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Geometry and inertia of the human body – review of research

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The paper is devoted to such morphological quantities of the human body as: (1) geometric, i.e., linear, planar, and spatial; (2) inertial, especially – mass, density, radius of centre of mass, moment of inertia and its radius. Description of quantities was given, material used (live subjects, cadavers, models), and methods utilised: mechanical and electromechanical, optical, geometric (for inertia quantities), penetrating, calculation, modelling. The most important results were given, especially for inertial quantities.

Keywords: human body, geometry, inertia, research, review

1. Introduction

From biomechanics' viewpoint, human morphology can be described in terms of the following problems: biomaterials, structure, construction, geometry, and inertia (Erdmann, 1990). The purpose of this paper is to present some investigations concerning geometric and inertial quantities of the human body. They were described in the previous papers by Erdmann (1985), Jensen (1993), and the others.

2. Geometric quantities

Geometric quantities can be divided into: (a) linear quantities (straight and curvilinear) – lengths of segment and curve; (b) planar quantities – area of surface, planar angle; (c) spatial quantities – capacity of tank, volume of solid, spatial angle.

Figures 1A and 1B present different straight lines representing the lengths of body segments. The lines with curvature allow us to show, e.g., a vertebral column or circumference of a body segment. Area of surface enables us to present the surface of foot's contact with the floor, or a surface perpendicular to the direction of movement

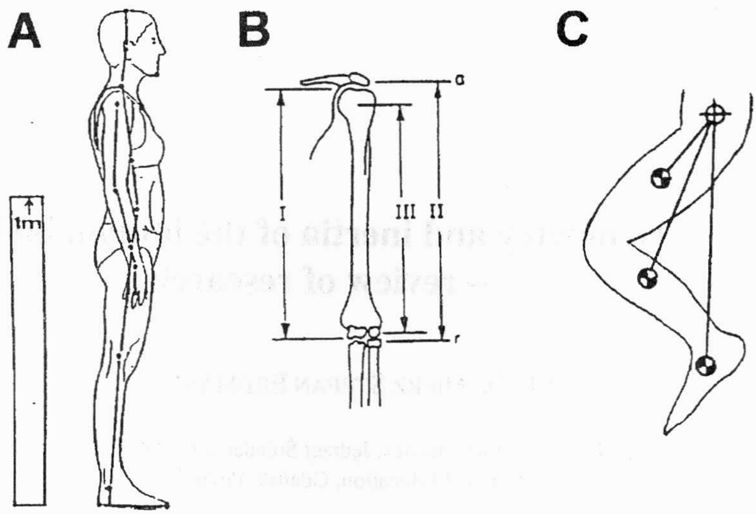


Fig. 1. Dimensions of the body: A – segments of the entire body; B – arm lengths (I – absolute, II – anthropometric, III – kinematic); C – the distances a for calculating the moment of inertia

(for air resistance calculations). Capacity of tanks can be related to heart or lungs and volume of solids, e.g., to the volume of a entire body for density calculations.

3. Inertial quantities

When one wants to change a state of movement of the body he always has to deal with a resistance of that body called "inertia". Inertial quantities are as follows: (a) mass, (b) density, (c) radius of centre of mass, (d) moment of mass (static moment), (e) moment of inertia, (f) radius of gyration, and (g) deviation moment.

Mass is a measure of inertia of the object in translation movement. It depends on the density and volume of the object:

$$m = \rho \times V, \tag{1a}$$

$$m = \int_a^b \rho V dV, \tag{1b}$$

where: m – mass, ρ – density, V – volume.

Density of a body depends on the kind of material from which the body is composed. It can be calculated by conversion of equation:

$$\rho = \frac{m}{V}. \tag{1c}$$

Radius of centre of mass (position vector of the centre of mass) is a distance between centre of mass and the reference system acquired. This system can be attributed to the body or can be found outside the body.

Moment of mass (static moment) is a product of the mass m and the radius of centre of the mass d:

$$Mm = m \times d. \tag{2}$$

Moment of inertia can be described in three manners:

a) moment of inertia of a point (mathematical pendulum) I_P:

$$I_P = m \times l^2, \tag{3}$$

where l is the length of weightless thread;

b) moment of inertia, where the axis of rotation crosses the centre of mass of the body (central moment of inertia I_C):

$$I_C = \sum_{i=1}^{n} m \times d_i, \tag{4}$$

where d is the distance between the mass element m and the axis of rotation C;

c) moment of inertia, where the axis of rotation passes beyond the centre of mass of the body (non-central moment of inertia I):

$$I = I_C + (m \times a^2), \tag{5}$$

where a is the distance between the axis crossing the centre of mass and the axis of rotation (Steiner's theorem).

For example, the moment of inertia of the lower extremity in relation to the frontal hip axis A (Fig. 1C) is a sum of moments of inertia of its parts:

$$I_{\mathrm{A.LE}} = I_{\mathrm{A.T}} + I_{\mathrm{A.C}} + I_{\mathrm{A.FT}}, \tag{6}$$

where: LE – lower extremity, T – thigh, C – calf, FT – foot.

Moments of inertia of the above parts are as follows:

$$I_{\mathrm{A.T}} = I_{\mathrm{C.T}} + (m_T \times a_{\mathrm{T}}^2),$$

$$I_{\mathrm{A.C}} = I_{\mathrm{C.C}} + (m_{\mathrm{C}} \times a_{\mathrm{C}}^2),$$

$$I_{\mathrm{A.FT}} = I_{\mathrm{C.FT}} + (m_{\mathrm{FT}} \times a_{\mathrm{FT}}^2).$$

Radius of gyration is a distance between the axis of rotation and a place where the entire mass of the body could be concentrated, so the moment of inertia calculated from the mass of the body and its radius of gyration would be equal to that moment of inertia where the mass is in real place (not concentrated). One can obtain: (a) central radius of gyration k_C and (b) non-central radius of gyration k:

$$k_C = \sqrt{\frac{I_C}{m}}, \tag{7}$$

$$k = \sqrt{\frac{I}{m}}. \tag{8}$$

Moment of deviation is a product of the mass m of the body and two radii of the centre of mass x and y in the orthogonal system of two axes X and Y:

$$Md = m \times x \times y. \tag{9}$$

4. Material

Usually in geometric and inertial investigations, three approaches are adopted, since living persons, cadavers, and models have been tested.

First investigations probably dealt with living subjects, but no written evidence can be found. Those investigations consist of the measurements of a body height and a body mass. Measurements of the other geometric and inertial quantities developed in the last 150 years are well documented. For cadaver studies both fresh and embalmed samples of the entire body and its parts were carried out. The first dissection of a cadaver for obtaining inertial data was performed by Harless (1860). Current mechanical approach to the modelling is adopted during research on accident prevention. The first mathematical models of the human body based on geometric and inertial quantities were developed in the XIX century by Harless (1860), who presented the body segments as a set of geometric shapes, and in the XX century by Matsui (1958), Hanavan (1964), Hatze (1980).

5. Methods

In order to obtain geometric and inertial quantities the following methods are used: (1) mechanical and electromechanical, (2) optical (for geometry), (3) geometric (for inertia), (4) penetrating, (5) calculation, (6) modelling. We can also find some advice on how to prepare the subject.

5.1. Preparation of the subject

Before measurement procedure the subject should be prepared properly. Living subject should be in undergarments only, without shoes, etc., except of special measurements. During geometric measurements the subject should stay motionless which is not easy for a longer period of time, thus it is advisable to conduct the measurements relatively quickly.

For comparison purpose a mass of the entire body should be measured in the morning when the subject relieved himself and did not eat anything.

When investigations with cadavers are carried out in order to measure inertial values the cadavers are often divided into segments. The line of division crosses the joints. In order to prevent the liquid tissues from loosing the cadavers are frozen before segmentation.

5.2. Mechanical and electromechanical methods

For straight dimensions mechanical or electromechanical anthropometer is used plus rulers, bow-compasses, slide caliper and caliper for measuring skin-fat folds. For curvilinear dimensions a measuring tape is usually used.

Surface quantities, e.g., foot-prints, are measured by means of an ink print apparatus. For measurements of volume an immersion method is used.

In order to estimate a mass/weight of the body, many scales of different types are applied. If the weight is measured using spring scale, where earth's gravity deformed springs, a mass can be calculated according to the following equation:

$$m_x = m_0 \frac{g_n}{g_x}, \tag{10}$$

where: m_x – the mass searched, m_0 – weight reading from the scale, g_n – normal Earth's acceleration (9.80665 m/s^2), g_x – Earth's acceleration at a measurement location.

A scale can be used also for the measurement of density of solids. Using the Archimedes law one can measure weight of an object in an air, and then in a water. Then, the following equation is utilized:

$$V = \frac{Q_a - Q_w}{\gamma_w}, \tag{11}$$

where: V – volume, Q_a – weight in an air, Q_w – weight in a water, γ_w – specific weight of water. Next, Eqs. (1c) and (9) should be used.

Levers, except of using them in scales, are also suitable for searching for location of centre of mass/gravity. Here two-sided (Borelli, 1680) and one-sided (Du Bois Reymond, 1900) levers are used. Basler (1931) introduced triangle board for two-dimensional location of centre of gravity. In order to locate faster the centre of mass, pantographs (Fischer, 1906; Erdmann, 1988), templates (Walton, 1970; Erdmann, 1979), and special division device (Erdmann, 1987) were introduced.

For investigating the moment of inertia a pendulum or accelerometer can be used:

$$I = \frac{D \times T^2}{4\pi^2}, \tag{12}$$

where: I – moment of inertia, D – moment of force acting on a pendulum, T – period;

$$I = \frac{d \times M}{d\alpha}, \tag{13}$$

where: I – moment of inertia, dM – instant moment of force, $d\alpha$ – instant angular acceleration.

5.3. Photo-methods

Since the XIX century photo-methods have been used for measurement purposes. At first photography has been developed and then film and television. In new instruments structured lighting (Yokoi et al., 1984; Lewis and Sopwith, 1986), moire (Turney et al., 1991), and laser (Koch and Koch, 1991) techniques are applied. The photo-method is utilized especially for obtaining geometric data. In the picture one can point out specific anthropometric landmarks as well as the length of particular segments. The picture allows us also to estimate an area of surface of the body necessary for calculation of air resistance, and with special images (e.g. stereophotogrammetry) for obtaining volumetric data.

5.4. Geometric methods

Using some of geometric methods for obtaining inertial data we accept the following assumptions: (1) density of the bodies investigated is known, (2) a body is of the same density within the entire volume, (3) the centre of volume has the same location as the centre of mass.

Geometric methods are based on volume measurements (voluminometry) of the entire body or its parts. When immersing method is used for the measurement of body parts these parts can be immersed every 1 cm at a time and then mass, location of centre of mass and moment of inertia can be calculated.

The geometry of the body parts may also be established using a measuring tape. It is possible to measure the circumferences every 1 or 2 cm along longitudinal axis and then to calculate the area of surface. Multiplying that surface by the height of the body part one can obtain volume of that body part.

Clauser et al. (1969), Zatsiorsky and Seluyanov (1979), Erdmann (1995) used geometric measurements. The data obtained were applied in regression equations.

5.5. Penetrating methods

Penetrating methods are based on the following physical phenomena: 1. Penetrating radiation, i.e. gamma rays and X-rays, directed toward an object being scanned.

The difference in the intensity of rays before and after penetration of an object gives an information on its density. Gamma rays were used by Zatsiorsky and Seluyanov (1979) and X-rays were used by, e.g., Huang and Wu (1976) and Erdmann (1995, 1997). 2. Ultrasounds used mostly for medicine purposes and for biomechanics purposes (Dąbrowska et al., 1985). 3. Magnetic resonance (especially nuclear, NMR).

5.6. Calculation methods

Such calculation methods as indirect methods are based on the data obtained due to direct examination of both cadavers and living bodies. The coefficients used here are relative data with reference to measured values.

The first group of coefficients are those obtained from geometric measurements, for example:

$$\lambda.\mathrm{le} = \frac{l.\mathrm{le}}{L}, \tag{14}$$

where: λ.le – coefficient of the length of lower extremity (relative length), l.le – absolute value of the length of lower extremity, L – entire body length.

The second group of coefficients are those calculated based on inertial values. The most important are:

- the coefficient of mass of body segment (relative mass) μ:

$$\mu = \frac{m}{M}, \tag{15}$$

where: m – absolute value of the mass of body segment, M – mass of the entire body;

- the coefficient of the radius of centre of mass of the body segment (relative radius) δ:

$$\delta = \frac{d}{l}, \tag{16}$$

where: d – absolute value of radius of centre of mass of the body segment, l – length of the body segment;

- the coefficient of the radius of gyration of the body segment (relative radius) κ:

$$\kappa = \frac{k}{l}, \tag{17}$$

where: k – absolute value of the radius of gyration of the body segment, l – length of the body segment.

Transformation of Eqs. (15)–(17) yields the following absolute values:

$$m = \mu \times M, \tag{15a}$$

$$d = \delta \times l, \tag{16a}$$

$$k = \kappa \times l. \tag{17a}$$

Another form of calculations are regression equations with dependent variables calculated from independent variables and from some parameters. The regression equations, which allow calculation of mass of the body segment based on the entire body mass (Basler 1957) and, in addition, on geometric quantities (Clauser et al. 1969, Zatsiorsky and Seluyanov 1979), are presented:

$$\text{mass of calf} = (0.11 \times W) - 1.90, \tag{18}$$

where W is the weight of the entire body;

$$\text{mass of calf} = (\text{calf circumference} \times 0.111) + (\text{B..ti} \times 0.047)$$

$$+ (\text{ankle circumference} \times 0.074) - 4.208, \tag{19}$$

where B..ti is the distance from the basis (B) to anthropometric landmark (ti) on upper edge of medial tibial condyle;

$$\text{mass of calf} = 0.0675x_1 + 0.0145x_2 + 0.2050x_3 - 6.0170, \tag{20}$$

where: x_1 – length of tibia, x_2 – breadth of ankle, x_3 – mean circumference of calf.

Similar regression equations for calculating a radius of centre of mass and a radius of gyration for several body segments are given.

Indirect location of the centre of mass of group of segments and of the entire body can be established by means of: (1) the method of the sum of mass (graphic method); (2) the method of the sum of moments of mass (analytic method).

Graphic method is based on the theorem of the centre of the mass of two objects A and B if the following relationship exists:

$$m_1 \times a = m_2 \times b, \tag{21}$$

where: m_1, m_2 – mass of objects, a, b – distances between centres of mass of objects and common centre of mass.

Analytical method utilizes the theorem of the sum of moments of mass: the sum of moments of mass equals the moment of sum of mass. Here an object is divided into the segments. For each segment's centre of mass the coordinates according to rectangular reference system are found. A multiplication of masses and coordinates according to the X- and Y- axes is performed and a sum of the products is obtained. Dividing a sum of the products by the entire mass of a body for both axes enables obtaining the coordinates of a centre of mass of the entire body.

5.7. Modelling methods

Taking into account geometric and inertial problems we can distinguish: physical and mathematical models. Physical models are utilised, for example, in experiments of car-crash simulations where one needs dummies built in such a way as the human beings are built. Those dummies allow us to obtain the location of centre of mass and the radii of gyration (King and Mertz, 1973). Due to mathematical models we are able to present a human body in terms of numbers and also to present the body segments as geometric figures (ellipsoids, cylinders, cones, frustums) (von Meyer, 1873; Matsui, 1958; Hanavan, 1964; Hatze, 1980; Erdmann, 1995, 1997).

6. Results

6.1. Dimensions of the body

Human body grows from about 0.5 m (newborns) up to (in Poland, mean data) about 1.59 (women) and 1.73 (men) (Zenkteler et al., 1983). Monographs dealing with geometric dimensions of people of different ages were presented, e.g., by: Górny (1965), Wolański (1979), Malinowski (1976), and Snyder et al. (1977). Sometimes a length of the body differs significantly from the mean value. Adult dwarfs have a length of up to about 0.5 m, and giants have a length of up to about 2.4 m (women) and 2.7 m (men) (McWhirter, 1981).

Some authors (Wolański et al., 1975; Brajczewski et al., 1983; Nowak, 1993; Batogowska and Słowikowski, 1994; Snyder et al., 1977) present the dimensions of the human body in special configurations which depend on the positions adopted by people in their everyday life or work. Jensen (1993) presented geometric models of the human body by means of elliptical cylinders.

6.2. Mass

A mass of a newborn infant is about 3.5 – 5.0 kg. The smallest mass of a newborn infant was about 0.75 kg (polipregnancy and birth before a proper time). In the case of the heaviest men and women, a mass reaches a value of hundreds of kilograms. Despite reaching adulthood by a man, his mass differs significantly. This is seen especially when comparing different sport groups. For example, according to Gundlach (Drozdowski 1972) marathon runners have the mass of about 60 kg, while shot putters have the mass twice as much – about 120 kg.

One can observe different values taking into account a mass of body segments. Of all parts of the entire body, an abdomen is relatively the heaviest (16%) and the lightest is a hand (less than 1%) (Erdmann, 1995). The mass proportions of various body segments change in the course of growing. For example, a relative mass of a head of a 3-year-old child has a value of about 17% according to the mass of the whole body, for a 10-year-old child it is about 11% (Yokoi et al. 1984), and for an adult – about 5%. A mass of body segments depends also on the kind of work performed by a subject, played sport discipline, state of illness, etc.

6.3. Density

Of all human tissues a lung tissue has the lowest density, i.e., 0.56 g/cm^3 (Erdmann and Gos 1990). Only a few tissues, including fat tissue (0.94 g/cm^3) are characterised by a density below 1.0 g/cm^3, 2/3 of the investigated tissues were in the range of 1.11 and 1.20 g/cm^3. The highest mean density of a solid tissue, as reported by Clauser et. al. (1969), was 1.89 g/cm^3 for a compact bone. The density of liquid tissues ranged from 0.928 for bile to 1.057 g/cm^3 for blood tissue.

Harless (1860) was the first, who presented the densities of various parts of a body. One hundred years later Dempster (1955) presented some important data. The density of body parts ranged from 0.92 for thorax to 1.17 g/cm^3 for a right hand.

A density of the entire body differs depending on a body build. The lowest density (1.01 g/cm^3) is characteristic of the people with the biggest volume of fat tissue, Brožek et al. (Drillis and Contini, 1966). The biggest density of the entire body reaches 1.11 g/cm^3, Brožek and Keys (Dempster, 1955).

6.4. Location of centre of mass

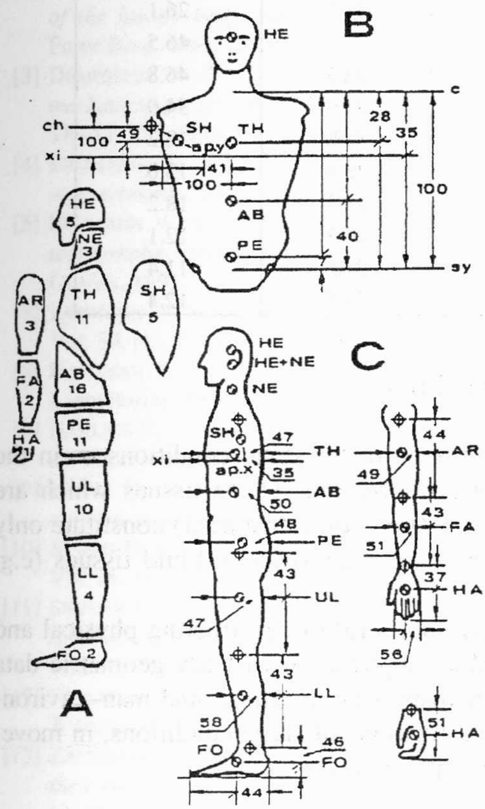
The centre of mass of the entire erected body is situated between the hip axis and the promontorium. For American children, detailed, absolute and relative data were given by Snyder et al. (1977). They stated that the centre of mass in relative values for 3-year-old child is situated 57.6% from a sole, where 100% is a height of the body. For 18-year-old boys and girls this value equalled 56.1 and 55.5%, respectively.

Location of the centre of mass of body segments is mostly given with reference to the neighbouring joints or ends of segments. It started with investigations conducted by Harless (1860). Other important data were presented by Braune and Fischer (1889), Bernstein et al. (1931), and Dempster (1955). Figures 2A–C show the data from Erdmann's paper (1995) which deals with relative mass and relative radii of the centre of mass of body segments.

6.5. Moments of inertia and their radii of gyration

Santchi et al. (1963) investigated the moments of inertia of a human body for 8 different configurations. Their experiments were carried out on 66 men (mean age 33.2 years, mean body mass 75.5 kg, mean stature 176.3 cm) who adopted an erect posture. Santchi et al. reported that central moments of inertia according to the following main axes: X (sagittal), Y (frontal), and Z (longitudinal) equalled 13.00, 11.64, 1.28 kg×m^2, respectively.

The biggest central moments of inertia were: the X- and Y-axes – an erect posture with the upper extremities raised above the head: 17.18 kg×m^2 and 15.48 kg×m^2, respectively, and the Z axis – an erect posture with upper and lower extremities angulary abducted (eagle posture): 4.14 kg×m^2. The smallest central moments of inertia were: the *X*- and *Y*- axes – sitting posture with the thighs raised and the calfs flexed: 4.42 kg×m^2, and 4.29 kg×m^2; the *Z*-axis – an erect posture with upper extremities raised above the head: 1.25 kg×m^2. Other important investigations of the moments of inertia of the entire body were carried out by Hanavan (1964), Hochmuth (1971), and Chandler (1975).



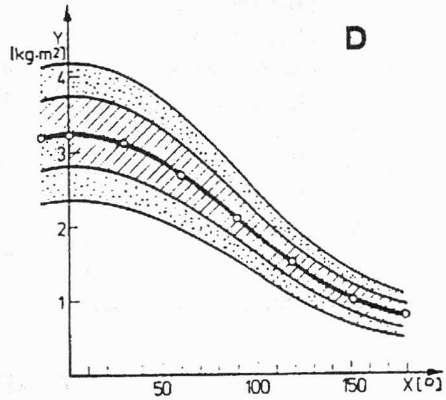


Fig. 2. Relative (%) data of the parts of the entire body of mass (A) and radii of centre of mass for frontal (B) and sagittal (C) planes; D – mean values of moment of inertia of the lower extremity (*Y*-axis) according to knee angle (*X*-axis) for 100 young men (physical education students), lines – one, and dots – two standard deviations

Braune and Fischer (1982) and Fischer (1906) were the first who investigated the moments of inertia of body segments. In the second half of the XX century those investigations were also conducted by Dempster (1955) and Chandler (1975). They investigated cadavers and used pendulum as a research instrument. Buisset and Pertuzon (1968) and also Cavanagh and Gregor (1974) applied a quick release method, while Zatsiorsky and Seluyanov used gamma rays, and Erdmann (1987) conducted anthropometric measurements and applied calculation method (Figure 2D).

Some authors except the moments of inertia give also the radii of gyration in absolute and in relative values. Those relative values are presented according to the length of segments in the Table (Zatsiorsky and Seluyanov, 1979).

Table. Central radii of gyration of body segments. Lengths of body segments = 100%

No.	Body segment	Radius of gyration according to the axis		
		Sagittal	Frontal	Longitudinal
1	Head and neck	30.3	31.5	26.1
2	Upper trunk	50.5	32.0	46.5
3	Medium trunk	48.2	38.3	46.8
4	Lower trunk	35.6	31.9	34.0
5	Arm	32.8	31.0	18.2
6	Forearm	29.5	28.4	13.0
7	Hand	28.5	23.3	18.2
8	Thigh	26.7	26.7	12.1
9	Calf	28.1	27.5	11.4
10	Foot	25.7	24.5	12.4

7. Applications

All geometric and inertial data can be utilized under static conditions or in the movement with small accelerations without any problem. Those tissues which are moving during bigger accelerations and decelerations (digestive tract) constitute only about 5% of the mass of the entire body (they have air spaces). Fluid tissues (e.g. blood) behave in a similar way.

The geometric and inertial data are especially useful in constructing physical and mathematical models of the human body. Main applications of body geometric data can be seen in the analyses of the man–mechanism, man–machine, and man–environment systems. Inertial data are needed in investigations of static conditions, in movement analyses, in load examining and in accidents' analyses.

8. Closure

Due to acceleration of growth of consecutive populations one needs still new data which refer to geometry and inertia of the human body. The dimensions of the bodies of children and youths from the previous years and from the 90s of this century are highly differentiated. This phenomenon can be explained by better living conditions of the present populations.

The authors of inertial tests examined some specific populations – mostly elderly people (investigations with cadavers) or young, healthy adults (students, sportsmen). Data of other populations are also needed. One should include some change of available data, while using inertial values relating to the different population he is dealing with.

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