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Mechanical properties of human tooth approximated with overdamped oscillators*

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Numerical calculations and analysis of the tension states in hard tissues of tooth are very interesting and important issues from the orthodontist's point of view. Orthodontic methods of treatment can have a dramatic effect on children's long-term dental health and facial appearance. Research done in this field shows that due to understanding mechanics of masticatory organ it is possible to attain the intentional aim of therapy in shorter time and avoid to undesired complications.

The behaviour of alveolodental ligament when the force was applied allows the authors to look at human tooth affected by damped harmonic oscillator. In this study, periodontograph was successfully achieved from harmonic oscillators properties.

Key words: oscillator, teeth, periodontograph

1. Introduction

A quasi-dynamic biomechanical masticatory system behaves in compliance with environmental laws. Numerical calculation and analysis of this system are very interesting and important issue from the orthodontist's point of view. Orthodontic methods of treatment can have a dramatic effect on children's long-term dental health and facial appearance.

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The main aim of this paper was to interpret a periodontograph, which was achieved experimentally in previous research [1], and to confirm it in theoretical way. The authors were trying to find similarities between human tooth and overdamped harmonic oscillator. A healthy tooth, from the biomechanician's point of view, shows overdamped harmonic oscillation patterns [2]. Theoretical calculations for damping factor cannot replace experiments carried on samples in order to determine the material properties of the periodontal ligament, but they can help to simplify them. Nowadays mathematical modelling and computing are far more popular and profitable than experiments on samples extracted from animals. Simulations replace especially those experiments which cannot be carried out or which are difficult to carry out on biologic material.

The approach is based on the analysis of theoretical model related to experimental observations. To facilitate the calculations a special computer program was produced.

Analysis of the tension states in hard tissues of tooth could be obtained by means of tensometric load cells. The authors were trying to design simple tensometers, which can be used in experimental research in that field in the future.

2. Periodontal ligament

Periodontal ligament plays the most significant role in physiological tooth mobility. The mechanical properties and elastic behaviour of periodontal tissues are decisive factors in understanding initial tooth mobility and bone remodelling process in orthodontics [3].

The periodontal ligament is a soft biological tissue that controls tooth movement under physiological loads by joining tooth and alveolar bone. It keeps the tooth in tooth-socket and allows it to chew and munch. Periodontal ligament consists of collagen fibres passing obliquely from the alveolar bone towards the apex of the tooth; the fibres 'sling' the tooth in position against pressure on its surface [4], [5]. Its various components differ in their material properties. Due to the combination of fluid and collagen fibres the periodontal ligament shows viscoelastic behaviour typical of soft biological tissues. It is non-linear and time-dependt. It also depends on loading history [3].

The most important task of dentition is food crumbling. In this case, physiological forces act on teeth and tooth-sockets. Average forces applied to the alveolodental ligament during chewing vary between 150 and 300 N. It was affirmed that the forces exerted on molar teeth could reach 800 N [1].

3. Mobilities of tooth and possible loads

Considering the load in two possible directions we have two typical mobilities of teeth. Every tooth is characterised by horizontal and vertical mobility [6].

Horizontal mobility of tooth is connected with the force applied in horizontal direction, along the main axis of tooth. In this situation, due to fibres, the root transfers the force on to the tooth-socket. The tooth is pushed to the tooth-socket but when the force quits acting on the tooth, it will quickly return to its initial position. This mobility varies in microns (μm). The displacement of the lower left first molar measured by transducer Type M-3 in the palatal and apical directions during clenching varied between 40 and 136 microns [7].

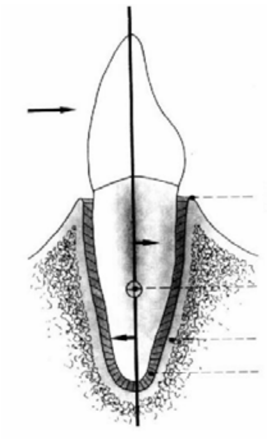

Fig. 1. Collagen fibres in periodontium and the tooth's behaviour when the force is applied in vertical direction [1]

Vertical mobility of tooth is connected with the force which acts in vertical direction of tooth as is shown in figure 1. In this case, we can treat the tooth with one root as a double-arm lever with the centre of rotation on 2/3 of tooth length. In two-dimensional computer models of the upper incisor, it is usually located at 42% of the root length measured from the alveolar crest. [8]

During nastication the first lower left molar showed displacement ranging from 11 to 71 microns and from 17 to 39 microns in the lingual direction and in the buccal direction, respectively [7].

These forces and displacements caused by loads have been examined since the 17th century [9], [10]. Many scientists tried to measure physiological forces acting in human oral cavity. Unfortunately, the lack of proper apparatuses made the results of their investigation unreliable.

Up to now the mobility degree has been assessed due to application of static loads and a subsequent measurement of the resultant displacement. The results obtained with the measurement technique proposed by Muhlemann (1967) were validated by O'Leary et al. (1964) and by Persson and Sweson (1980). This approach, however, has not been clinically disseminated both because of a high cost of the equipment and, above all, because of the difficulty in performing the measurements [11].

In 1998 in Japan, the tooth displacement transducer, Type M-3, with magnesensors was constructed. It was fixed to the labial surface of the anterior first molars using a paraocclusal splint [7]. Displacement of molars was measured. The experiments have not been continued.

In CASTELLINI's work [11], the ratio of the maximum of the tooth displacement to the input force peak has been considered to be the mobility degree index. Dynamic loads have been applied to teeth with a small hammer and measured with a load cell. The resultant displacement of tooth has been measured with a Laser Doppler vibrometer that allows easy and versatile non-contact measurements with high accuracy and sensitivity.

The author claims that due to this technique it will be possible to measure a pathological mobility of the tooth, before it becomes evident and troublesome.

Some of experiments were done on rats using a non-contacting displacement detector and a homeostatic clamp which immobilises the jaw [12].

The average eruption rates of the rat incisor were estimated to be 516 microns/24 hr at 37 Celsius degrees of the rectal temperature. The maximum pushing force was estimated to reach 9 mN.

In our experiment, a new piece of apparatus is constructed. It is very cheap and simple but unfortunately it has not stipulated basic safety regulations, hence it must be improved in the future.

4. Tensometric load cell

Analysis of the tension states in hard tissues of tooth could be carried out by means of tensometric load cells. Sample tensometers were designed.

Tensometric load cell is the instrument that is used to measure linear strains and tensions. There are many kinds of electrical tensometers: electric resistance tensometer (it determines resistance as the stress), capacity strain gauge (changes in the capacity measured by tension) and inductive tensometer.

In this paper, all three groups of tensometers were constructed, but only capacity strain gauges gave reliable results. They were made from various materials, e.g. the leaf from head's truck of hard disc, rubber from the inner tube and a piece of wire (figure 2).

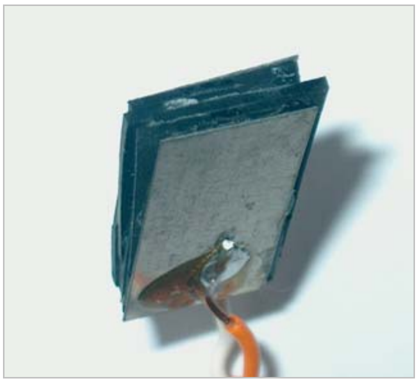

Fig. 2. One of the tensometers constructed

The authors made some necessary measurements to check an ultimate compressive strength of the tensometer. In figure 3, the capacity characteristic (electrical capacity which depends on compressive force) of one of the tensometers being constructed is shown.

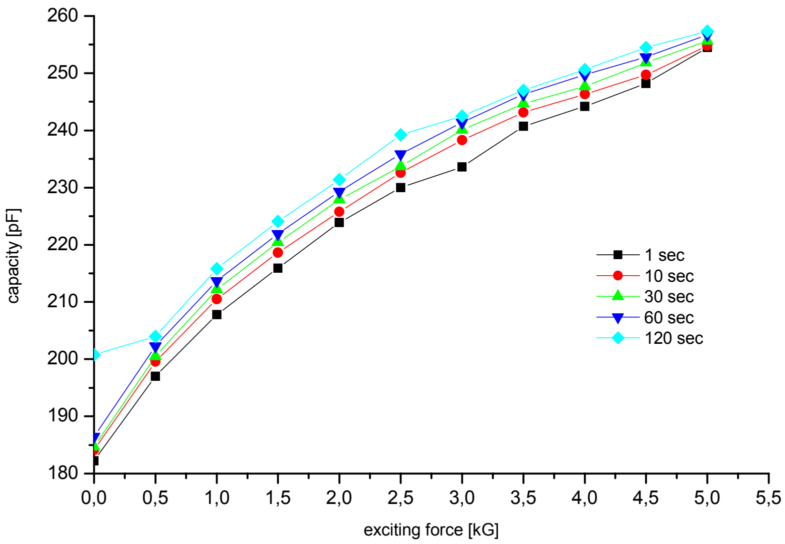

Fig. 3. Capacivity characteristic of tensometer
(different lines represent different measurement times – from 1 to 120 sec)

Overload of this tensometer was not observed in a range of the forces examined. This means that this load cell is sensitive to the physical load applied.

5. Periodontograph

Periodontograph is a graph consisted of two parts (figure 4). The first part (A) shows the displacement of tooth in a jaw versus the force applied (a constant increase of force in time-unit), the second part (B) shows the way in which the tooth recovers its initial position.

The curve in the part A of figure 4 is divided into two parts. The first one (I) is called *primary tooth mobility* that lasts untill the maximum stress of periodontium is reached. Later, when the force is still applied, alveolar process becomes deformed.

In the first part (A), on the *y*-axis there is aberration in micrometers (μm), while on the *x*-axis there is force (*N*). In the second part (B), there is time on the *x*-axis. Hooke's law is only fulfilled when the forces are smaller than 0.5 N. When the force is greater than 4 N the saturation of periodontograph takes place which is caused by the deformation of alveolar process. The part B shows a decrease in displacement with time. Force relaxation and hysteresis of periodontal ligament depend on loading history.

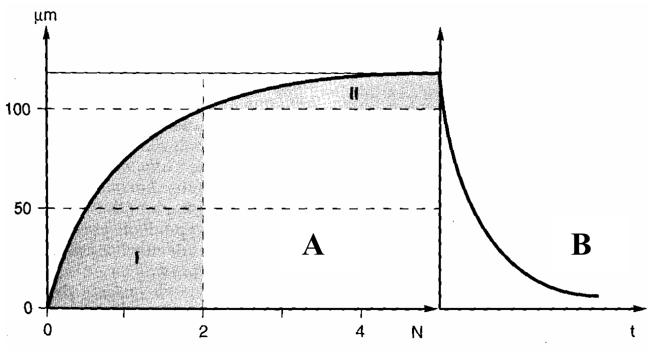

Fig. 4. Periodontograph [1]
(I – primary tooth mobility, II – secondary tooth mobility)

Periodontograph is a goldmine for dentists. Periodontograph of molar tooth shows its smaller movements because the area of its root in comparison to that of other teeth is bigger. When the tooth is changed by parodontosis it behaves differently. After force application, aberration increases very quickly. When the force is no longer exerted on the tooth it either slowly returns to the initial position or never returns to it.

Periodontograph has different shapes for different cases so it is very important to find its shape. Unfortunately it cannot be done experimentally because of technical problems. That is why we tried to model it due to computing.

6. Oscillator

The healthy teeth in different generations of men and women, whose age ranged from their teens to their forties, showed similar damped oscillation patterns. Harmonic oscillator is one of the most important and the simplest systems in physics. It is applied as the first approximation in many complicated calculations and modelling. Due to the properties of harmonic oscillator the periodontograph and tooth behaviour were explained.

One-dimensional harmonic oscillator with time-dependent exciting force is defined as follows [13]:

$$\ddot{x}(t) + 2\gamma \dot{x}(t) + \omega_0^2 x(t) = F(t)\,, \tag{1}$$

where:

$F(t)$ – arbitrary exciting force which is expressed by the Fourier transform,

γ – damping factor,

ω_0 – natural frequency of an oscillator; $\omega_0 = \sqrt{k/m}$.

It was observed that the second part of periodontograph behaves similarly to an overdamped harmonic oscillator in which damping factor (γ) is much higher than natural frequency (ω_0). Taking advantage to the fact that this exciting force is expressed by the Fourier transform, the following total solution was obtained:

$$x(t) = Ax_1(t) + Bx_2(t) + \int_{-\infty}^{\infty} G(t,t')F(t')dt'\,, \tag{2}$$

where:

A, B – arbitrary constants which take into account initial conditions,

$G(t, t')$ – the Green function of an oscillator which assumes the following form:

$$G(t,t') = \begin{cases} \dfrac{\mathrm{e}^{-\gamma t(t-t')} \sinh\left[\sqrt{\gamma^2 - \omega_0^2}\,(t-t')\right]}{\sqrt{\gamma^2 - \omega_0^2}} & \text{for } t > t', \\ 0 & \text{for } t < t'. \end{cases} \tag{3}$$

When the third expression is inserted into the second one, the following equation is obtained:

$$x(t) = Ax_1(t) + Bx_2(t) + \int_{t_0}^{t} \frac{\mathrm{e}^{-\gamma t(t-t')} \sinh\,[\sqrt{\gamma^2 - \omega_0^2}\,(t-t')]\, F(t')dt'}{\sqrt{\gamma^2 - \omega_0^2}}\,. \tag{4}$$

This is the final form of an overdamped harmonic oscillator solution. In this expression, there is any exciting force but in further calculations the following form of the load was used:

$$F(t) = F_0 e^{-\alpha t} . \quad (5)$$

This force was chosen for the calculations because it has adiabatic decaying. Such a form of the force is often used in modelling and quite well reflects the reality.

7. Computer program

The approach is based on the analysis of theoretical model related to experimental observation [1]. To facilitate the calculations a special computer program in Fortran 77 was produced. The data achieved show that periodontograph is well reproduced for the following parameters used in programming: $t_0 = 0$ sec, $dt = 0.05$ sec (time raster), $\omega_0 = 0.5$ sec^{-1}, $F_0 = 1$ N, $\alpha = 3$ sec^{-1}, $m = 0.5$ kg. The variable m mentioned above is the only modelling parameter and it has nothing in common with mass of tooth. The force magnitude was chosen according to periodontograph published by Lehmann and Hellwig. Hypothesis that damping factor (γ) is much bigger than natural frequency (ω_0) is realised by the program produced.

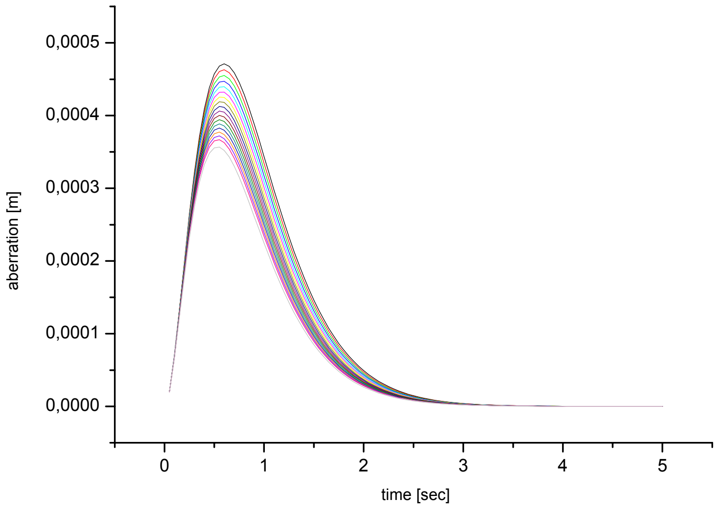

Fig. 5. Displacement versus time for different friction factors γ (in the range from 0.55 to 1.55 sec^{-1}) obtained from computer program

8. Conclusions

Masticatory organ is a quasi-dynamic system. Despite the complexity of this system, an overdamped harmonic oscillator with an exciting force could approximate a single tooth. The authors satisfactorily reconstructed a periodontograph of a single healthy tooth from harmonic oscillator properties. The best fitting was obtained for $\gamma = 2.3\ \mathrm{sec}^{-1}$. This means that damping properties of periodontal ligament, the softest tissue of the tooth, are equal to $2.3\ \mathrm{sec}^{-1}$. If the tooth is changed by parodontosis, the friction factor is too small to obtain the fitting to oscillation pattern.

Understading the mechanism of periodontograph is of key importance in diagnosis of masticatory organ. Different adequately fitted forces or other parameters of equation (4) could model pathological cases such as parodonthosis in the oral cavity. There is no need to test periodontal ligament experimentally using biological samples from animals or viscoelastic material.

In orthodontics, external loads are applied to the tooth crowns using orthodontic appliances. Since stresses and strains in the periodontal tissue, caused by an initial tooth movement, stimulate alveolar bone remodelling and thus orthodontic tooth movement, their knowledge is fundamental to the selection of an optimal force system for target tooth movement. For this reason, typical properties of the periodontal ligament have to be known [14].

The objective of orthodontic treatment is to achieve as precise and rapid tooth movement as possible, without provoking such undesired complications as bone remodelling and root resorption. To enable the application of an optimal orthodontic force that meets these requirements, knowledge of the biomechanics of tooth and material properties of the periodontal ligament is a must [3].

Loads and aberrations could be measured by means of tensometer. Its construction is not a simple task. A special cover of the tensometer must make the contact of the wires with saliva impossible. Tensometers designed for experiments in the oral cavity have to comply with safety regulations.

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