Acta of Bioengineering and Biomechanics
Vol. 1, No. 1, 1999

A mathematical model of bone remodelling under overload and its application to evaluation of bone resorption around dental implants

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This paper deals with the formulation of a mathematical model allowing us to describe mechanical bone remodelling process and rapid bone resorption under overload. For this purpose, physiological signal transmission processes of remodelling from mechanical stimuli to the change of bone density are described by $n + 1$ sequential evolution equations with $n + 1$ macroscopic internal state variables. In the normal physiological situation, the value of internal variable in the k-th step approaches the value of the variable in the $(k - 1)$-th step, but under overload conditions the target value in the k-th step reduces to a value much smaller than in the normal situation, which represents the loss of physiological balance. The value of the internal variable in the last step specifies the balance level of bone density, which is the target of the current bone density. The simulation results showed that this model could describe a time-dependent process of bone remodelling including bone resorption. Finally, the proposed model was applied to problems of bone resorption around artificial implants. The simulation results predicted the bone resorption qualitatively.

Keywords: bone remodelling, mathematical modelling, bone resorption

1. Introduction

Several mathematical models of bone remodelling have been proposed so far (ex. Van Rietbergen et al., 1993). Most of them, however, aimed at describing the morphological optimum structure of bone. Thus, they are applicable to a prediction of the static morphological structure of the bone, but not necessarily adequate to describe the evolution of bone remodelling in the time process. Tanaka et al. (1998) proposed a mathematical model of macroscopic internal bone remodelling by taking account of physiological remodelling processes. They succeeded in describing the evolution of bone remodelling in the process of time.

In this study, we extended the above model so as to describe the rapid bone resorption observed in mandibulae around artificial implants (Olsson, 1995 and Jung et al., 1996). Then we performed simulations to evaluate the bone resorption around implants.

2. Formulation of the model

The purpose of our model is to represent a phenomenological remodelling process from mechanical stimuli to the change of bone density based on the concept of a macroscopic internal state variable theory.

To propose the model, we first accept the following assumptions:

- Only macroscopic internal bone remodelling is considered; cancellous bone is approximated as a hypothetical continuum. The change of trabecular structures is taken into account as the density change of the continuum. Furthermore, the macroscopic change in bone shape is not considered.
- The model is formulated for the cases of uniaxial loading conditions.
- A linear elastic material represents the mechanical properties of the continuum.
- Bone is not formed under static loading conditions.
- Rapid bone resorption occurs under overload conditions.

Firstly, based on the models reported in the literature (e.g. Van Rietbergen et al., 1993) mechanical stimuli S are defined by

$$S = \left| \frac{\sigma \dot{\varepsilon}}{\rho} \right|, \tag{1}$$

where σ and $\dot{\varepsilon}$ are the uniaxial stress and strain rate, and ρ is the mass density.

Next, we assume that the transmission process from the mechanical stimuli to the change of bone density is approximated by $n + 1$ sequential internal processes, and that these processes are represented by a set of macroscopic internal state variables R_k $(k = 0, 1, ..., n)$. The variable R_0 expresses the accumulation of the stimuli, while R_k $(k = 1, ..., n)$ is a variable that represents the $(k + 1)$-th internal process. The evolution equations of R_k are formulated in the form

$$\dot{R}_0 = S - r_0 R_0^l, \tag{2}$$

$$\dot{R}_k = r_k \left[R_{k-1} - \left\{ 1 + \beta_{tk} \langle \varepsilon - \varepsilon_{t0} \rangle + \beta_{ck} \langle \varepsilon_{c0-\varepsilon} \rangle \right\} R_k \right], \tag{3}$$

where r_0, l, r_k, β_{tk} and β_{ck} are material constants, and ε_{t0} and ε_{c0} are thresholds for tensile and compressive strains beyond which the abnormal bone resorption occurs. The symbol $\langle \rangle$ denotes the Macauley bracket defined by $\langle x \rangle = xU[x]$, where $U[x]$ is the unit step function with $U[0] = 0$.

In normal remodelling processes, the strain value stays within the threshold ε_{t0} or ε_{c0}. In this case, the variable R_k approaches R_{k-1} as found in Eq. (3). In the abnormal situations caused by overloads, the variable R_k approaches the value $R_k/(1 + \beta_{tk}\langle \varepsilon - \varepsilon_{t0} \rangle + \beta_{ck}\langle \varepsilon_{c0} - \varepsilon \rangle)$

much smaller than R_{k-1}. This abnormal process in each step leads to the decrease of the value of R_n and induces the reduction of bone density as shown in the following.

The evolution equation of the bone density ρ is established so that the ρ approaches the target density ρ_t determined by the current value of R_n:

$$\dot{\rho} = \alpha(\rho)\{\alpha_f\langle\rho_t - \rho\rangle - \alpha_r\langle\rho - \rho_t\rangle\}, \tag{4}$$

where α_f and α_r are the constants that specify the rates of bone formation and bone resorption. The normalized function $\alpha(\rho)$ ($0 \le \alpha(\rho) \le 1$) represents the effects of the surface area of microscopic trabecular bone on the bone remodelling (Van Rietbergen et al., 1993). By use of the value of R_n, the parameter ρ_t is formulated as follows:

$$\rho_t = \rho_{\min} + (\rho_{\max} - \rho_{\min})\left[1 - \exp\left\{-(hR_n)^2\right\}\right], \tag{5}$$

where $\rho_{\max}$ and $\rho_{\min}$ are the maximum and minimum bone densities, and h is a constant.

Finally, based on the literature (Carter et al., 1997) we represented the elastic modulus E of bone by a cubic expression of ρ in the following

$$E = g\rho^3, \tag{6}$$

where g is a constant.

3. Simulations and discussion

To examine the adequacy of the proposed model, we simulated the experimental results according to Rubin et al. (1984). The applied to roosters' ulna the repeated loads at 0.5 Hz for 6 weeks. The numbers of cycles in a day were 0, 4, 36, 360 or 1800. In this experiment, they examined the relation between the number of cycles and the bone density.

The material constants of the proposed model, except the thresholds ε_{t0}, ε_{c0} and the values β_{tk}, β_{ck}, were identified by referring to these results. Furthermore, according to Frost (1992), woven bones are formed when over 0.3% strain is applied to bones. Based on this information, we assumed that the rapid bone resorption begins with the woven bone formation. The thresholds ε_{t0}, ε_{c0} and the values β_{tk}, β_{ck} were identified so as to satisfy this assumption.

Figure 1 shows the comparison of the predictions with the experiments by Rubin et al. (1984). The proposed model reproduced the normal remodelling responses quite well.

Next we performed finite element analyses of a mandible with implants to evaluate the strain around the implants. The effects of the distance between implants and the direction of loads were examined. From the results, the largest compressive strains around implants were in the range between –0.2% and –0.35%. Based on these, we simulated the bone resorption around the implants. The repeated loads with triangular

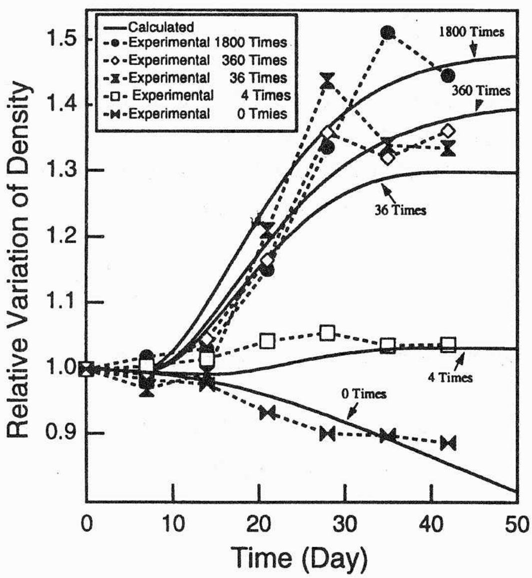

Fig.1. Comparison of model predictions with experiments by Rubin et al. (1984)

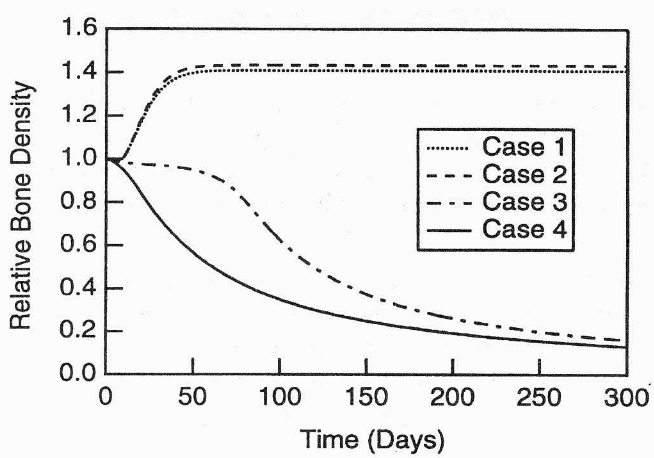

Fig. 2. Time variation of mandibular bone density around implants

waveform are applied based on the information by Mioche et al. (1995). The simulation conditions are shown in the Table. Figure 2 shows the simulation results of bone

Table. Loading conditions

	Case 1	Case 2	Case 3	Case 4
Initial strain amplitude (%)	0.25	0.25	0.3	0.3
Number of cycles/day	600	1200	600	1200

resorption. The results show that the model can represent qualitatively the rapid bone resorption around an implant.

References

[1] Carter D.R., Hayes W.C., *The Compressive Behaviour of Bone as a Two-Phase Porous Structure*, J. Bone and Joint Surgery, 1977, Vol. 59-A, pp. 954–962.

[2] Frost H.M., *The Role of Changes in Mechanical Usage Set Points in the Pathogenesis of Osteoporosis*, J. Bone and Mineral Research, 1992, Vol. 7–3, pp. 253–261.

[3] Jung Y.C., Han C.H., Lee K.W., *A 1-Year Radiographic Evaluation of Marginal Bone Around Dental Implants*, International J. Oral and Maxillofacial Implants, 1996, Vol. 11, pp. 811–818.

[4] Mioche L., Peyron M.A., *Bite Force Displayed during Assessment of Hardness in Various Texture Contexts,* Archives of Oral Biology, 1995, Vol. 40, pp. 415–423.

[5] Olsson M., *MkII-A Modified Self-Tapping Branemark Implant: 3-Year Results of a Controlled Prospective Pilot Study*, International J. Oral and Maxillofacial Implants, 1995, Vol. 10, pp. 15–21.

[6] Rubin C.T., Lanyon L.E., *Regulation of Bone Formation by Applied Dynamic Loads*, J. Bone and Joint Surgery, 1984, Vol. 66-A, pp. 397–402.

[7] Tanaka E., Yamamoto S., Aoki Y., Okada T., Yamada H., *A Constitutive Model to Describe Bone Remodeling Process*, Proceedings, 5th JUSC Conference on Biomechanics, Sendai, Japan, 1998, pp. 160–161.

[8] Van Rietbergen B., Huiskes R., Weinans H., Sumner D.R., Turner T.M., Galante J.O., *The Mechanism of Bone Remodeling and Resorption Around Press-Fitted THA Stems*, J. Biomechanics, 1993, Vol. 26–4, pp. 369–382.