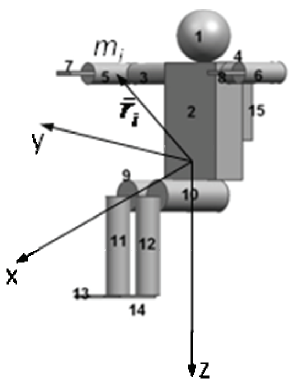

Fig. 2. Digitization of a man's body along with the impact of the masses of the individual elements on the total body mass

Vector $\mathbf{r}_i$ represents the distance between the center of mass of the i-th element and the origin O of the xyz reference system (figure 2):

$$\bar{r}_i = x_i\bar{i} + y_i\bar{j} + z_i\bar{k}\ . \tag{1}$$

Location of the center of mass of the jumper–parachute system is described by the following dependencies:

$$x_C = \frac{S_{0x}}{m} = \frac{\sum_{i=1}^{15} m_i x_i}{\sum_{i=1}^{15} m_i},$$

$$y_C = \frac{S_{0y}}{m} = \frac{\sum_{i=1}^{15} m_i y_i}{\sum_{i=1}^{15} m_i}, \tag{2}$$

$$z_C = \frac{S_{0z}}{m} = \frac{\sum_{i=1}^{15} m_i z_i}{\sum_{i=1}^{15} m_i}.$$

The moment of inertia of the jumper's body was calculated in reference to Ox axis related to the body:

$$J_{x0} = \sum_{i=1}^{15}[m_i(y_{ci}^2 + z_{ci}^2) + J_{ci}], \tag{3}$$

where:

m_i – the mass of the i-th part of a jumper's body,

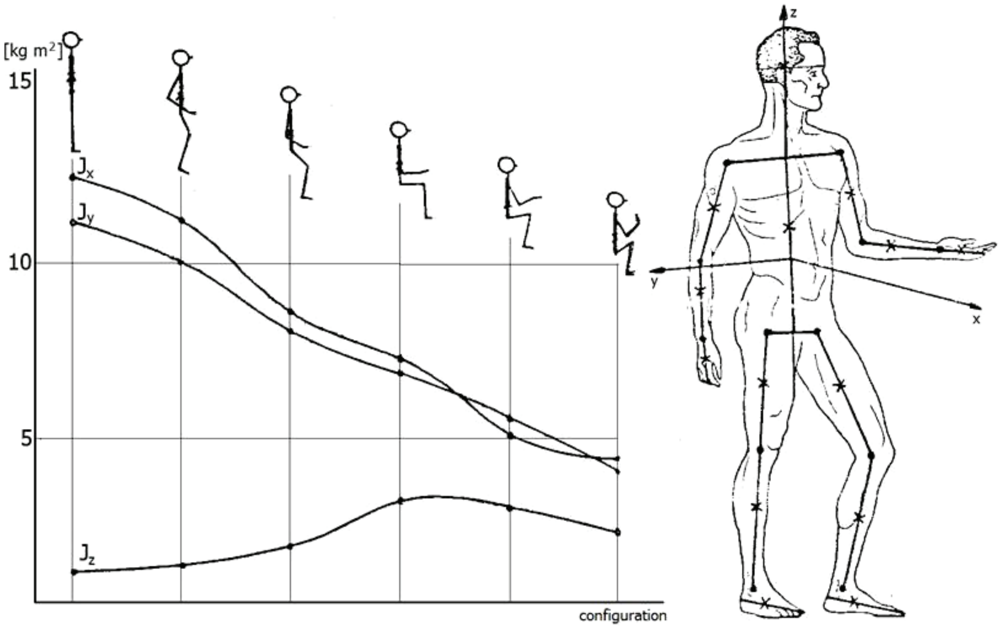

Fig. 3. Moments of inertia of the jumper's body, depending on the configuration [2]

x_{ci}, z_{ci} – the coordinates of the center of mass of the i-th body part,

J_{ci} – the moment of inertia of the i-th element in relation to the center of mass of the jumper.

The moments of inertia in the reference y- and x-axes of the xyz reference system closely related to the jumper were formulated analogically (figure 3). Changes of the moments of inertia, depending on the body configuration, are presented in figure 2.

Deviation moments of inertia were calculated analogically to the primary moments:

$$J_{xy} = \sum_{i=1}^{15} [m_i x_i y_i + J_{xyci}],$$

$$J_{yz} = \sum_{i=1}^{15} [m_i y_i z_i + J_{yzci}], \quad (4)$$

$$J_{xz} = \sum_{i=1}^{15} [m_i x_i z_i + J_{xzci}].$$

2.2. Kinematic relations of a parachute jumper

The mathematical model of a parachute jumper was developed in a relative, central reference system closely associated with the man's body. The origin of this system is located in the center of its mass (figures 2 and 4).

Vector of temporary linear velocity (figure 4) is given by

$$\overline{V_0} = U\bar{i} + V\bar{j} + W\bar{k}, \quad (5)$$

and vector of angular velocity (figure 4) by

$$\overline{\Omega} = P\bar{i} + Q\bar{j} + R\bar{k}. \quad (6)$$

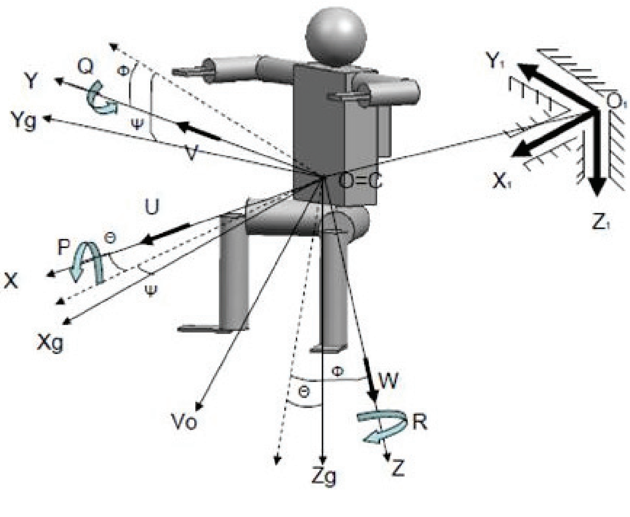

Fig. 4. Reference systems and kinematic parameters of a parachute jumper

The angles of rotation Φ, Θ, Ψ represent the location of the reference system closely connected with the Oxyz rocket in relation to a gravitational system O$x_g y_g z_g$ perpendicular to the immobile inertial system O$_1 x_1 y_1 z_1$.

$$\begin{bmatrix}\dot{\phi}\\ \dot{\theta}\\ \dot{\psi}\end{bmatrix}=\begin{bmatrix}1 & \sin\phi\tan\theta & \cos\phi\tan\theta\\ 0 & \cos\phi & -\sin\phi\\ 0 & \sin\phi\sec\theta & \cos\phi\sec\theta\end{bmatrix}\begin{bmatrix}P\\ Q\\ R\end{bmatrix}=\Lambda_{\Omega}^{-1}\begin{bmatrix}P\\ Q\\ R\end{bmatrix}. \tag{7}$$

Kinematic relations between the elements of linear velocity $\dot{x}_1, \dot{y}_1, \dot{z}_1$ measured in an inertial system $O_1x_1y_1z_1$ and the elements of velocity U, V, W in an Oxyz reference system related to the rocket are as follows:

$$\begin{bmatrix}\dot{x}_1\\ \dot{y}_1\\ \dot{z}_1\end{bmatrix}=\begin{bmatrix}\cos\psi\cos\theta & \cos\psi\sin\theta\sin\phi-\cos\phi\cos\psi & \cos\psi\sin\theta\cos\phi+\sin\phi\cos\psi\\ \sin\psi\cos\theta & \sin\psi\sin\theta\sin\phi-\sin\phi\cos\psi & \sin\psi\sin\theta\cos\phi-\sin\phi\cos\psi\\ -\sin\theta & \sin\phi\cos\theta & \cos\phi\cos\theta\end{bmatrix}\times\begin{bmatrix}U\\ V\\ W\end{bmatrix}. \tag{8}$$

Angle of attack:

$$\alpha=\arctan\frac{W}{U}, \tag{9}$$

angle of glide:

$$\beta=\arcsin\frac{V}{V_0}. \tag{10}$$

2.3. Dynamic movement equations

General equations representing a parachute jumper's movement were formulated by adopting the basic equations of dynamics, assuming that the jumper is a system with six degrees of freedom and their movement is spatially considered.

Derivative of momentum **Π** against time is expressed by

$$\frac{\delta\overline{\Pi}}{\delta t}+\overline{\Omega}\times\overline{\Pi}=\overline{F}, \tag{11}$$

and derivative of momentum $\mathbf{K}_0$ against time by

$$\frac{\delta\overline{K}_0}{\delta t}+\overline{\Omega}\times\overline{K}_0+\overline{V}_0\times\overline{\Pi}=\overline{M}_0, \tag{12}$$

while:

$$\overline{\Pi}=m(\overline{V}_0+\overline{\Omega}\times\overline{r}_C). \tag{13}$$

During a freefall the jumper is subject to gravitational and aerodynamic forces and moments.

The elements of the external forces (figure 4) are as follows:

$$\overline{F}_0=X\bar{i}+Y\bar{j}+Z\bar{k}, \tag{14}$$

and the elements of the moments of the external forces (figure 4) as:

$$\overline{M}_0=L\bar{i}+M\bar{j}+N\bar{k}. \tag{15}$$

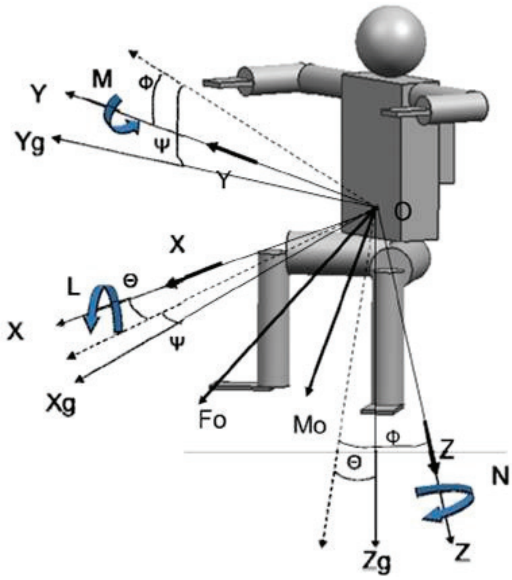

Fig. 5. Forces and moment of external forces

The forces of gravity of the jumper in a gravitational system are as follows:

$$\begin{bmatrix}X^g\\ Y^g\\ Z^g\\ \hline L^g\\ M^g\\ N^g\end{bmatrix}=\begin{bmatrix}-mg\sin\theta\\ mg\cos\theta\sin\phi\\ mg\cos\theta\cos\phi\\ \hline mg(y_C\cos\theta\cos\phi-z_C\cos\theta\sin\phi)\\ mg(-z_C\sin\theta-x_C\cos\theta\cos\phi)\\ mg(x_C\cos\theta\sin\phi+y_C\sin\theta)\end{bmatrix}, \tag{16}$$

where the vector of location of the center of mass of the jumper is expressed by the dependence:

$$\bar{r}_C=x_C\bar{i}+y_C\bar{j}+z_C\bar{k}. \tag{17}$$

Aerodynamic forces acting on the jumper in the gravitational system are as follows:

$$\begin{bmatrix} X^a \\ Y^a \\ Z^a \\ \hdashline L^a \\ M^a \\ N^a \end{bmatrix} = \begin{bmatrix} -\frac{1}{2}\rho S V_0^2 (C_{xa}\cos\beta\cos\alpha + C_{ya}\sin\beta\cos\alpha - C_{za}\sin\alpha) + X_Q Q \\ -\frac{1}{2}\rho S V_0^2 (C_{xa}^A \sin\beta^A - C_{ya}^A \cos\beta^A) + Y_R R_A \\ -\frac{1}{2}\rho S V_0^2 (C_{xa}\cos\beta\sin\alpha + C_{ya}\sin\beta\sin\alpha + C_{za}\cos\alpha) + Z_Q Q \\ \hdashline -\frac{1}{2}\rho S V_0^2 [C_a (C_{mxa}\cos\beta\cos\alpha + C_{mya}\sin\beta\cos\alpha - C_{mza}\sin\alpha)] + L_R R \\ -\frac{1}{2}\rho S V_0^2 [C_a (C_{mxa}\sin\beta - C_{mya}\cos\beta)] + M_Q Q \\ -\frac{1}{2}\rho S V_0^2 [C_a (C_{mxa}\cos\beta\sin\alpha + C_{mya}\sin\beta\sin\alpha + C_{mza}\cos\alpha)] + N_R R \end{bmatrix}, \tag{18}$$

where:

C_{xa}, C_{ya}, C_{za} – the coefficients of aerodynamic resistance, lateral and lift forces,

C_{mxa}, C_{mya}, C_{mza} – the coefficients of forces of aerodynamic tilting, inclining and deflecting moments,

C_a – a mean aerodynamic chord,

$\mathbf{r}_A = [x_A, y_A, z_A]^T$ – the vector and the coordinates of the position of point A of reduction in aerodynamic forces acting on the jumper in Oxyz system.

Dynamic equations of motion are as follows:

- the equation of longitudinal motions:

$$m(\dot{U} + QW - RV) - S_x(Q^2 + R^2) - S_y(\dot{R} - PQ) + S_z(\dot{Q} + PR) = -mg\sin\theta - \frac{1}{2}\rho S V_0^2 (C_{xa}\cos\beta\cos\alpha + C_{ya}\sin\beta\cos\alpha - C_{za}\sin\alpha) + X_Q Q, \tag{19}$$

- the equation of lateral motions:

$$m(\dot{V} + RU - PW) + S_x(\dot{R} + PQ) - S_y(P^2 + R^2) - S_z(\dot{P} - QR) = mg\cos\theta\sin\phi - \frac{1}{2}\rho S V_0^2 \times (C_{xa}\sin\beta - C_{ya}\cos\beta) + Y_R R, \tag{20}$$

- the equation of climbing motions:

$$m(\dot{W} + PV - QU) - S_x(\dot{Q} - PR) + S_y(\dot{P} + QR) - S_z(Q^2 + P^2) = mg\cos\theta\cos\phi - \frac{1}{2}\rho_h S V_{\mathrm{A}}^2 \times (C_{xa}\cos\beta\sin\alpha + C_{ya}\sin\beta\sin\alpha + C_{za}\cos\alpha) + Z_Q Q, \tag{21}$$

- the equation of tilting motions:

$$I_x\dot{P} - (I_y - I_z)QR - I_{xy}(\dot{Q} - PR) - I_{xz}(\dot{R} + PQ) - I_{yz}(Q^2 - R^2) + S_y(\dot{W} + VP - UQ) + S_z(PW - UR - \dot{V}) = mg(y_C\cos\theta\cos\phi - z_C\cos\theta\sin\phi) - \frac{1}{2}\rho S V_0^2 \big[y_A(C_{xa}\cos\beta\sin\alpha + C_{ya}\sin\beta\sin\alpha + C_{za}\cos\alpha) - z_A(C_{xa}\sin\beta - C_{ya}\cos\beta) + C_a(C_{mxa}\cos\beta\cos\alpha + C_{mya}\sin\beta\cos\alpha - C_{mza}\sin\alpha)\big] + L_R R, \tag{22}$$

- the equation of inclining motions:

$$I_y\dot{Q} - (I_z - I_x)PR - I_{xy}(\dot{P} + QR) - I_{yz}(\dot{R} - PQ) - I_{xz}(R^2 - P^2) - S_x(\dot{W} + VP - UQ) + S_z(\dot{U} - VR + QW) = -mg(z_C\sin\theta + x_C\cos\theta\cos\phi) - \frac{1}{2}\rho_h S V_A^2 \big[z_A(C_{xa}^A\cos\beta^A\cos\alpha^A + C_{ya}\sin\beta\cos\alpha - C_{za}\sin\alpha) - x_A(C_{xa}\cos\beta\sin\alpha + C_{ya}\sin\beta\sin\alpha + C_{za}\cos\alpha) + C_a(C_{mxa}\sin\beta - C_{mya}\cos\beta)\big] + M_Q Q, \tag{23}$$

- the equation of deflecting motions:

$$I_Z\dot{R} - (I_x - I_y)PQ - I_{yz}(\dot{Q} + PR) - I_{xz}(\dot{P} - QR) - I_{xy}(P^2 - Q^2) + S_x(\dot{V} - WP + RU) - S_y(\dot{U} - RV + WQ) = mg(x_C\cos\theta\sin\phi + y_C\sin\theta) - \frac{1}{2}\rho S V_0^2 [x_A(C_{xa}\sin\beta - C_{ya}\cos\beta) - y_A(C_{xa}\cos\beta\cos\alpha + C_{ya}\sin\beta\cos\alpha)] + N_R R. \tag{24}$$

Equations (19)–(24) together with dependencies (5)–(10) constitute a mathematical model describing the motion of a parachute jumper during freefall.

3. Results

3.1. Tests of the jumper's model carried out in wind tunnel

Tunnel tests were carried out for three configurations of the sitting position of the parachute jumper's model (figure 6) equipped with a sports suit, helmet with a crown for acoustic altimeter and the parachute model in the "back-back" system (main and reserve parachutes placed in one bag on the back). The jumper's model tested was hung on a frame covering the entire measurement space. A symmetry axis of the model at zero approach angle coincided with the axis of the tunnel.

Tunnel tests were carried out at the dynamic pressure q = 100 mm H_2O = 980 Pa and the speed V = 40 m/s. The height of the model h = 0.5 m and the reference area $S = 0.25$ m^2.

Dimensionless aerodynamic coefficients were determined on the basis of the dependence given in [2], [7], [9]:

- aerodynamic lift P_z:

$$C_z = \frac{P_z}{\frac{1}{2}\rho V_0^2 S}, \tag{25}$$

- aerodynamic resistance P_x:

$$C_x = \frac{P_x}{\frac{1}{2}\rho V_0^2 S}, \tag{26}$$

- tilting moment M_a:

$$C_m = \frac{M_a}{\frac{1}{2}\rho V_0^2 Sh}. \tag{27}$$

The parametric identification of the parachute jumper allows one, by applying a mathematical model, to carry out numerical calculations and simulation for such a jumper during the flight with a closed parachute – both free and controlled.

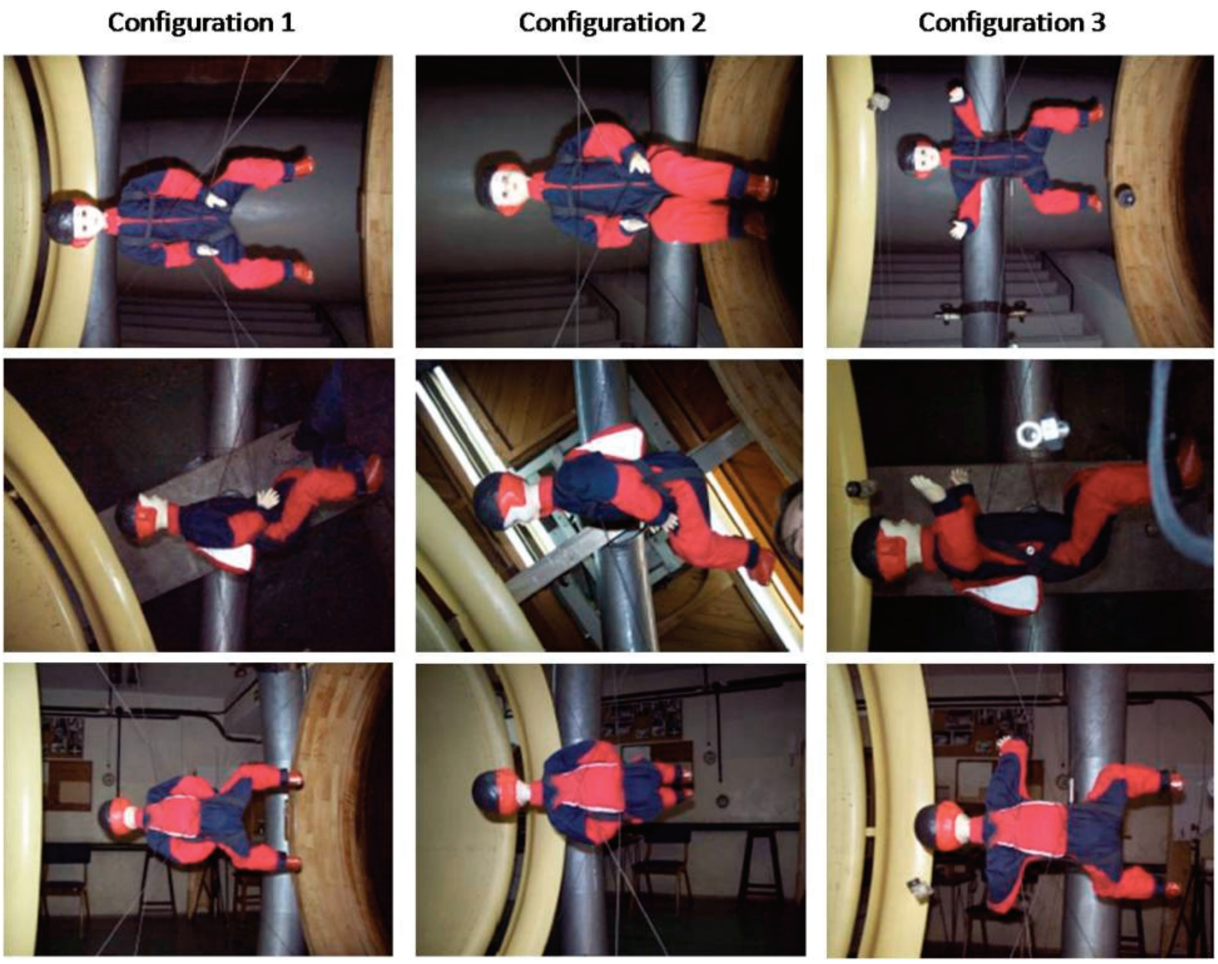

Fig. 6. Silhouette of the parachute jumper in the wind tunnel

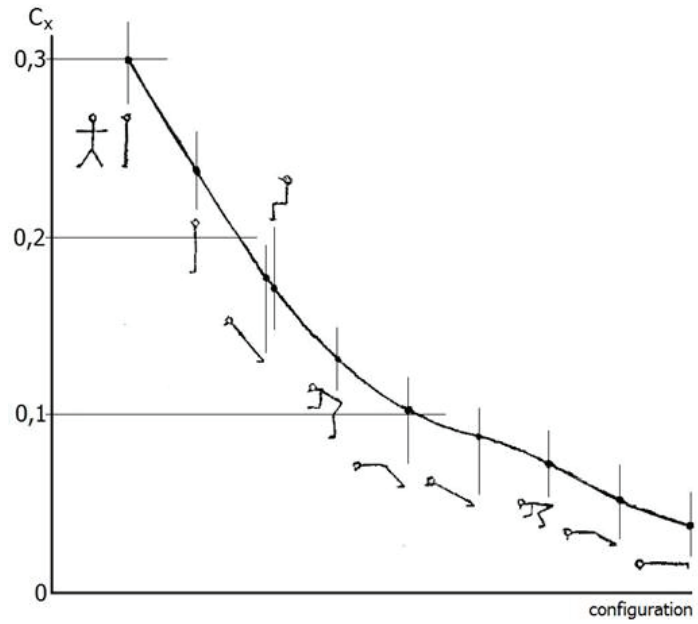

Fig. 7. Changes in aerodynamic resistance coefficient
depending on the configuration of the body in the flow [2]

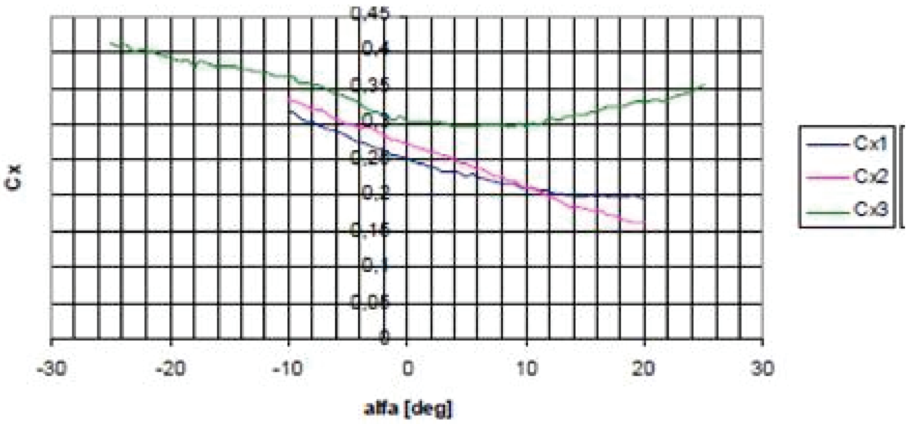

Fig. 8. Changes in the coefficient of aerodynamic resistance acting on the jumper
in the function of the approach angle for three configurations

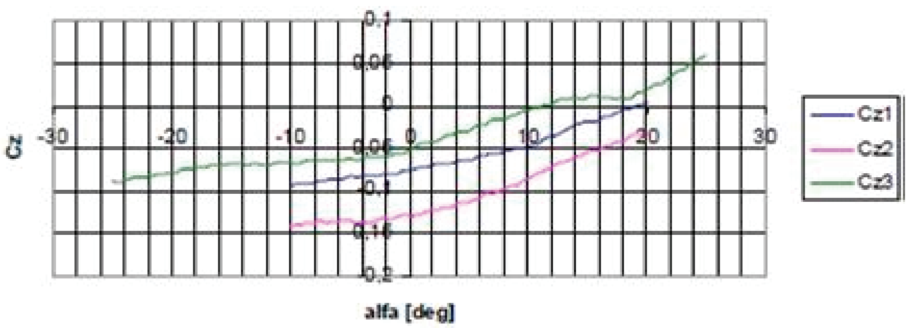

Fig. 9. Changes in the coefficient of lifting force acting on the jumper
in the function of approach angle

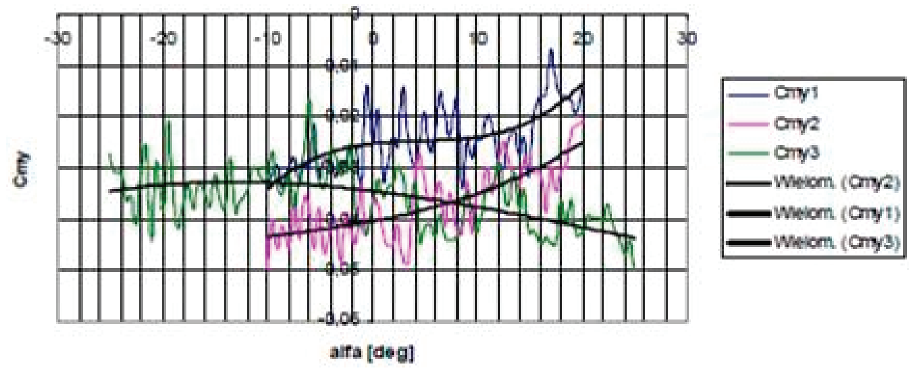

Fig. 10. Changes in the coefficient of inclining moment in the function of approach angle

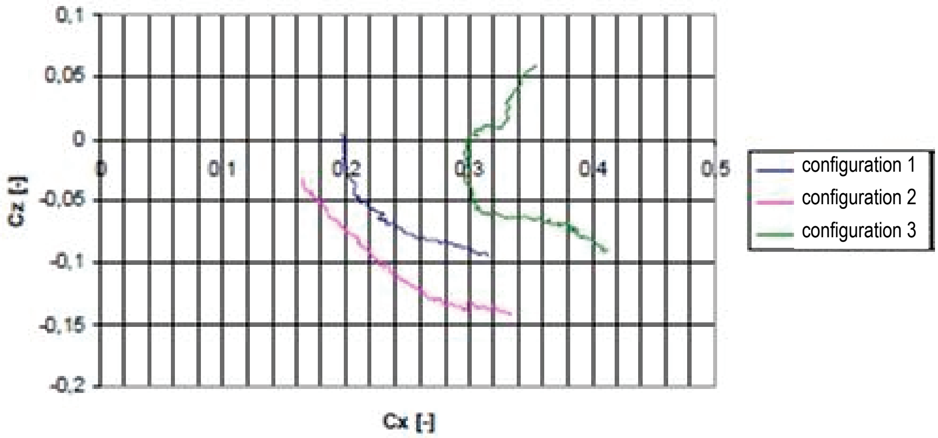

Fig. 11. Polar of the parachute jumper

3.2. Numerical simulation of the movement of a parachute jumper

Test numerical simulation has been carried out for the jumper with the weight m = 70 kg and the height h = 1.7 m after jumping out without the initial velocity (V_C = 0) at the altitude H = 2000 m in the body configuration shown in figure 12. The results obtained have been presented graphically in figures 13–16.

The calculations presented show that for such a configuration of the jumper during freefall from the altitude of 2000 m, the 320-m deviation from the vertical occurs (figure 15). During freefall a jumper accelerates until they reach the speed limit which takes place when the aerodynamic resistance balances the body weight. After reaching this speed the jumper, whilst approaching the ground, reduces the limit fall speed due to the increase in the air density.

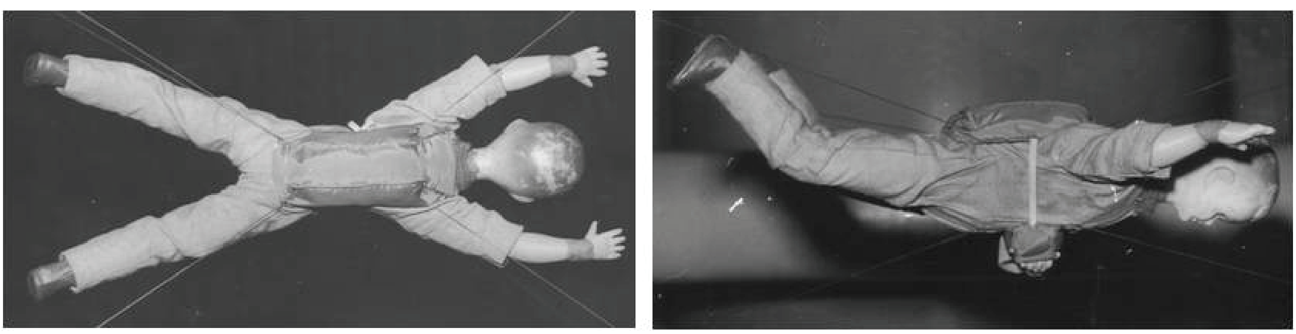

Fig. 12. Test silhouette of the parachute jumper for the purposes of simulation tests

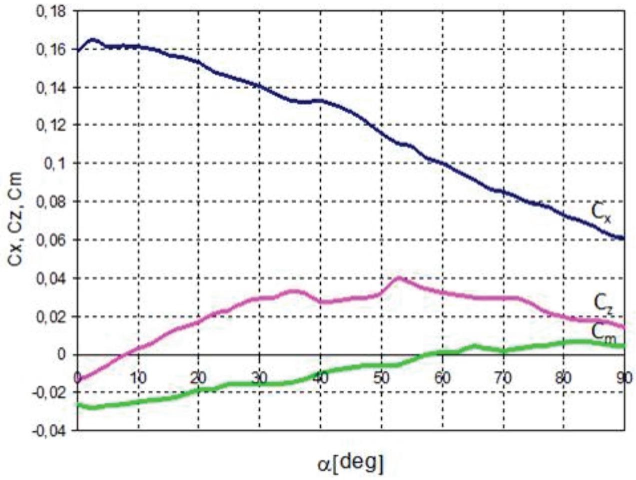

Fig. 13. Aerodynamic characteristics of silhouette in simulation tests

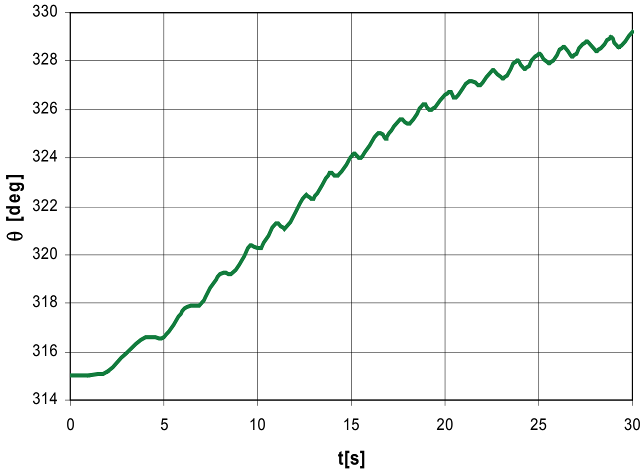

Fig. 14. Change of the angle of the jumper's body position during freefall

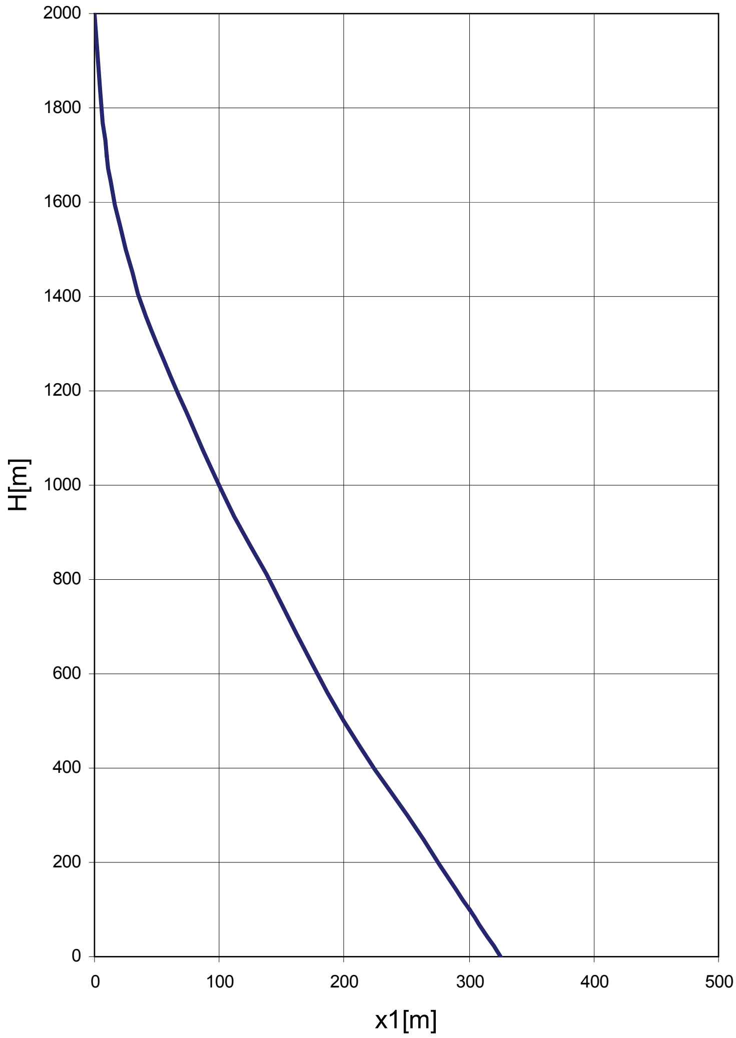

Fig. 15. Trajectory of the jumper's movement during freefall

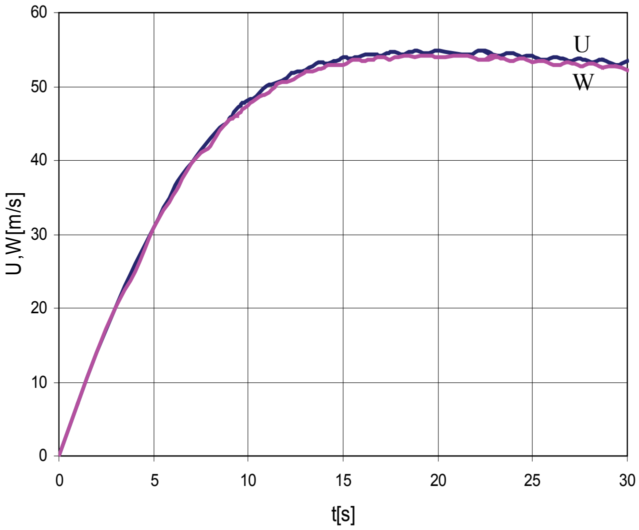

Fig. 16. Change of the components of jumper's velocity *U*(*t*) and *W*(*t*) during freefall

4. Discussion and final conclusions

In case of emergency, and particularly when the aircraft gets substantially damaged due to explosion, even an experienced jumper faces a considerable psychological resistance to a jump [3], [5], [6]. This results in the delay in the decision to leave the aircraft independently and too often to open a parachute too early. Minor problems occur in the case of planned jumps with the delayed opening of the parachute. This results in the popularity of sports such as Skydiving and its variant – freefall, which is the art of controlled movement in flight.

The tests in the wind tunnel have been carried out on the parachute jumper's model for different configurations of their body in a sitting position. The tests have shown that the configuration of the arms and legs of the jumper significantly affects the jump. The body rotations change the velocity of various elements of the body, depending on the distance from the center of mass.

Evaluated mathematical model of the dynamics of a parachute jumper allowed the analysis of the jumper's movement with a closed parachute.

The numerical calculations show that the jumper who falls freely accelerates until he reaches the speed limit, and then, whilst approaching the ground, reduces the limit fall speed due to the increase in the air density. During the jump from a higher altitude, the effects of reduction of the fall speed and the damping of fluctuations are much greater.

Obituary: Justyna Moniuszko

On April 10, 2010, the co-author of this article, engineer Justyna Moniuszko, died tragically in the presidential plane crash near Smolensk. In this tragic day, Justyna performed her duties as a flight attendant.

Aviation was her great passion. In Białystok, while still being a schoolgirl, she became a member of the Aero Club. It was there where she was jumping with a parachute and started flying a glider. She was a very active jumper – she jumped over 250 times. In 2010, she graduated from the Warsaw University of Technology, Faculty of Power and Aeronautical Engineering, as an engineer and continued her education at the supplementary MA/MCs studies. We said goodbye to that cheerful, hardworking girl with great regret and sadness. The information about her death shocked us. Fly high, Justyna!

Acknowledgement

This work was supported by the Polish Ministry of Science and Higher Education – project ON501 003534.

References

[1] KĘDZIOR K., KOMOR A., MARYNIAK J., MORAWSKI J., *Zastosowanie modelowania i symulacji komputerowej w biomechanice ruchu*, Problemy Biocybernetyki i Inżynierii

Biomedycznej, tom 5, Biomechanika, Polska Akademia Nauk, Wyd. Komunikacji i Łączności, Warszawa, 1990, 99–115.

[2] ŁADYŻYŃSKA-KOZDRAŚ E., MARYNIAK E., *Dynamika skoczka spadochronowego w fazie swobodnego spadania z zamkniętym spadochronem – modelowe badania aerodynamiczne, modelowanie i symulacja numeryczna*, Zeszyty Naukowe Akademii Marynarki Wojennej, rok XLVIII, nr 169 K/1, Gdynia, 2007, 275–288.

[3] MARYNIAK A., *Proces decyzyjny w sytuacji awaryjnej – analiza psychologiczna*, Mechanika w Lotnictwie, ML-XI 2004, Wydawnictwo PTMTS, Warszawa, 2004, 137–144.

[4] MARYNIAK J., *Static and dynamic investigations of human motion*, Mechanics of biological solids, Euromech colloquium 68, Varna, 1975, Bulgarian Academy of Sciences, Sofia, 1977, 151–174.

[5] MARYNIAK J., MARYNIAK A., ŁADYŻYŃSKA-KOZDRAŚ E., FOLTE U., *Katapultowanie – możliwości, problemy i modelowanie*, Nauka–Innowacje–Technika, NIT, październik/grudzień 2004, nr 5 (7), 28–45.

[6] MARYNIAK J., ŁADYŻYŃSKA-KOZDRAŚ E., *Katastrofy lotnicze – przyczyny, skutki, bezpieczeństwo*; [w:] *Perspektywy i rozwój systemów ratownictwa, bezpieczeństwa i obronności w XXI wieku*, pod redakcją Z. Kitowskiego i J. Lisowskiego, Gdynia, 2003, 7–27.

[7] MONIUSZKO J., *Modelowanie dynamiki skoczka spadochronowego z uwzględnieniem doświadczalnej identyfikacji parametrycznej, geometrycznej, masowej i aerodynamicznej*, Praca dyplomowa inżynierska, Politechnika Warszawska, Warszawa, 2009.

[8] MORECKI A., EKIEL J., FIDELUS K., *Biomechanika ruchu*, PWN, Warszawa, 1971.

[9] NIZIOŁ J., MARYNIAK J. (red.), *Mechanika techniczna.* Tom II. *Dynamika układów mechanicznych*. Część V. *Dynamika lotu*, Wyd. Komitet Mechaniki PAN, IPPT PAN, Warszawa, 2005, 363–472.