

Goldmann applanation tonometry – not as good as gold

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A thesis that linear mechanics does not apply to the analysis of cornea load during Goldmann applanation tonometry measurement and that the concept of surface tension in the lacrimal fluid is an ineffective attempt at circumventing the associated problems is put forward. The fundamental problem emerging during numerically simulated measurement of pressure on the eyeball, whose dimensions are considered to be calibrated, stems from the fact that the flattening of the cornea at the nominal intraocular pressure leads to a critical state in which the shell loses stability. The consequences are far-reaching. The Goldmann tonometer performs well at low intraocular pressure, but above the nominal pressure its readings are always understated. The cause of the error is not the tonometer itself (its readings can be even very accurate). It is shell “solution” called Imbert–Fick law which is faulty.

Key words: cornea, biomechanical model, material parameters, Goldmann applanation tonometry, IOP

1. Introduction

Although the problem of an axially symmetric ellipsoidal elastic shell with variable thickness, loaded with internal pressure and flattened over a small surface, whose solution is analyzed below, is a strictly mechanical problem, it has the most interesting application in ophthalmology. Goldmann applanation tonometry (GAT) is an intraocular pressure (IOP) measuring method based precisely on such a case of shell loading, and the accuracy of the measurement closely depends on the interpretation of the results.

GAT consists of a set of conditions to be met in order to carry out a correct measurement, a measuring instrument and formulas for correcting the pressure reading. Although there are many varieties of the instrument, not always based on the Goldmann procedure, all the results obtained by means of them are ultimately referred to GAT (their readings are compared with those of the applanation tonometer). Therefore it is not easy to overemphasize the importance of

this IOP measuring technique. The theoretical description of shell deformation mechanics plays a fundamental role in GAT, but the solution of the problem appears to be completely different from what we have imagined it to be for more than a century.

1.1. Goldmann applanation tonometry (GAT)

The measurement of intraocular pressure is similar to the case of shell loading described earlier. The cornea is flattened over a strictly defined small surface and the average pressure p_G , understood to be a ratio of the force to the surface area of the flattening, is the measure of the IOP. For the flattened area of a membrane, the equilibrium condition assumes the following form (p means IOP)

$$p_G = p. \quad (1)$$

When slightly flattened, the cornea closely resembles a membrane, and equation (1) as applied to IOP determination is called the Imbert–Fick law. This law

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has been used since the times of Maklakoff and Fick, i.e. since the end of the 19th century.

However, precise measurements made by Goldmann in the middle of the 20th century showed that the results obtained deviated from the Imbert–Fick law [1]. Therefore he introduced the so-called calibration values for the eye parameters, i.e. central corneal thickness CCT, the axial radius of external corneal curvature R and the applanation zone diameter D , at which the Imbert–Fick law was to be satisfied:

$$\text{CCT} = 0.520 \text{ mm}, \quad R = 7.8 \text{ mm}, \quad D = 3.06 \text{ mm}. \quad (2)$$

Obviously, the eyes of patients in clinical practice rarely met simultaneously the conditions relating to CCT and R and Goldmann's successors had to further improve the IOP measurement technique by introducing corrections. The associated problems have persisted till today: in applanation tonometry there are neither generally accepted correction tables which would allow one to convert p_G measured for different R or CCT nor a theoretical description of the deformation of the cornea.

The task that Goldmann set (or perhaps we set it ourselves) boils down to determining how the $p_G(p)$ function depends on the two parameters, i.e. CCT and R . The applanation zone diameter D also affects p_G , but it is strictly controlled (constant) during measurement.

The effect of CCT and R on p_G measured by a tonometer has been studied for years, beginning with research [2], [3], assuming a priori the validity of law (1) and diameter $D = 3.06$ mm. Attempts to question or verify the above assumption, even if they were made, have been unnoticed. Is the assumption really so obvious that it does not need to be tested? The cornea is not an ideal membrane, and we all agree with this, but if it so, then why should it become similar to an ideal membrane at calibration parameters?

1.2. The problem of calibration values

In the literature on the subject, one can find a series of discourses why the applanation diameter of 3.06 mm was chosen by GOLDMANN [4]–[7]. In fact, the digits in the decimal places, i.e. 0.06, do not increase measurement accuracy, but merely make it easier to convert the force measured in grams into the pressure in mm Hg. He justified the choice on the basis of experimental results, but this does not explain at all why equality (1) holds. The only argument for the choice of this diameter found in the literature is the effect of surface tension in the lacrimal fluid. GOLDMANN [1] assumes that the external force flat-

tening the corneal apex over the calibration diameter overcomes intraocular pressure resistance and bending resistance. The latter, however, does not appear in the balance of forces (1) since it is counterbalanced by the force originating from the surface tension in the lacrimal fluid wetting the tip of the tonometer (figure 1b). Thus, only at calibration diameter D the two forces (the one bending the shell and the one resulting from surface tension) balance out:

$$\frac{\pi D^2}{4} p_b = \pi D S. \quad (3)$$

The variable p_b represents this part of the external pressure, which causes the flattening of only the corneal apex, and S is the surface tension in the tear film. The pressure p_b is assumed to be constant and independent of p , it can be most easily determined if there is no internal pressure

$$p_b = p_G|_{p=0}. \quad (4)$$

After Goldmann this line of reasoning was followed by ADLER [6] and many others. Unfortunately, the argument that equalities (3) and (4) hold true has not been supported by any shell solutions (Young's solution [8], which ORSSENGO and PYE [5] refer to when describing their model, is a linear solution).

ELSHEIKH et al. [7] demonstrate that the surface tension force changes the measured p_G by 0.45 mm Hg (of course, at $D = 3.06$ mm), so it seems to be much lower than the actual measuring accuracy. The designers of new variants of tonometers when they compare their readings with those of the Goldmann applanation tonometer (called “true, gold-standard IOP”) are quite satisfied with a convergence below 3 mm Hg. This accuracy of IOP measurement fully satisfies ophthalmologists. Hence one may conclude that 0.45 mm Hg represents a rather insignificant measurement disturbance. Then why correct it at all? If the assumption about the effect of surface tension in the tear film is valid, the readings differing by no more than 1 mm Hg should be obtained for any, not too large (let us say up to 6 mm), diameter D and at any IOP.

The fact that the successive generations of researchers have stuck to the diameter of 3–4 mm for over a century means that the reason must be more serious than the surface tension force in the lacrimal fluid. This is not the only doubt about the calibration dimensions. In the author's opinion, the problem is more general: why would the cornea be subject to law (1) exclusively at the calibration dimensions? If the radius R has an effect on p_G and at the same time the surface tension of the tear film does not actually determine the

diameter $D = 3.06$ mm, then there is only one conclusion: the mechanics of this phenomenon is unknown and there is no theoretical justification for the calibration values (2). That is why the attempts to correct p_G with regard to R and CCT repeatedly lead to ambiguous results.

However, today the numerical solution of this kind of elastic shell is possible and the knowledge of the mechanical parameters of the eye turns out to be sufficient to build a realistic eyeball model for GAT simulations. The results of investigating the p_G function dependent on the p variable and parameter D , obtained by means of such a model, are presented below.

2. Numerical model

2.1. Geometry

The geometric parameters of the model were adopted in accordance with the Gullstrand–Le Grand standard [9] for the human eyeball. The cornea dimensions and other eyeball parameters are given in table 1.

Table 1. Eyeball parameter values

Parameter	Value
Axial radius of anterior corneal curvature	$R=7.8$ mm
Axial radius of interior corneal curvature	$R_{in}=6.49$ mm
Central corneal thickness	CCT=0.52 mm
Peripheral corneal thickness adjacent to limbus	PCT=0.75 mm
Diameter of cornea	11.5 mm
Diameter of outer corneal surface	12.5 mm
Scleral thickness in polar region	1.0 mm
Scleral thickness on equator	0.6 mm
Scleral thickness near limbus	0.8 mm
Nominal intraocular pressure	2135 Pa (16 mm Hg)

Both anterior and posterior corneal profiles are modelled with an ellipse whose eccentricity e equals 0.5 [10]:

$$z(x) = \frac{1}{e^2 - 1} \cdot \left[\sqrt{\rho^2 + x^2 \cdot (e^2 - 1)} - \rho \right],$$

where ρ denotes the axial radius of curvature and is given as 7.8 and 6.49 mm for the anterior and posterior corneal surfaces, respectively (see table 1).

2.2. Material

Let us describe the material characteristic by the function proposed by WOO et al. [11]:

$$\sigma = A(e^{\alpha\varepsilon} - 1), \quad \varepsilon \geq 0. \quad (5)$$

A comprehensive review and evaluation of the results obtained using this function were carried out by FUNG [12]. The model parameter values are given in table 2. In order to make the cornea model behaviour similar to that ascribed to the membrane, let us also assume that $\sigma = E_0 \varepsilon$, $\varepsilon < 0$, where $E_0 = A\alpha$.

Consequently, as classic Goldmann's measurements indicate, in the absence of p the cornea only slightly resists the force flattening the apex. (The material for which $E_{\text{compress}} = 0$ is called a *cable type material*. A shell made of such a material shows longitudinal rigidity which follows from $E_{\text{tension}} > 0$, but it is devoid of flexural rigidity.)

For stress other than zero the secant longitudinal modulus of elasticity is the ratio of stress to strain in simple tension

$$E_{\text{secant}} = \frac{\sigma}{\varepsilon}. \quad (6)$$

In order to make the model as realistic as possible, the cornea was embedded in a deformable sclera whose elasticity modulus is a multiple of that of the cornea. Let us then assume that the sclera material curve is given by the equation:

$$\sigma = A(e^{5\alpha\varepsilon} - 1), \quad (7)$$

which means a fivefold increase in the secant longitudinal modulus of elasticity of this material relative to that of the cornea, for the same stress. This value of the ratio of the sclera modulus to the cornea modulus follows from the research done by WOO et al. [11], and also from our investigations [10].

Such parameters A and α of the cornea material (table 2) were adopted that the resultant secant elasticity modulus E_{secant} was close to its values recognized by most of the researchers. The values of the exponent α , determined by NASH et al. [13], are in a range from 34 to 82. WOO et al. [11] initially obtained a slightly lower value, i.e. $\alpha = 28$ ($A = 5.4$ kPa), but their later studies, described in survey work by ETHIER et al. [14], corrected this figure to $\alpha = 48.3$ ($A = 1.75$ kPa). The present author's research [15], [16] corroborates the corrected results obtained by WOO et al. The selected values of the parameters

A and α ensure that the model satisfied law (1) precisely at the nominal IOP:

$$p_G = p = 2135 \text{ Pa (16 mm Hg)}. \quad (8)$$

The numerical model satisfying condition (8) may have its cornea made of a material whose parameters A and α are in a relatively wide range but there is a strictly defined functional dependence between them. For two of the selected values of the parameter α , staying within the limits experimentally determined by NASH et al. [13], the corresponding parameters A were selected in such a way as to satisfy condition (8). All the important specifications of the materials M_a and M_b are given in table 2.

Table 2. The choice of two optimum materials for the GAT test

Symbol of material	A (kPa)	α	E_0 (MPa)	E_{secant} (MPa)	ν
M_a	0.2	61.6	0.0123	0.267	0.49
M_b	0.8	39.0	0.0312	0.239	0.49

The parameter E_{secant} is calculated from (6) for the stress of 20 kPa which appears in the corneal apex at the nominal IOP. In the case of the complex state of stress in the eye shells, it is necessary to determine one more material constant – the Poisson ratio ν , which for calculation purposes is assumed to be close to 0.5 (characteristic of an incompressible material).

2.3. Model discretization

The model was solved using the finite element method. Because of the symmetry of the eyeball,

plane2D, quadrilateral and 8-node body of revolution elements were used. The mesh density in the particular model areas is shown in figure 1. In the solutions, the material curve asymmetry (the tension curve is different from the compression curve) was taken into account. Also the pressure direction was updated as the deformation of the structure increased. The model is fully physically and geometrically nonlinear. The finite element modelling was implemented in *Cosmos/M* system, which is standard and commercially available software.

3. Results

3.1. Dependence of p_G on r and p

Figure 2 shows five series of solutions for the model in which the cornea is made of the material M_a and also one series for the material M_b at $p = 16 \text{ mm Hg}$. Each series illustrates the increase of external p_G with the applanation zone radius r at the fixed internal p (the model deformation is shown in figures 1b and 5c). The surface tension force in lacrimal fluid is not taken into account here.

The successive series illustrate corneal apex flattening for the fixed (ever higher) p of 0, 5, 16, 32 i 40 mm Hg. By intersecting the curves with the straight line $r = 1.53 \text{ mm}$ one can read the corresponding “measured” values of p_G . In this way, the curve for the model made of material M_a in figure 3 was obtained.

The curve for the model made of the material M_b , shown in figure 3, does not differ much from the curve for the model made of M_a , which means that the

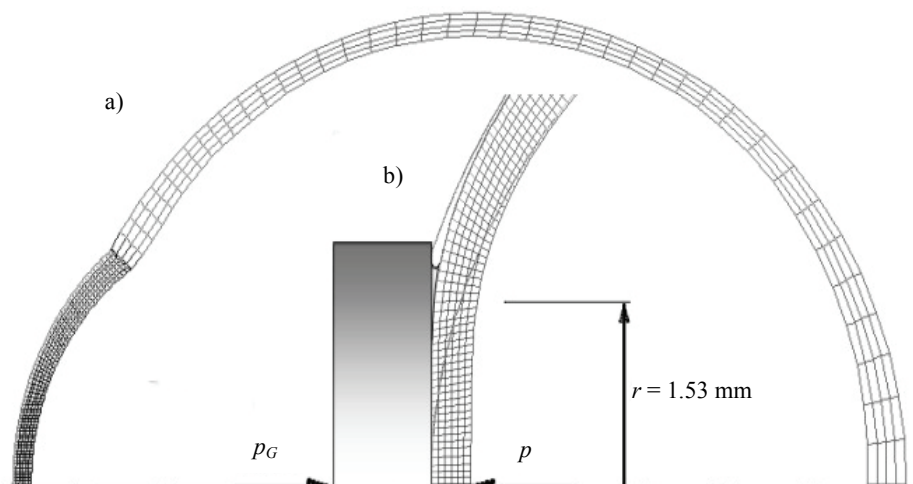


Fig. 1. Finite element grid (a), flattened corneal apex at distance $r = 1.53 \text{ mm}$ from axis (b). The surface tension force in the lacrimal film pulls the tonometer tip towards the cornea (the force is not taken into account in the calculations)

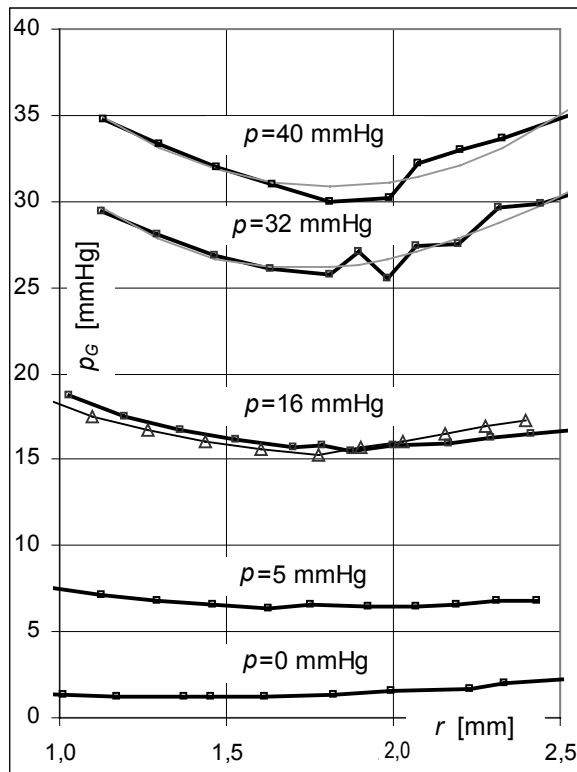


Fig. 2. The pressure p_G is calculated as a function of applanation zone radius r . Thin gray lines represent approximations to the data by a polynomial of order two. The series of solutions for material M_a are marked with squares. The curve obtained at $p = 16$ mm Hg for the model made of material M_b (marked with triangles) is included for comparison

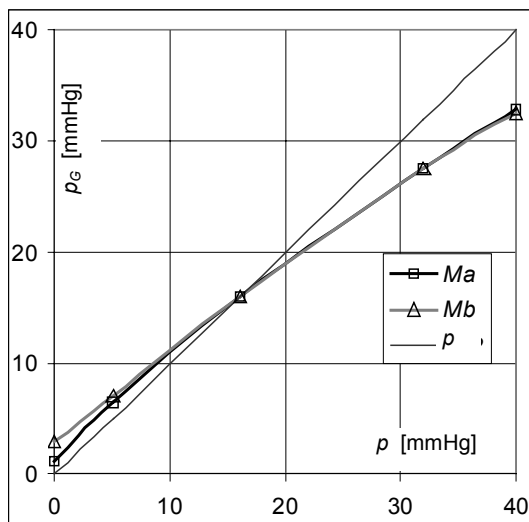


Fig. 3. Solutions for the model with the cornea made of materials M_a and M_b

other graphs are similar, as are the ones shown in figure 2 for this material. The only noticeable difference in figure 3 is around the origin, $p = 0$. When the exponent α is then increased, p_G approaches zero. But any

increase in p , even by 5 mm Hg, makes the p_G measurement results indistinguishable from each other, despite the fact that the exponents α of the two materials assume nearly the largest and the smallest values of the range determined by Nash et al. The effect of the constant A is limited here to the fulfillment of condition (8).

3.2. Example solution

Let us examine the last point on the curve in figure 3 for such a structure made of the material M_a and loaded with the highest considered pressure. When the corneal apex is flattened over the calibration diameter at $p = 40$ mm Hg, $p_G = 32.8$ mm Hg. This means that if one excises the flattened disc being affected by the two pressures, then a shearing force acting on the circumference towards the inside of the eye would be needed to make the disc stable. Such a force is actually exerted by the part of the cornea, which surrounds the disc. This effect is at variance with Goldmann’s assumption and our intuition. Let us “freeze” this configuration by immobilizing the model nodes on the disc’s edge. After the flattened fragment of the model is removed, the nodes are immobilized (in the positions occupied under the load p) on the cross section thus created and the shell model is solved again for $p = 40$ mm Hg. The reactions in the nodes, running along the symmetry axis, turn out to be directed towards the *outside* of the model (their total value stems from the pressure difference between the two sides).

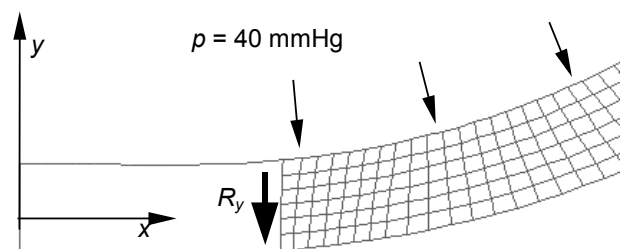


Fig. 4. When the flattening zone is removed, in order to keep the edge of the hole in unchanged position y , one must apply force R_y directed towards the *outside* of the cornea. A substantial part of p (as much as 7.2 mm Hg) must support this edge from the inside. Without this force the edge of the hole would displace *into* the cornea, as shown in fig. 5d

So how will the part of the cornea, left after the hole is cut, deform when the constraint imposed on the displacement of the nodes located on the edge is removed whereby they can freely move parallel to the axis? Note that the nodes movement perpendicular to this axis is still not allowed and reaction $R_x \neq 0$. A solution of this

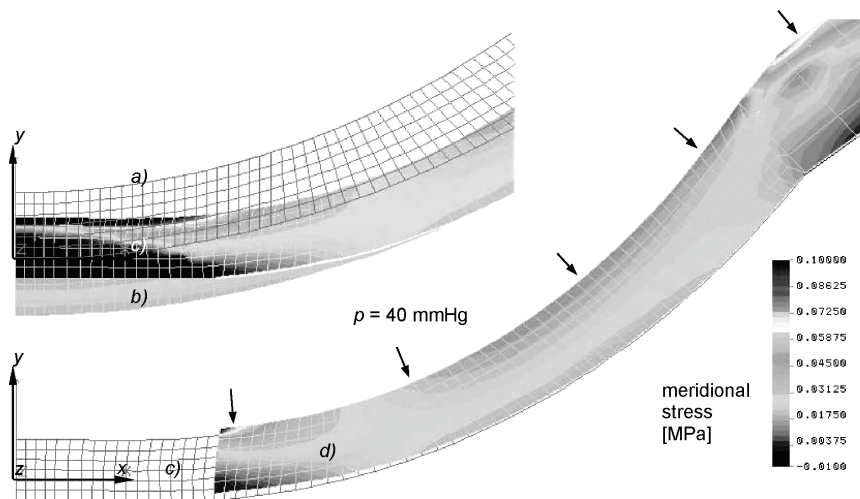


Fig. 5. Model solutions in the successive stages of loading. The upper part of the figure shows the apex configuration of the cornea not subjected to any load (a), loaded with $p = 40$ mm Hg (b) and subsequently flattened (c). The lower part shows model (c) with the appplanation zone as a finite element grid and the configuration of the cornea without the flattened disc and loaded with the same pressure. The nodes on the edge of the hole are deprived of the freedom of displacement in direction x

problem, in the form of a displacement field, is shown in figure 5d.

It is apparent that the removal of constraints along the y -axis leads to virtual movement of this edge *into* the model, despite the pressure $p = 40$ mm Hg directed towards the *outside*. In order to maintain the continuity of the shell along the appplanation zone boundary, the force R_y (figure 4) associated with the differential pressure of 7.2 mm Hg between p_G and p is needed. A faultlessly measuring appplanation tonometer indicates then 32.8 mm Hg.

4. Discussion

4.1. Why $D \approx 3$ mm

The results presented above allow one to understand why the appplanation diameter adopted by Maklakoff and later by Goldmann is close to 3 mm. The p_G curves in figure 2 have the same characteristic shape, reaching a minimum once $r = 1.5$ mm is exceeded. In the author's opinion, the existence of this extremum is the main reason for the general acceptance of the calibration diameter (2).

In the case of curve $p_G(r)$ for the model made of the material M_a , obtained while flattening the corneal apex under $p = 16$ mm Hg (the middle curve in figure 2), the deviation of p_G from p is less than 0.5 mm Hg in a very wide range of variation of r (from 1.4 to 2.4 mm). The range of r for the material

M_b is 1.3–2.2 mm. One can acknowledge that the appplanation zone calibration diameter $D = 3.06$ mm not only ensures that the calibration eye satisfies equality (8), but also that p_G is measured close to its minimum, even at p different from the nominal one (here 0, 5, 32 and 40 mm Hg).

The $p_G(r)$ relation is nonlinear, which makes the selection of measuring point difficult. The imposition of the constant D during measurement, by Fick and later by Goldmann, eliminates this problem and the choice of the diameter value is not accidental – the function $p_G(r)$ is nearly constant in its vicinity. Consequently, in a certain range of the variation of r , the ratio of the force to the flattening area is practically constant. In clinical examinations this means high tolerance of the measured p_G to variations in flattening zone area.

Contrary to the assumption that D is constant, this tolerance is very helpful here. In reality, measurements on different eyeballs are performed at different appplanation zones precisely because D is always equal to 3.06 mm. Similarity between two different corneal shells is determined by their *relative* dimensions. Therefore if the second cornea's curvature radius R is by 10% larger, then also its appplanation zone diameter (and thickness) should be larger by as much as of order for p_G to be the same. Mathematical solutions for shells are not sensitive to scale – they are identical regardless of the magnification of the structure.

Equally important is the fact that p_G determined in the vicinity of the extremum turns out to be independent of individual differences in cornea elasticity. According to figure 2, the minimum p_G for the material

M_b appears at a somewhat smaller (by about 0.2 mm) radius r than for M_a , and the deviation of p_G from p is slight in a wide range of variation of this radius. It likewise happens at pressures different from the nominal pressure. Thus close to $r = 1.53$ mm, p_G values measured on corneas differing only in their material are nearly identical.

Goldmann was not the first who dealt with this problem. Before him, at the end of the 19th century, Maklakoff in 1885 built a tonometer and used it to measure IOP, but he would kinetically produce the load by dropping a small flat weight on the cornea of the supine patient. Thus he did not impose any applanation area diameter but a force. The value of this force was such that the applanation area diameter – different for different patients – was in a range of 3–4 mm [6]. There are no doubts, however, that half a century later Goldmann chose diameter (2) for his much more precise instrument based on the high tolerance of the p_G result to applanation area variation close to a diameter of 3 mm, observed by Maklakoff (mainly on the basis of the fact that p_G and p balance out at this diameter when the measurement is performed at 16 mm Hg, ± 4 mm Hg). Maklakoff experimentally adjusted his IOP measuring technique to the phenomenon whose mechanical expression are the shell solutions shown in figure 2. Thus the choice of the diameter (similar to that of the other calibration dimensions) has nothing to do with surface tension.

4.2. Corneal apex flattening resistance is negative

According to the solutions shown in figure 3, obtained for the materials M_a and M_b which are distant from each other (and also for the other materials tested by the author, but not included in this comparison), the $p_G(p)$ function is always convex upwards. This means that for a model satisfying condition (8), always $p_G > p < 16$ mm Hg and conversely: $p_G < p > 16$ mm Hg (at an intraocular pressure lower than the nominal one, the measured pressure is higher than the real one, and above the nominal pressure the opposite is true: the measured pressure is lower than the real one).

It is hard to intuitively accept the fact that $p_G < p$ since this means that the flattened area near the axis is pulled by the remaining bent part of the cornea instead of (as it appears) being pushed by it. This upsets the view of the deformation mechanics associated with GAT, built up since the times of Goldmann.

The belief that in order to maintain cornea flattening over the diameter of 3.06 mm an additional pressure p_b , always directed to the inside of this shell, is needed derives from the intuition acquired when solving geometrically linear structures. A footbridge loaded with pedestrians is such a structure. When another pedestrian is added, this produces additional displacement having the same value as that when the footbridge is loaded with only this single pedestrian – the displacements produced by the successive loads acting on this structure are superposed. The situation is completely different in the case of a geometrically nonlinear structure. A column supporting a heavy load (close to the critical load at which it will undergo buckling) can easily be deformed even by a small shearing force. The additional displacement caused by this force is much larger than that when no load acts on the column from above. The displacements produced by the forces successively applied to a structure loaded in this way cannot be superpositioned since the structure is not geometrically linear.

For the same reason it is erroneous to try to superpose displacements (and internal forces – stresses or moments) originating from p_G when the internal pressure p is equal to or higher than the nominal one since under this load the cornea's shell loses its stability. Starting from the equilibrium point (8) a corneal shell flattening is spontaneous since the bending force needed for this equals zero. Above this point the resistance of the shell during flattening, overcome by the pressure difference between p_G and p , decreases so radically that it becomes negative. The pressure acting from within on the peripheral zone of the cornea facilitates the flattening of the corneal apex since the latter undergoes tension, and the displacements connected with this flattening arise much more readily (as in the case of the column under the critical load, which then easily deforms).

The problem of calibration values turns out to be actually the problem of shell stability. The consequences are far-reaching. The Goldmann tonometer performs well at low IOP, but above the equilibrium pressure (8) its readings are always *understated*. The cause of the error is not the tonometer itself (p_G can be measured even very accurately by this instrument). It is shell “solution” (1) which is faulty. Even though the Imbert–Fick law is a correct condition of equilibrium in the applanation zone, the latter has no decisive effect on p_G . The remaining part of the cornea, being under pressure, plays a more active role during flattening than is commonly believed and the consequences in GAT are, it seems, incorrectly interpreted. The bending rigidity of the corneal shell (and the re-

lated p_b) is noticeable only at p close to zero. Then, as figure 3 shows, the material parameters have the strongest effect on this rigidity. But when p exceeds half of the nominal value, the measured p_G mainly depends on the internal pressure – bending resistance no longer counts. This radically alters the functional dependence of the external pressure upon the internal pressure and as it follows from the above considerations, the function becomes unpredictable, which is illustrated in the *Example solution*.

This novel deplanation mechanism reveals an adverse phenomenon, so far unnoticed in GAT, i.e. the deviation of p_G from p in the calibration eye becomes negative above the equilibrium point (8), and at p twice as high as the nominal one it is already as high as the dimensional deviations caused by the largest departures of CCT and R from the calibration values. Staying with the idea of calibration values, one can say that the deviation of p_G from p depends on p to the same extent as on CCT and r . This means that condition (8) defines still another calibration parameter.

5. Conclusions and final remarks

Researchers who investigate the cornea material parameters differ in their opinions about the value of the (secant) modulus of elasticity, which, of course, is reflected in the values of A and α in equation (5). Most researchers [5], [11], [17], [18], including the present author, favour the “low-modulus” cornea ($E_{\text{secant}} \approx 0.3$ MPa). This result is usually obtained from numerical simulations of experiments on the intact cornea or the eyeball. A representative of experimenters (whose results mostly come from measurements) favouring the “high-modulus” cornea ($E_{\text{secant}} \approx 8$ MPa) is HJORTDAL [19], [20], but there are also reports according to which the modulus reaches tens of MPa [21], [22]. Besides the cornea material parameters adopted here, also the geometry of the cornea and the model fixing conditions may arouse controversy. In each of the cases, one can find models in the literature which do not satisfy the assumptions made here. But all the different approaches to the eyeball model structure have no significant effect on the conclusions emerging from the presented solutions, particularly on the shape of the $p_G(p)$ curve shown in figure 3. The investigations carried out by the author have shown that:

1. There is no such a biologically acceptable material which can ensure that the eyeball model will satisfy law (1) in the whole physiological range of intraocular pressure (the straight line in figure 3).

2. In the GAT numerical simulation, the difference between the “measured” p_G and the internal p clearly depends on the level of the latter.

3. If assumption (8) is fulfilled, the measured p_G above this equilibrium pressure is understated.

4. The calculated correction of the tonometer reading p_G taken in non-calibration conditions should be based on the solutions which take into account the critical state of the shell subjected to pressure, since the structure under such a load is geometrically non-linear and the Goldmann assumptions derived from linear mechanics do not apply to it.

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