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The description of the human knee as four-bar linkage

HENNING DATHE[1, 2]*, RICCARDO GEZZI[2], CHRISTOPH FIEDLER[3], DIETMAR KUBEIN-MEESENBURG[2], HANS NÄGERL[2]

[1] University Medical Center, Department of Medical Informatics, Georg-August-University, Göttingen, Germany.
[2] Joint Biomechanical Research Group, Department of Orthodontics, Georg-August-University, Göttingen, Germany.
[3] Lima Corporate, Udine Italy.

Purpose: We investigate the dependence of the kinematics of the human knee on its anatomy. The idea of describing the kinematics of the knee in the sagittal plane using four-bar linkage is almost as old as kinematics as an independent discipline. We start with a comparison of known four-bar linkage constructions. We then focus on the model by H. Nägerl which is applicable under form closure. *Methods:* We use geometry and analysis as the mathematical methods. The relevant geometrical parameters of the knee will be determined on the basis of the dimensions of the four-bar linkage. This leads to a system of nonlinear equations. *Results:* The four-bar linkage will be calculated from the limits of the constructively accessible parameters by means of a quadratic approximation. *Conclusions:* By adapting these requirements to the dimensions of the human knee, it will be possible to obtain valuable indications for the design of an endoprosthesis which imitates the kinematics of the natural knee.

Key words: *human knee anatomy, knee kinematics, four-bar linkage, analytical description*

1. Background

The use of mechanics to address biological issues initially occurred in parallel, starting with Aristotle (384BC–322BC) and continuing with Leonardo da Vinci (1452–1519), Giovanni Alfonso Borelli (1608–1679), Wilhelm/Eduard Weber (1795–1891) before finally separating itself and becoming an independent discipline shortly before the turn of the last century. See, for example, [25], Chapter 1.2 for a historical survey. Franz Reuleaux (1829–1905) [26], [27] and Wilhelm Braune (1831–1892)/Otto Fischer (1861–1917) [4] were particularly influential in the German-speaking area.

An overview of the use of gears and kinematics in biological mechanical systems can be found in [27], in the third section, "Kinematics in the Animal Kingdom". There is also a good new survey of four-bar linkage models in biology and an interesting classification of gears in M. Muller in ref. [21].

A four-bar linkage model of the knee was first mentioned by Zuppinger in 1904, which can recently be found in ref. [30]. However, at no point does he talk of the continuous guiding function of the cruciate ligaments, but rather highlights that in each case two of the four cruciate and collateral ligaments take on a temporary guiding function. This results in a total of six guiding options. Nevertheless, the cruciate ligament four-bar linkage is given as an example shown in Fig. 1.

Straßer [28] shows that this four-bar linkage cannot be generally applicable. This is what Straßer writes in his textbook on muscle and joint mechanics, in the chapter on changing the position of the starting point and changing the tension of the cruciate ligaments on page 370 et seq.

* Corresponding author: Henning Dathe, University Medical Center, Georg-August-University, Department of Medical Informatics, Von-Siebold-Straße 3, 37075 Göttingen, Germany. Tel: ++49 551 39-172512, e-mail: hdathe1@gwdg.de
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"The elliptical fixed centrode (of the Zuppinger four-bar linkage, the author) and the associated tendons are moved a little forwards in the femur positioned in an upright manner and its intended rigid overall continuation.

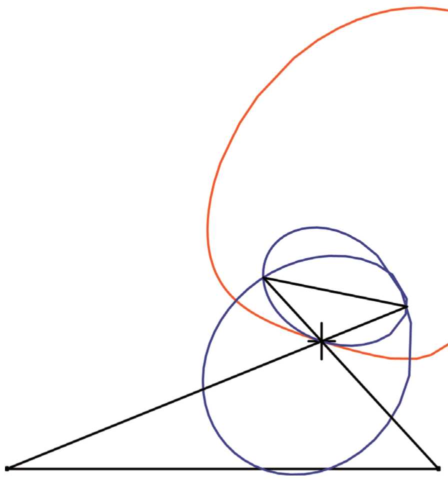

Fig. 1. Zuppinger's four-bar linkage in the dimensions according to Straßer [28].
The fixed centrode (tibia) is red, the moving centrode (femur) is blue. The instantaneous centre of rotation (black cross) is always just under one femur condyle radius away from the joint cavity

Accordingly, the fixed centrode is not significantly different to the evolutes of the rotational slip which really occurs. The diversity of the moving centrode in the tibia (probably the measurements by Fischer, the author) (in their rigidly intended continuation) from the straight pole line of the pure rotational slip which crosses it in an increasing forwards manner at an angle of 70–80° is considerably more pronounced.

In contrast to the assumption by Zuppinger, this view demonstrates that in the actual bending and stretching movement, even if we disregard the last part of flexion/extension, the two cruciate ligaments cannot remain almost equally tensioned. It should be noted here that the cruciate ligaments always remain crossed...

Different author's assumptions about the change in tension in the cruciate ligaments appear to be contradictory..."

Nevertheless, the cruciate ligament four-bar linkage model comes back several times, by Kapandji (1970) [18] (with direct reference to Straßer), Huson 1973 [15] and by Menschik [19] in collaboration with Jank [16].

If the cruciate ligaments really had a purely holding function, they would be thicker and would not have any mechanical receptors. The latter would be superfluous in rigid rods anyway. Furthermore, hardly any usable clinical output has been published about the knee which was actually developed and implanted by Menschik [29] – an indirect but important counterargument to this construction. Ultimately, Zuppinger directly refers to an *unloaded* knee joint.

H. Nägerl therefore developed an alternative four-bar linkage model on the basis of bones covered in cartilage as the most rigid structure in the knee. This describes the situation under force closure, in other words the standing phase of movement, see [24]. This is described in greater detail in the following chapter.

This model is depicted in Fig. 2.

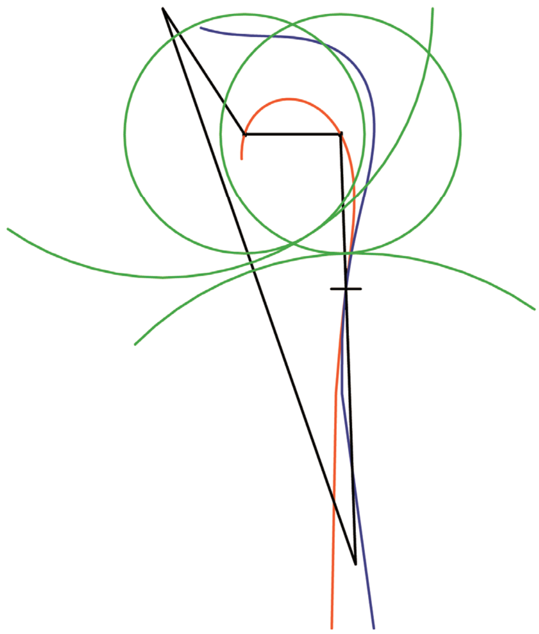

Fig. 2. The Nägerl four-bar linkage with the measurements from [24].
Fixed centrode in red (femur), moving centrode in blue (tibia), sagittal radii in green. The instantaneous centre of rotation is found at the level of the joint cavity, which means that at this stage the condyles roll on each other

It is possible to check the suitability of both models using measurements. One way of opting for or against four-bar linkage models is a measurement of the centrodes. It should be noted that for the purposes of achieving reproducible measurements of fixed centrodes in the knee, Fischer placed his test subjects face down on a table and fixed the thigh [7]. By this, he achieved a fixed centrode *distal to (below)* the femoral condyles. Later considerations by Freudenstein et al. [8] make the incorrect assumption of a lateral flat tibia and therefore achieve other centrodes. In any case, these authors distinguish *both* centrodes as belonging to either the femur or the tibia.

A further way of selecting the suitable model is the isometric point method by Benjes et al. [3]. However, their work did not lead to a clear result. In our opinion, the isometric point pairs shown in Fig. 3 are vaguely sufficient for a hybrid four-bar linkage comprising an unphysiological extended anterior cruciate ligament and a slightly proximal extended lateral chain. In particular, the load situation achieved during the measurement remains unclear in the work.

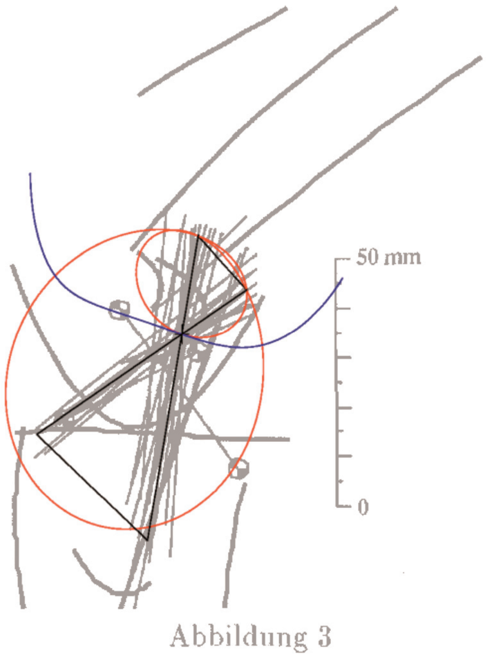

Fig. 3. The approximate four-bar linkages experimentally determined by Benjes et al. [3]

This work aims to theoretically quantify the Nägerl four-bar linkage construction. The method should be beneficial to develop further four-bar linkages in a similar form. The second section will provide a mathematical summary of the constructive specifications which are given as an equation for each parameter. Four of these equations can be used in order to calculate the four sizes of the four-bar linkage. A suitable mathematical method must be developed for this. In the third chapter we derive analytically the four-bar linkage, with simplified but elucidating assumptions. Without assumptions of this kind, we are reliant on purely numerical methods, which will be a paper on its own. On the one hand, the starting point for this study is a description of the kinematics of the knee, but on the other hand, it should serve as a model for the development of endoprostheses which imitate the kinematics of the natural knee.

2. Method: The quantitative description of the four-bar knee joint

The sagittal section through the knee is made both on the medial and on the lateral compartment. This results in approximately circle-shaped contours, whereby only the medial side of the tibia is concave, all of the others are convex [24]. Under contact, this results in limitations of movement. The distances of the central points of the contours which contact the radii remain constant during the movement. This results in a double hinge both on the medial and lateral sides, also known as dimeric link-chain. The length of this dimeric link-chain is determined by the radii. With the designations

Table 1. The first set of length parameters

RFM	Radius Femur medial
RFL	Radius Femur lateral
RTM	Radius Tibia medial
RTL	Radius Tibia lateral

we obtain the lengths of the chains

Table 2. The second set of length parameters

M = RTM – RFM	medial chain
L = RTL + RFL	lateral chain

A projection on the plane of movement, for example, the sagittal plane, results in two further chains from the central points of the radii. One belongs to the femur and one belongs to the tibia. Perspective effects can play a considerable role here. The medial and lateral compartments are about 50 mm apart and the intercondylar axis needs not be perpendicular to the plane of movement. We wish to abbreviate the new chains as follows:

Table 3. The third set of length parameters

F	femoral chain
T	tibial chain

The specifications of the four lengths (*F*, *M*, *T*, *L*) can be used to determine the sagittal kinematics of the knee.

By using plaster models of the knee and sagittal x-rays, we were able to measure these lengths, because the anatomical fixation method [5] allowed us to mould hard tissues to true-scale. The following reference values (in the form of average ± standard deviation) were obtained

$$(F, M, T, L) \approx (7.9 \pm 3.8 \text{ mm}, \quad 44.9 \pm 15 \text{ mm},$$

$$125 \pm 20 \text{ mm}, \quad 77.9 \pm 16 \text{ mm}) \qquad (1)$$

These values are used as a starting point here.

In this paper, a systematic mathematical method will be introduced which can be adopted to calculate this four-bar linkage from constructive and biomechanical specifications.

The general idea of this work is to present the significant sizes concerning the knee as functions of the four gear dimensions. The resulting system of equations will then be examined in terms of its solvability.

All of the figures in this article are in millimetres.

2.1. Constructive specifications

The radii of the femoral condyles have often been measured, see, for example [12]. Since the contact tracks continue to run centrally adjacent to the inner limitation of the menisci, they have a smaller radius than the two condyle rollers. We therefore use the following values:

Table 4. The first two numerical values

RFM:= 20	Radius Femur medial
RFL:= 20	Radius Femur lateral

This results in our first two equations

$$RTL := L - 20, \qquad (2)$$

$$RTM := M + 20. \qquad (3)$$

Due to the anatomy of the knee, the highest point in the convex condyle (lateral) of the tibia should run above (proximal to) the concave condyle (medial). This protrusion can be modelled by the parameter

$$U := RTL + RTM - T, \qquad (4)$$

see Fig. 4. Using equations (2) and (3) results in

$$U = L + M - T. \qquad (5)$$

We therefore postulate $U > 0$.

Key to the kinematics of the four-bar linkage is the so-called Grashof parameter. It is essential to the complete revolution of a link against another link of the four-bar linkage, and determines the type of gear. It is calculated from the sum of the lengths of the shortest and longest gear links, minus the other two links. This is in our case

$$G := F + T - M - L, \qquad (6)$$

and we demand $G > 0$ in accordance with our first measurements. $F = G + U$ applies in particular. Based on our working group's experience with pig and cow knees, G should not be too large [11], [14].

2.2. Biomechanical specifications

In this section, requirements from biomechanics are translated into our gear model. The necessary approximations for this will only be discussed subsequently in the discussion in terms of their usability.

Rollback is frequently discussed in the literature, see, for example, [23] and the sources cited in this work. Our four-bar linkage should be able to fulfil at least the physiological requirements, otherwise the design is not appropriate for an endoprosthesis. In our model, this rollback corresponds to the continuous arc length on the tibia. These arc lengths are maximum for the position in which the lateral and femoral links are parallel (α) maximum, see Fig. 4a) or in which the medial and femoral links are parallel (β) maximum, see Fig. 4b).

These positions should always be more extreme than in the physiological deep flexion. Otherwise, the four-bar linkage will move backwards when these limits are exceeded and there will be no further rolling. The four-bar linkage is therefore the upper limit of the physiological knee.

The law of cosines, see Fig. 4a, gives

$$\cos(\alpha) := \cos(\angle(T, M)) = \frac{M^2 + T^2 - (F + L)^2}{2MT}, \qquad (7)$$

and

$$\cos(\beta) := \cos(\angle(T, L)) = \frac{L^2 + T^2 - (F + M)^2}{2LT} \qquad (8)$$

The arc lengths on the tibia therefore result in

$$BTM = \alpha \cdot RTM, \qquad (9)$$

$$BTL = \beta \cdot RTL. \qquad (10)$$

Reference values from the literature are used for the two rollbacks [23].

It is now possible to calculate the posterior camber of the concave medial tibial condyles compared to their lowest point. The following applies in deep flexion

$$H := RTM(1 - \cos(\alpha)) \qquad (11)$$

and $H > 0$ is necessary for small α.

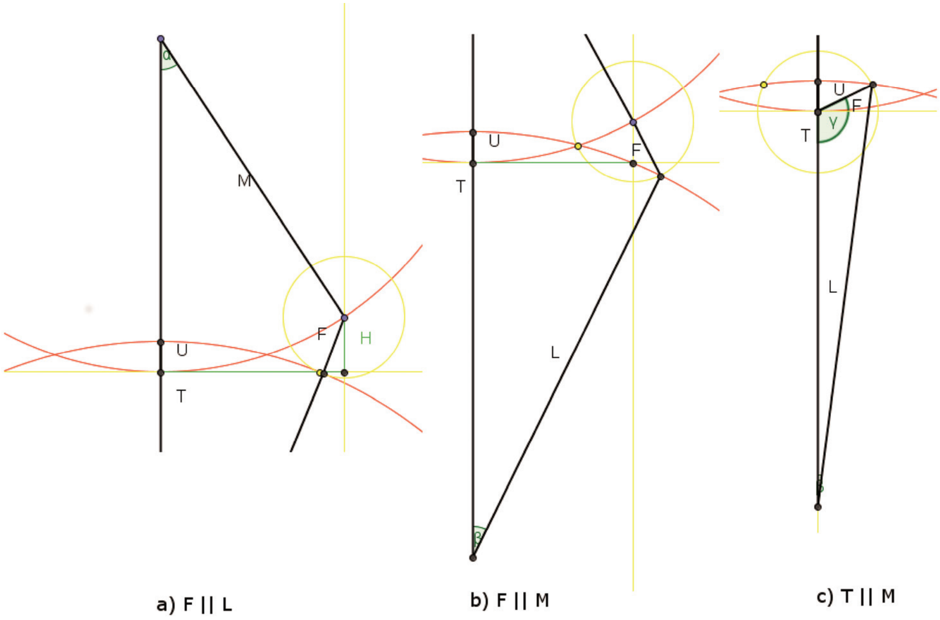

Fig. 4. The constructive parameters of the four-bar linkage. (a) and (b) correspond to the maximum arc lengths in deep flexion, (c) corresponds approximately to the situation when standing. Further details in the text

Furthermore, it is also possible to show the conditions for the position when standing. While standing, the line of action of the force goes through the medial compartment. When standing, therefore, both the medial link and the longitudinal axis of the tibia should be approximately vertical, see Fig. 5. Since the tibial axis is almost parallel to the tibial link in the chain, this results in a further limit position when standing. In this situation, the instantaneous centre of rotation is exactly $T - M$ below the medial centre of the femur. More specifically, $T - M > RFM$ should apply. In this position, the initial locations of the central points of the femur can also be determined relative to one another. This occurs again as a result of the cosine law, see Fig. 4c,

$$\cos(\gamma) := \cos(\angle(T,F)) = \frac{F^2 + (T-M)^2 - L^2}{2F(T-M)}. \quad (12)$$

This angle is highly dependent upon the perspective of the projection on the plane of movement, as the central points of the two condyles belong to different compartments, medial and lateral. The entire flexion angle ϕ between the femur and the tibia is now simply given as

$$\phi = \gamma - \alpha$$

and can be used as a further target parameter.

2.3. Typical values

The following dimensions arise for the constructive parameters using the values from x-rays and models in formula (1):

Table 5. The constructive parameters for the average values

Parameter	Value [mm]	Note
U	–2	Gap is not worthy of discussion
H	4.42	A little too high
BTM	24.10	Very large
G	10	May be, surprisingly large
BTL	24.59	Also very large

These values do not result in a knee which would be able to be constructed sensibly. This particularly unsatisfactory situation was the starting point for the further discussions.

An attempt was initially made to overcome these problems using skill, but this was unsuccessful. This meant that the distribution of the measured values also had to be taken into account.

It is known that 95% of all measurement values are within the range average value ±2* standard deviation of a normally distributed size. We realised this numerically using four FOR loops nested in one another for F, M, T and L. We then use this to calculate the constructive parameters where possible. The data set is further filtered using the sensible requirements ($U > 0$, $H < 5$, $BTM < 25$, $G > 0$); only approximately 0.26% of all data is left over. The solutions therefore seem to be very rare. From these, averages and standard deviations are calculated. This results in the following new situation: $F = 4.23 \pm 0.29$ is decreased, $M = 50.9 \pm 6.6$ is increased, $T = 123.1 \pm 10.2$ remains approximately the same, $L = 72.6 \pm 8.9$ also remains approximately the same, $U = 0.426 \pm 0.326$, $H = 3.95 \pm 0.47$, $BTM = 23.7 \pm 0.87$, $G = 3.81 \pm 0.53$, $\alpha = 19.3 \pm 2.0°$, $\beta = 13.6 \pm 1.6°$. These values will be used in the following.

3. Results: The calculation of the parameters of the four-bar linkage

Now, the description of the four-bar linkage knee given in the previous section will be used in order to obtain the parameters (F, M, T, L) from other parameter sets.

3.1. The approximation

Here, the four-bar linkage will be calculated analytically from the parameters U, H, BTM and G by means of equations (5), (11), (9) and (6). The approximation which leads to the solution is instructive and should therefore be carried out before the general numerical solution.

From (6) and (5), we immediately derive

$$F = G + U, \tag{13}$$

in other words, the femoral length can be calculated directly from G and U.

The non-linearity of the system of equations is essentially due to the simultaneous occurrence of α and $\cos(\alpha)$. Since, however, the cosine occurs in the form $1 - \cos(\alpha)$, we approximate

$$V := 1 - \cos(\alpha) \approx \frac{a^2}{2}, \tag{14}$$

in other words, $H \approx RTM\alpha^2/2$. Together with equation (9), this can be solved using α and RTM to obtain

$$\alpha \approx \frac{2H}{BTM} \tag{15}$$

and

$$RTM \approx \frac{BTM^2}{2H}. \tag{16}$$

Combined with equation (3), this results in

$$M = RTM - 20 \approx \frac{BTM^2}{2H} - 20. \tag{17}$$

In this way, the sizes F and M can be calculated.

We also get

$$V \approx \frac{\alpha^2}{2} = \left(\frac{2H}{BTM}\right)^2 \frac{1}{2} = \frac{2H^2}{BTM^2}, \tag{18}$$

in other words, the left hand side of (7) is known in principle. We are therefore looking at the right hand side. The expression

$$V = 1 - \cos(\alpha) = 1 - \frac{M^2 + T^2 - (F+L)^2}{2MT}$$
$$= \frac{2MT - (M^2 + T^2 - (F+L)^2)}{2MT}$$
$$= \frac{(F+L)^2 - (T-M)^2}{2MT} \tag{19}$$

is now only dependent on the unknowns L and T. Using equation (5), T can be eliminated in favour of L. Firstly, we multiply the denominator up to get

$$2VMT = (F+L)^2 - (T-M)^2$$

and after using

$$T = M + L - U \tag{20}$$

this results in

$$2VM(M + L - U) = (F+L)^2 - ((M + L - U) - M^2)$$
$$= F^2 + L^2 + 2FL - (L^2 + U^2 - 2UL)$$
$$= 2(F+U)L + F^2 - U^2. \tag{21}$$

This equation can be solved for L. From

$$F^2 - U^2 - 2VM(M - U) = L(2VM - (F+U))$$

we then ultimately come to the solution

$$L = \frac{F^2 - U^2 + 2VM(U - M)}{2(VM - (F+U))}. \tag{22}$$

Since L is now known, we can finally calculate T using formula (20). This completes the linear calculation of the knee gear.

3.2. A numerical example

The following new constructive parameters arise for values coming from Section 2.3 corrected using a certain degree of experience:

Table 6. The first trial for new constructive parameters

Parameter	Value [mm]	Note
U	1	Small
H	2	Can still be well constructed
BTM	20	OK but large
G	1	Close to the double dead-centre position

These are then used to determine the gear (*F*, *M*, *T*, *L*) = (2.00 80.00 168.21 89.21) which is almost at the double dead-centre position. It is now $\gamma = 119°$ and *BTL* = 12.4. The four-bar linkage is shown in Figs. 5 and 6. Here you can see the significance of the parameters *U* and *H* clearly.

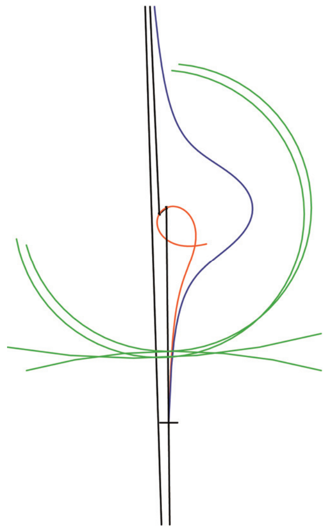

Fig. 5. The four-bar linkage achieved using linearization when standing. The instantaneous centre of rotation is at the level of the joint cavity, which means that the condyles roll on each other

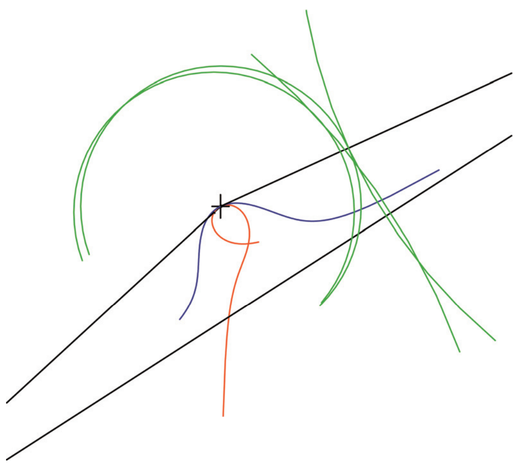

Fig. 6. The four-bar linkage achieved using linearization during deep flexion. The instantaneous centre of rotation is in the centre of the condyles, which means sliding with the approximate function of a hinge

Even slightly greater values for *G*, for example *G* = 2, lead to gears such as (*F*, *M*, *T*, *L*) = (3, 80, 130, 51) which have a lateral chain which is rather too small compared to *RFL* = 20. Decreasing the *BTM* to 18 also impacts the values (*F*, *M*, *T*, *L*) = (2.00, 61.00, 119.49, 59.49), again with a small lateral chain. This may be an indication of the lateral shape of the tibia, which has ever smaller radii in the direction of the centre of the joint.

4. Discussion

In principle, the question, which four-bar linkage describes the kinematics of the human knee optimally, could be answered by means of experimental measurements. Unfortunately, there are hardly any usable measurements available in the literature. Most measurements examine only one centrode. Because centrodes are known to roll on each other, the motion can then be reconstructed with given initial values. It follows that a measurement in the sense of kinematics has to include *both* centrodes, the femoral and the tibial one.

Furthermore, the articular surfaces must be in contact by force closure, as it is in the stance phase of gait. One advantage of the Nägerl-type four-bar linkage is that the instantaneous pole is around the stance phase *below* the center of the femoral condyles. This enables the articular surfaces to roll on each other, which minimizes friction during the stance phase. This is not possible for the other four-bar linkages.

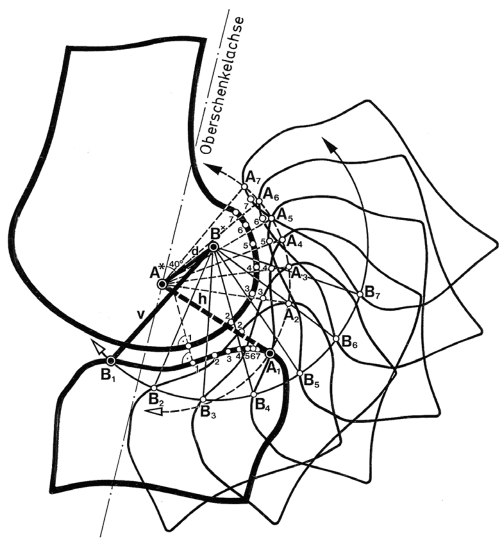

Fig. 7. The tibia constructed as envelope curve of Menschiks version [20] of Zuppingers four-bar linkage. The resulting surface geometry is concave overall

Fig. 8. A medial parasagittal section through a human knee by Fischer [7]. The convexity of the tibia is obvious and in contrast to Fig. 7. This has long time been not well understood. Since the kinematic of the knee cannot act against the bones, one is leaded to Nägerls four-bar linkage, which is the basis of this work

Although Zuppinger's original ideas were more general than stating one single four-bar linkage, the cruciate linkage construction has first been disproved by Straßer, see introduction. Nevertheless, Menschik rediscovers in his Book [20] this construction and enhances it to a construction principle of the legs of all vertebrates. He is aware of the incongruence of joint surfaces, which he correctly identifies as the envelope surfaces of the given motion [13]. He claims the cruciate ligaments as rigid and therefore as steering system of the whole motion. Using this model and a given femoral shape, the resulting tibial shape is concave overall, see Fig. 7, but the lateral side is known to be convex in reality, see Fig. 8.

The lateral convexity of the tibia is known to be important for the correct function of the knee [1]. The constructional mismatch in tibial geometry leads together with the ligament apparatus to a reduced mobility of typically 80–110° in flexion for the Menschik-prosthesis as reported in [29].

Benjes et al. [3] do not try to simplify their results to obtain a four-bar linkage. Interestingly, this model is close to the original predictions of Zuppinger using something close to the posterior cruciate ligament and a lateral ligament. The resulting centrodes are above the contact points, so there will be no rolling of the articular surfaces. Unfortunately, no load system was specified for the measurement, so it is hardly to compare it with other measurements. Benjes' real focus is on a spatial two-parametric model of the knee motion [2], and this is beyond the scope of this work.

In Nägerl's construction [24], the joint surfaces are also envelope surfaces. As the stiffest structure, they dominate the movement [22]. Since they are approximated to have a circular shape in the plane of motion, the mid-points of contacting circles have constant distances. So, the segments between these midpoints form rigid rods from which a four-bar linkage can be constructed. Now, the cruciate ligaments change their lengths during flexion, as occurs in reality. Furthermore, they contain mechanoreceptors which would be useless while having rigid ligaments [17]. So, most of the general methods in Menschik's book are correct, only the four-bar linkage is wrong. As a clear example of his errors, he tries to predict the lengths of the femur and the tibia from the ratio of lengths of the cruciate ligaments. The results are given with five digits after the decimal point in mm. But this is a severely unrealistic precision of 10Å, especially for a biological system.

Because of the known problems with the cruciate linkage construction, we focused on the Nägerl-type four-bar linkage. The results are in agreement with our own anatomical findings. Therefore, this work aimed to show the satisfiability of various different requirements on a Nägerl-type four-bar linkage. We looked for more accurate dimensions of the four-bar linkage by way of a suitable mathematical method.

Starting with the constructive specifications, we initially determined the four parameters (U, H, BTM, G) which are used to construct a four-bar linkage (F, T, M, L). We also showed that the distribution of the original initial values from equation (1) has to be taken into account in order to achieve sensible results. The calculation in Section 3 offers the possibility of improving these initial values by using a linearization.

The marginalities of the angles arise from the dead-point positions of the four-bar linkages and are therefore not arbitrarily determined. There is only a certain degree of freedom in terms of their dimensioning on the measurable parameters. The final rotation is a spatial effect and exceeds the present planar model. The formulation of the standing condition also requires a corresponding assumption, and should therefore be treated with a certain degree of caution. As an extension of the approach shown, the four-bar linkage can be viewed starting from a certain flexion in order to exclude the final rotation. However, this requires the mathematical modelling of the entire movement of the four-bar linkage and exceeds the scope of the present work. Furthermore, even the knee movement in flexion is only approximately planar.

The Nägerl-type four-bar linkage has been successfully applied as basis to the realization of endoprostheses [6]. The presented calculations, which are a missing link to understand these endoprostheses, are an indirect proof of the reliability of this type of construction. We therefore quantified the originally qualitative construction principle.

5. Conclusion

There are sensible solutions to construct an endoprosthesis. The practical implementation to develop an endoprosthesis requires further considerations, which are shown in ref. [6]. In particular, it includes an analytical description of the movement of the knee. The first successful clinical results of a multi-centric study on the knee developed by our working group are shown in [9] and [10].

An alternative way to treat the presented equations is to solve them numerically. This extends the flexibility of the method considerably and will be published in a separate work.

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