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Original paper

Investigation of statistical relationships between quantities describing bone architecture, its fractal dimensions and mechanical properties

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The paper presents linear, logarithmic and exponential regression tabecular bone indices, fractal dimensions and strength. The analysis of the above parameters was supported by determining non-parametric correlation coefficients: Spearman's ρ, gamma and Kendall's τ. The principal components' analysis (PCA) was also performed in order to reduce the number of indices describing the variance in the data set. The analysis showed the most independent indices: lacunarity (λ_m, λ_{min}, λ_{max}), BMD, Conn.D., SMI, DA, ρ_A and age.

Key words: *indices of bone architecture, lacunarity, independence of bone indices*

1. Introduction

In mechanical engineering, in a vast majority of cases, strength calculations are based only on the knowledge of the macro-geometry and the magnitude of the structure as well as on the time course of the load. The internal structure and its disturbances (including those introduced intentionally) are seldom taken into account, only being considered, for example, in the case of rolled materials, and – more frequently – for composite materials. In the cases where more complex phenomena are involved, such as material fatigue, creeping or relaxation, in strength calculations usually phenomenological relationships are used – which are often quite far from the physical nature of the destruction process, but allow us to perform the calculation in a tolerably effective way.

In biomechanical engineering, the knowledge of bone structure is of the utmost importance in determining its load-carrying ability, as bone is a fundamental construction material in humans and animals. Here, the material change due to the influence of many internal and external factors depends on the history of these processes, and – what is specially important – continuously reconstructs itself [21]–[24]. Among the factors that influence bone structure we can mention, first of all, the diseases experienced at present and in the past, the way of "making use" of the bone, the age and current activity of the "user", as well as the individual's characteristics [25]–[27].

If one relies solely on the simplified methods of structure evaluation, mainly those reduced to a fast assessment of the apparent bone density expressed in g/cm^2 (the so-called BMD), the diagnostic conclusions could be erroneous. For example, for a bone with a structure disturbed by pathological processes and for a bone of regular but ontogenetically loose structure, the responses to an X-ray image, which allows for determination of the BMD, might be quite similar. However, the differences between the bone strengths might be quite significant.

Therefore, in many year analyses of bone structure, i.e. the level of its architecture, a number of indices have been examined that take into account, among other things, the amount of adequately supported bone trabeculae,

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their thickness, etc. All this is done in order to fully characterize the bone structure and find its influence on the behaviour of bone subjected to load [18]–[20].

In many works published in recent years, their authors try to use fractal quantities in the quest for more effective methods for the description of the relationships between bone structure and its strength [15]–[17].

The aim of the present study is to determine the relationship between the structural indices, including the fractal quantities, and bone strength as a basis for further analysis. In order to obtain these bases, it is essential to determine which of the assessed indices are mutually independent and therefore which of them can be used in multifactor description of bone strength.

2. Indices describing bone architecture

In this work, we analyse the description of the trabecular bone tissue using basic indices commonly applied [2], [3], [8]. The indices are collected in table 1, 2–12.

In the professional literature, one commonly recognises these indices as those allowing us to carry out structural analyses of bone. The analysis was complemented by the BMD value (table 1, 1) determined based on X-ray images. Additionally, we included in the analysis the mean, minimal and maximal volumes of bone structures determined for the all 36 μm layers in each specimen considered (table 1, 16–18). These data are supplemented by the total bone volume V (table 1, 15) calculated as the sum of all layers of bone structures volume.

Among the fractal parameters (table 1, 20–26), we analyzed fractal dimension (3D) of the whole specimen and mean, maximal, minimal fractal dimensions (2D) and lacunarity for the all 36 μm layers in each specimen. The fractal dimensions are used as the measure of the degree of filling the specimen volume and the complexity of the bone structure. At the same time, the lacunarity value allows us to distinguish the specimens of the same

Table 1. Indices describing trabecular bone

1	BMD	Bone mineral density, (g/cm^2)
2	BS	Area of surface of the trabecular tissue that fills the specimen, (mm^2)
3	BS/BV	Quotient of BS and BV values – bone surface-volume-ratio, (mm^{-1})
4	BV	Volume of trabecular tissue contained in specimen, (mm^3)
5	BV/TV	Quotient of BV and TV values – bone volume fraction, (–)
6	Conn.D.	Number of joints between individual trabeculae per unit volume of specimen
7	DA	Degree of anisotropy of architectural structure between individual axes in specimen
8	SMI	Index defining type of specimen's architecture, i.e. showing whether the structure of trabular tissue is built of plate or rode-like trabeculae
9	Tb.N	Average number of continuous trabeculae (fixed on both ends) per unit surface area or unit volume of specimen, (1/mm)
10	Tb.Th	Average thickness of trabecula in specimen of trabecular tissue, (mm)
11	Tb.Sp	Average distance between trabeculae in specimen of trabecular tissue, (mm)
12	TV	Total volume of specimen, (mm^3)
13	US	Compression strength of specimen, (MPa)
14	W	Age
15	V	Bone volume for examined specimen, calculated when using scanning step of 36 μm, (mm^3)
16	v_{max}	Maximal bone volume in 36 μm layer of specimen, (mm^3)
17	v_{min}	Minimal bone volume in 36 μm layer of specimen, (mm^3)
18	v_m	Average bone volume in 36 μm layer of specimen, (mm^3)
19	F	Force for strain of specimen by 0.8%, specimen modelled based on 36 μm scans
20	λ_m	Average lacunarity of bone structure in 36 μm layer of specimen
21	λ_{min}	Minimal lacunarity of bone structure in 36 μm layer of specimen
22	λ_{max}	Maximal lacunarity of bone structure in 36 μm layer of specimen
23	$FD2D_m$	Average fractal dimension for 36 μm layer of specimen
24	$FD2D_{min}$	Minimal fractal dimension for 36 μm layer of specimen
25	$FD2D_{max}$	Maximal fractal dimension for 36 μm layer of specimen
26	FD3D	Fractal dimension for the whole specimen
27	ρ_A	Apparent density of porous material calculated in the same way as for nonporous materials, i.e. mass/volume, (g/cm^3)

Table 2. Results of calculations of linear regression coefficients for the data from all the specimens

	V	FD3D	v_{max}	v_{min}	v_m	F	λ_m	λ_{min}	λ_{max}	$FD2D_m$	$FD2D_{min}$	$FD2D_{max}$	W	BMD	US	BV	BV/TV	BS	BS/BV	Tb.N	Tb.Th	Tb.Sp	Conn.D.	SMI	DA	ρ_A	Indicator
	–	0,95	0,92	0,89	0,98	0,81	–0,7	–0,61	–0,69	0,92	0,79	0,91	–0,05	0,67	0,68	1	0,97	0,87	–0,88	0,87	0,89	–0,88	0,53	–0,89	0,12	0,55	V
		–	0,91	0,88	0,96	0,74	–0,76	–0,62	–0,78	0,97	0,84	0,94	–0,1	0,68	0,63	0,95	0,96	0,88	–0,86	0,96	0,83	–0,98	0,66	–0,86	0,15	0,56	FD3D
			–	0,77	0,94	0,71	–0,65	–0,6	–0,64	0,85	0,65	0,93	–0,08	0,6	0,51	0,93	0,93	0,77	–0,87	0,83	0,88	–0,84	0,5	–0,86	0,1	0,5	v_{max}
V	–			–	0,92	0,86	–0,73	–0,54	–0,73	0,91	0,93	0,78	–0,05	0,79	0,76	0,88	0,91	0,74	–0,79	0,85	0,81	–0,83	0,55	–0,84	0,32	0,41	v_{min}
FD3D	6	–			–	0,83	–0,72	–0,6	–0,71	0,94	0,81	0,91	–0,06	0,73	0,67	0,97	0,99	0,8	–0,9	0,89	0,92	–0,89	0,54	–0,92	0,19	0,5	v_m
v_{max}	6	6	–			–	–0,66	–0,53	–0,62	0,78	0,76	0,71	–0,2	0,72	0,8	0,78	0,81	0,58	–0,76	0,67	0,79	–0,67	0,35	–0,84	0,24	0,21	F+–
v_{min}	6	6	4	–			–	0,84	0,9	–0,71	–0,66	–0,65	0,12	–0,52	–0,57	–0,7	–0,7	–0,73	0,52	–0,77	–0,53	0,75	–0,47	0,7	–0,32	–0,39	λ_m
v_m	6	6	6	6	–			–	0,68	–0,5	–0,39	–0,53	0,13	–0,37	–0,42	–0,61	–0,58	–0,62	0,47	–0,59	–0,48	0,55	–0,28	0,59	–0,17	–0,28	λ_{min}
F	6	6	6	6	6	–			–	–0,77	–0,72	–0,69	0,13	–0,52	–0,54	–0,69	–0,7	–0,7	0,63	–0,76	–0,58	0,81	–0,46	0,7	–0,41	–0,36	λ_{max}
λ_m	3	3	3	3	3	0	–			–	0,92	0,93	–0,08	0,74	0,71	0,92	0,94	0,85	–0,84	0,94	0,81	–0,96	0,68	–0,83	0,18	0,54	$FD2D_m$
λ_{min}	0	0	0	0	0	0	6	–			–	0,73	–0,1	0,75	0,74	0,78	0,81	0,73	–0,69	0,84	0,66	–0,84	0,65	–0,7	0,3	0,41	$FD2D_{min}$
λ_{max}	4	6	4	3	4	3	6	3	–			–	–0,08	0,61	0,59	0,92	0,91	0,84	–0,84	0,88	0,82	–0,92	0,63	–0,81	0,01	0,59	$FD2D_{max}$
$FD2D_m$	6	6	6	6	6	6	3	0	6	–			–	–0,11	–0,14	–0,05	–0,02	–0,13	0,02	–0,11	0,03	0,09	–0,15	0	0,02	0,17	W
$FD2D_{min}$	6	6	3	6	6	6	0	0	3	6	–			–	0,8	0,66	0,75	0,5	–0,62	0,69	0,64	–0,64	0,46	–0,7	0,35	0,31	BMD
$FD2D_{max}$	6	6	6	6	6	6	0	0	5	6	6	–			–	0,65	0,65	0,57	–0,55	0,61	0,56	–0,6	0,39	–0,59	0,22	0,33	US
W	0	0	0	2	0	0	0	0	0	0	1	0	–			–	0,96	0,89	–0,87	0,89	0,88	–0,89	0,55	–0,89	0,1	0,56	BV
BMD	5	3	4	5	5	5	0	0	0	4	3	3	1	–			–	0,79	–0,89	0,91	0,91	–0,9	0,57	–0,92	0,19	0,5	BV/TV
US	2	1	0	3	2	3	0	0	0	3	3	0	0	6	–			–	–0,61	0,9	0,57	–0,89	0,74	–0,68	–0,04	0,61	BS
BV	6	6	6	6	6	6	3	0	5	6	6	6	0	4	1	–			–	–0,7	0,97	0,78	–0,28	0,87	0,23	–0,42	BS/BV
BV/TV	6	6	6	6	6	6	3	0	5	6	6	6	0	5	2	6	–			–	0,67	–0,96	0,82	–0,79	0,13	0,54	Tb.N
BS	6	6	6	6	6	4	6	3	6	6	5	6	0	3	0	6	6	–			–	–0,73	0,23	–0,89	0,22	0,4	Tb.Th
BS/BV	6	6	6	6	6	6	0	0	3	6	3	6	0	3	0	6	6	3	–			–	–0,72	0,79	–0,17	–0,56	Tb.Sp
Tb.N	6	6	6	6	6	6	3	0	6	6	6	6	0	3	3	6	6	6	6	–			–	–0,38	0,16	0,42	Conn. D.
Tb.Th	6	6	6	6	6	6	0	0	3	6	3	6	0	4	0	6	6	3	6	6	–			–	–0,3	–0,35	SMI
Tb.Sp	6	6	6	6	6	6	3	0	6	6	6	6	0	3	2	6	6	6	6	6	–				–	–0,17	DA
Conn. D.	1	3	0	1	1	1	0	0	2	3	3	3	0	0	0	1	1	6	0	6	0	4	–			–	ρ_A
SMI	6	6	6	5	6	5	0	0	3	6	3	6	2	5	0	6	6	4	6	6	6	6	0	–			
DA	3	3	3	6	3	3	3	0	3	3	3	0	0	0	0	3	3	1	3	3	3	3	0	3	–		
ρ_A	6	6	6	6	6	6	0	0	3	6	6	6	0	3	3	6	6	6	6	6	6	6	3	3	3	–	
Indicator	V	FD3D	v_{max}	v_{min}	v_m	F	λ_m	λ_{min}	λ_{max}	$FD2D_m$	$FD2D_{min}$	$FD2D_{max}$	W	BMD	US	BV	BV/TV	BS	BS/BV	Tb.N	Tb.Th	Tb.Sp	Conn. D	SMI	DA	ρ_A	

Table 3. Juxtaposition of rank sums for linear regression and two nonlinear regressions (exponential and logarithmic) for osteoporotic specimens

fractal dimensions which differ in the amount and structure of their pores.

3. Subject and method of investigations

The investigations were carried out on 42 cylindrical specimens from human femoral bones acquired following the procedure of the implantation of femoral joint prosthesis. A half of the material collected originated from subjects with diagnosed osteoporosis, and the other half from patients suffering from coxarthrosis. The latter were treated as the reference specimens which can be found in the work [7].

The structural indices were assessed based on examinations performed in Eindhoven on a μCT80 type computer microtomograph. The BMD value was measured by means of a LUNAR (USA) densitometric apparatus, and the strength US (table 1, 13) was determined using a MTS (USA) testing machine. The force F (table 1, 19) was numerically calculated for the strain $\varepsilon = 0.8\%$. The calculations were made with the use of FEM, in the ANSYS 11.0 system environment. For these analyses, isotropic material properties described by the quantities $E = 10$ GPa and $\nu = 0.3$ were accepted.

The calculations of fractal dimension and lacunarity were conducted using a box-counting algorithm with variable mesh size in the software ImageJ with the module FracLac [12]–[14].

The statistical calculations were aimed at determining mutual relationships between the quantities considered. In this assessment, we assumed three different models: a linear regression model and two nonlinear ones (exponential and logarithmic). The values of correlation coefficients were calculated for each of the models [1], [11]. Then we determined nonparametric correlation coefficients: Spearman, and additionally gamma and Kendall tau. The principal components' analysis (PCA) was also performed, with the intention of reducing the number of indices describing the variance in the data set. The calculations were performed using Statistica software (version 8.0).

4. Results of the examinations and their analysis

The results of the linear and nonlinear regression calculations for all cases of associations between the indices considered are juxtaposed in the correlation tables. The calculations were carried out for three sets of the data, separately for the indices of specimens diagnosed as the coxarthrosis and those diagnosed as the osteoporotic, and the third group analyzed consisted of the data from all the specimens together.

Table 2 shows, as an example, the results of calculations of linear regression for all the data. Bright gray background highlights the correlation coefficients with absolute value up to 0.5, a dark gray background from 0.5 to 0.7. Other cells contain the correlation coefficients whose values are greater than 0.7.

The correlation coefficients, calculated in the manner described, were given the following ranks:

- 0 – if the absolute value of the correlation coefficient was $\mathrm{mod}(r) \le 0.5$,
- 1 – if the absolute value of the correlation coefficient was in the interval of $0.5 < \mathrm{mod}(r) \le 0.7$,
- 2 – if $\mathrm{mod}(r) > 0.7$.

In the first case, no correlation was assumed, in the second case – a weak correlation, and in the third – a significant correlation between the indices examined. The results of the analysis of the calculations for all the regressions examined are juxtaposed in table 3. In this table, a sum of ranks of the correlation coefficients is shown for each pair of the indices examined [4]. Due to the assumed rank values, these sums range from 0 to 6. It was assumed that the sums of ranks of the values from 0 to 1 indicate that neither a linear relationship, nor any of the examined nonlinear relationships between the quantities exist (table 3, cells with light gray background). Conversely, the sum values in the range from 5 to 6 indicate the existence of such relationships (table 3, cells with dark gray background and white numbers). Example calculations for the specimens diagnosed as osteoporotic are juxtaposed in table 3.

Table 4. Indices of high degree of independence

	Index	Coxarthrosis	Osteoporosis	Total
1	Lacunarity λ_m	–	11	–
2	Lacunarity λ_{min}	7	22	9
3	Lacunarity λ_{max}	8	–	–
4	Age W	25	24	25
5	BMD	14	–	–
6	Conn.D.	16	16	12
7	SMI	7	–	–
8	DA	15	7	25
9	ρ_A	15	–	14

The analysis of the above tables allows us to select the indices which are most frequently independent of the others. These indices are collected in table 4 for all

the analyzed regressions and all three groups of data. The numbers in the table refer to the number of indices which proved uncorrelated with that given in the left column.

The indices presented in table 4 are only those which proved independent of at least 7 other indices. As we see, they can be divided into three groups: indices of lacunarity, indices of bone structure, and the age index.

In order to confirm the results obtained, we determined a matrix of nonparametric Spearman correlation coefficients [5], [9], [10]. First, the results were compared with the matrices of nonparametric gamma and Kendall tau correlation coefficients, which testified to the consistency of the tests. Then the values of the Spearman correlation coefficient were associated with appropriate values from table 3 in order to assess the consistency of both analyses. The following percentage consistencies were found:

- for coxarthrosis specimens, 69.7%,
- for osteoporotic specimens, 84.0%,
- for all specimens, 85.5%.

5. Principal component analysis (PCA) [6]

The principal component analysis was based on determining the eigenvectors of the matrices of linear correlation coefficients for the data of the osteoporotic specimens, coxarthrosis specimens and for all the specimens together. The determined eigenvalues of the matrices of correlation coefficients define the principal components explaining the variance of the source data. The lower the eigenvalue, the smaller the proportion of the source data variance explained by a specific component. The eigenvalues in table 5 are calculated for the matrices composed of the data of the osteoporotic and coxarthrosis specimens, and of all the specimens together.

As can be seen in table 5, the first seven eigenvalues for the coxarthrosis and combined specimens are greater than unity, as are the first six eigenvalues for osteoporosis. According to the Kaiser test, only the eigenvalues which are greater than unity describe the variance of source data better than an arbitrary original index. Thus, only the first seven components associated with appropriate eigenvalues in table 5 were accepted for further analysis. From the same table, it can also be learnt that the selected components explain about 94% of variance of the source data in the cases of osteoporosis and coxarthrosis, and about 94% for the set of data containing all specimens. This means that the overall number of thirty-six indices describing the source data can be, with a minor loss in accuracy of the model, compressed into seven statistically-independent principal components.

The principal components define an orthogonal system of coordinates on which can be linearly projected the index values obtained from the source data. This means that the original variables are linearly

Table 5. Important eigenvalues of linear correlation matrices for osteoporotic, coxarthrosis, and all specimens, together with cumulated percentage of explained variance of source data

Component	Osteoporosis		Coxarthrosis		Together	
	Eigenvalue	Cumulated % of explained variance	Eigenvalue	Cumulated % of explained variance	Eigenvalue	Cumulated % of explained variance
1	36.61229	70.4083	34.69909	66.7290	36.11668	69.4552
2	4.23725	78.5568	5.77942	77.8433	3.60453	76.3870
3	3.15042	84.6153	3.02056	83.6521	2.90996	81.9830
4	2.63253	89.6779	2.36719	88.2044	2.31200	86.4292
5	1.51405	92.5895	2.19376	92.4231	2.02710	90.3275
6	1.38598	95.2548	1.21661	94.7628	1.37890	92.9792
7	0.57766	96.3657	1.00202	96.6897	1.01773	94.9364
8	0.52354	97.3726	0.50522	97.6613	0.72978	96.3398
9	0.41706	98.1746	0.45266	98.5318	0.42028	97.1480
10	0.37949	98.9044	0.28198	99.0741	0.37709	97.8732
11	0.19152	99.2727	0.18375	99.4274	0.29857	98.4474
12	0.14166	99.5451	0.11837	99.6551	0.21724	98.8651
13	0.08964	99.7175	0.09515	99.8381	0.13406	99.1229
14	0.04315	99.8005	0.05230	99.9386	0.11402	99.3422
15	0.03807	99.8737	0.02638	99.9894	0.09963	99.5338
16	0.02744	99.9265	0.00553	100.0000	0.07715	99.6822

Table 6. Factor loadings for osteoporosis (absolute values)

Index	Component 1	Component 2	Component 3	Component 4	Component 5	Component 6	Component 7
V_{36}	**0.97**	0.10	0.11	0.08	0.12	0.04	0.01
FD3D	**0.99**	0.04	0.10	0.06	0.02	0.00	0.03
v_{max}	**0.93**	0.09	0.26	0.01	0.12	0.01	0.08
v_{min}	**0.90**	0.13	0.22	0.24	0.09	0.01	0.10
v_m	**0.96**	0.16	0.11	0.11	0.07	0.00	0.02
F_{36}	**0.87**	0.31	0.02	0.08	0.11	0.05	0.30
λ_m	**0.61**	**0.70**	0.01	0.29	0.03	0.01	0.11
λ_{min}	0.36	**0.68**	0.32	0.34	0.20	0.28	0.07
λ_{max}	**0.78**	0.49	0.08	0.05	0.25	0.03	0.05
$FD2D_m$	**0.98**	0.06	0.09	0.16	0.08	0.01	0.01
$FD2D_{min}$	**0.84**	0.07	0.39	0.03	0.30	0.00	0.13
$FD2D_{max}$	**0.93**	0.06	0.10	0.28	0.04	0.03	0.10
W	0.49	0.01	0.47	0.40	0.36	0.46	0.01
BMD	**0.75**	0.41	0.22	0.20	0.26	0.26	0.05
US	**0.59**	0.24	**0.51**	0.17	0.07	0.46	0.12
BV	**0.97**	0.06	0.12	0.08	0.13	0.04	0.02
BV/TV	**0.96**	0.16	0.12	0.08	0.10	0.02	0.02
BS	**0.89**	0.35	0.05	0.11	0.17	0.09	0.06
BS/BV	**0.86**	0.31	0.36	0.01	0.12	0.02	0.04
Tb.N	**0.96**	0.17	0.08	0.12	0.11	0.00	0.01
Tb.Th	**0.86**	0.36	0.32	0.11	0.00	0.02	0.03
Tb.Sp	**0.95**	0.15	0.00	0.21	0.02	0.06	0.03
Conn.D.	**0.53**	0.35	0.47	**0.54**	0.16	0.07	0.04
SMI	**0.87**	0.12	0.26	0.10	0.17	0.19	0.24
DA	**0.65**	0.08	0.09	**0.52**	0.38	0.19	0.22
ρ_A	**0.88**	0.08	0.18	0.14	0.06	0.06	0.25

Table 7. Factor loadings for coxarthrosis (absolute values)

Index	Component 1	Component 2	Component 3	Component 4	Component 5	Component 6	Component 7
V_{36}	**0.95**	0.01	0.14	0.04	0.13	0.02	0.08
FD3D	**0.98**	0.16	0.03	0.02	0.01	0.07	0.02
v_{max}	**0.83**	0.06	0.48	0.04	0.10	0.02	0.21
v_{min}	**0.91**	0.23	0.15	0.16	0.01	0.08	0.18
v_m	**0.97**	0.08	0.18	0.09	0.04	0.06	0.01
F_{36}	**0.79**	0.49	0.04	0.17	0.21	0.03	0.16
λ_m	**0.84**	0.26	0.07	0.43	0.13	0.01	0.05
λ_{min}	**0.77**	0.30	0.20	0.42	0.10	0.05	0.04
λ_{max}	**0.67**	0.40	0.22	0.46	0.20	0.11	0.13
$FD2D_m$	**0.97**	0.07	0.14	0.11	0.00	0.04	0.01
$FD2D_{min}$	**0.83**	0.01	0.47	0.19	0.04	0.10	0.09
$FD2D_{max}$	**0.92**	0.16	0.22	0.00	0.03	0.06	0.11
W	0.01	0.14	0.28	0.34	**0.85**	0.05	0.22
BMD	**0.54**	0.30	0.50	0.33	0.14	0.14	0.44
US	**0.69**	0.30	0.44	0.12	0.10	0.32	0.15
BV	**0.95**	0.07	0.16	0.02	0.12	0.01	0.06
BV/TV	**0.96**	0.05	0.14	0.15	0.06	0.13	0.07
BS	**0.81**	0.45	0.04	0.17	0.12	0.00	0.11
BS/BV	**0.86**	0.32	0.28	0.19	0.09	0.12	0.02
Tb.N	**0.88**	0.35	0.12	0.02	0.17	0.22	0.09
Tb.Th	**0.82**	0.39	0.31	0.23	0.09	0.07	0.01
Tb.Sp	**0.94**	0.27	0.09	0.06	0.07	0.06	0.04
Conn.D.	0.49	**0.75**	0.20	0.14	0.19	0.26	0.05
SMI	**0.86**	0.43	0.12	0.07	0.00	0.17	0.09
DA	**0.53**	**0.74**	0.18	0.14	0.11	0.07	0.16
ρ_A	**0.55**	0.39	0.00	0.32	0.31	0.50	0.17

Table 8. Factor loadings for all examined specimens (absolute values)

Index	Component 1	Component 2	Component 3	Component 4	Component 5	Component 6	Component 7
V_{36}	**0.96**	0.03	0.16	0.04	0.07	0.06	0.03
FD3D	**0.99**	0.09	0.01	0.03	0.02	0.10	0.00
v_{max}	**0.90**	0.06	0.22	0.15	0.17	0.09	0.06
v_{min}	**0.92**	0.19	0.02	0.17	0.09	0.07	0.07
v_m	**0.97**	0.05	0.15	0.01	0.07	0.01	0.07
F_{36}	**0.82**	0.32	0.03	0.14	0.20	0.25	0.14
λ_m	**0.79**	0.16	0.39	0.36	0.13	0.13	0.02
λ_{min}	**0.64**	0.15	0.26	**0.60**	0.07	0.25	0.13
λ_{max}	**0.81**	0.19	0.32	0.25	0.14	0.10	0.13
$FD2D_m$	**0.98**	0.06	0.01	0.15	0.04	0.07	0.01
$FD2D_{min}$	**0.87**	0.08	0.16	0.34	0.16	0.03	0.02
$FD2D_{max}$	**0.93**	0.20	0.13	0.03	0.10	0.07	0.01
W	0.09	0.06	**0.53**	0.15	0.78	0.05	0.24
BMD	**0.74**	0.26	0.05	0.40	0.06	0.18	0.10
US	**0.71**	0.19	0.14	0.35	0.03	0.48	0.01
BV	**0.96**	0.06	0.14	0.06	0.07	0.03	0.04
BV/TV	**0.97**	0.04	0.16	0.01	0.03	0.03	0.11
BS	**0.87**	0.33	0.19	0.11	0.01	0.01	0.01
BS/BV	**0.87**	0.16	0.40	0.01	0.15	0.08	0.11
Tb.N	**0.94**	0.17	0.18	0.03	0.07	0.12	0.12
Tb.Th	**0.85**	0.21	0.45	0.02	0.14	0.01	0.02
Tb.Sp	**0.95**	0.15	0.09	0.00	0.07	0.18	0.03
Conn.D.	**0.62**	0.50	0.39	0.23	0.09	0.19	0.28
SMI	**0.89**	0.23	0.20	0.10	0.06	0.02	0.15
DA	0.20	0.80	0.15	0.02	0.31	0.31	0.21
ρ_A	**0.55**	**0.51**	0.14	0.10	0.29	0.22	0.44

correlated with the principal components – the new variables. The attempt to interpret the principal components needs these correlations to be defined, which can be done by determining the so-called factor loadings. The factor loadings defined for the assumed numbers of principal components (seven, in this case) are listed in tables 6, 7 and 8 for osteoporosis, coxarthrosis, and all the investigated specimens, respectively.

Inspection of the factor loading values shown in tables 6-8 leads to the conclusion that almost all the components present in the data set correlate well with the first principal component. Undoubtedly, age is an exception to this rule; lacunarity, BMD, US, Conn.D., DA, and ρ_A can be, to some extent, also counted as exceptions. Strong correlation of the majority of the components allows us to distinguish the group of indices, carrying common information about the bone structure, which can be associated with the first – the structural – principal component. The second, the third and the fourth components are characterized by medium values of factor loadings, which could indicate the fact that some of the indices carry additional information about the bone structure – information that does not appear in the first component. In the indices carrying such an information, lacunarity, US, Conn.D., DA, and ρ_A can be numbered. The fifth component exhibits the highest correlation with the age index, except perhaps for the case presented in table 6 – the data pertaining to osteoporosis. The sixth and seventh components do not exhibit any higher values of factor loadings for most of the indices.

6. Discussion and conclusions

The coefficients of correlation between the indices of bone structure and the mechanical properties are similar to those analyzed in other studies [19], [28], [29], irrespective of the purpose of their implementation.

The analysis of correlation coefficients for linear, logarithmic and exponential approximations showed that nine of the indices are the most independent ones, and these can be divided into three groups. The first group consists of lacunarity (λ_m, λ_{min}, λ_{max}). The second group is composed of other structural indices, i.e. BMD, Conn.D., SMI, DA, and ρ_A. The age index belongs to the third group. The analysis of non-parametric correlation coefficients (Spearman, gamma and Kendall tau) confirmed the previous conclusion (Item 1) with consistency greater than 70%.

HILDEBRANDT et al. [20] made similar analysis for the 5 indices, i.e. Tb.N, Tb.Th, Tb.Sp, SMI and DA. Tb.N, Tb.Th, Tb.Sp were calculated using two methods: the traditional – model-dependent and model-independent. These volumes were correlated linearly and logarithmically with BV/TV. For trabecular bone taken from different parts of the skeleton (iliac crest, lumbar spine 2nd, lumbar spine 4th, femoral head and calcaneal core) there were no correlations (assuming a mod (r) > 0.5) between BV/TV and DA (all specimens) and between Tb.Th (model-independent) and Tb.Th (the 2nd lumbar spine specimens). These results, although very fragmentary, coincide with ours. The essence of our research is to identify those indicators for which the correlations are weak. Such a search can also be found in the literature. For example, POTHUAUD et al. [19] correlated BV/TV with the skeleton graph configuration indicators such as the number of connection, termini vertices, branches, the number of loops and volumetric topological parameters. In all cases, the authors used (as the second variant) the values of selected indices in a standardized form. The calculations for the mechanical properties did not bring about the expected improvement of the determination coefficients R^2. This may testify to the important relationship between the quantities considered.

The principal component analysis (PCA) showed that many of the indices exhibit linear mutual relationship. Due to this fact, it was impossible to determine the full (36-element) eigenvector – see table 5. The principal component analysis showed that there could exist seven principal components that explain over 96% of variation of the source data. These describe the data set variability better than the original indices. The analysis of factor loadings revealed three principal components, the first of which can be associated with the structural indices such as BMD, Conn.D., DA, and ρ_A, as well as US. The second principal component also has a structural character, and is associated with the indices such as Conn.D., DA, US and ρ_A. The age index dominates in the third principal component.

One can state that the principal component analysis (PCA) does not allow us to define, in a unique way, new components that might facilitate description of the source data independently for coxarthrosis and osteoporosis specimens, or jointly for both groups. It seems, however, that the PC analysis confirms the basic correlation analyses, whose results are summarized in table 4.

It seems that in future work, a fabric tensor carrying a additional information combining anisotropy and topological parameters should be taken into account.

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