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Numerical approach to shape sensitivity analysis of femoral implants

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Implant separation from bone or cement is widely known serious drawback of bone endoprosthetics. It is believed that stress concentrations on the implant surface are responsible for the failure and that implant shape optimisation is the right way of the problem solution. An important tool in effective optimisation algorithms is the design sensitivity analysis (DSA). The paper presents the finite element formulation of the sensitivity problem. Three-dimensional model of femur with cementless implant is analysed.

Keywords: bone implants, shape sensitivity, optimisation, finite element method

1. Introduction

Bone implantation (endoprosthetics) is a standard surgical technique employed in treatment of joint failures, especially in hips and knees. One of the serious drawbacks of this technique is the separation of the implant from either bone tissue or cement occurring after certain period of time. The precise mechanism of the failure is not clear, however, unnatural stress distribution around the implant is considered the main reason for the degradation process.

Optimisation of the implant properties is widely believed to be the right way of improvement of endoprostheses reliability. Thus, among numerous papers devoted to numerical equilibrium analysis of the bone–implant systems, the issue of design optimisation becomes of increasing interest in the recent years [1, 2]. Optimisation of the implant shape seems the most promising in terms of the expected improvement. So far, two-dimensional linearly elastic models are only discussed which poses essential limitations to the results' precision and reliability.

In the present paper, a three-dimensional numerical model of a bone–implant system supplemented by the possibility of sensitivity analysis is presented. The design sensitivity analysis (DSA) [3] is a crucial tool in the most advanced and efficient optimisation algorithms. Femur with a cementless endoprosthesis is chosen as an example.

2. Finite element formulation

The equilibrium problem of the finite element method is expressed as follows

$$\mathbf{F}(\mathbf{q}) = \mathbf{Q}, \tag{1}$$

where **F** and **Q** denote internal and external nodal forces, respectively. **F** depends on the unknown nodal displacements **q** via geometric and constitutive relations

$$\mathbf{F}(\mathbf{q}) = \int_{\Omega} \mathbf{B}^T \sigma(\varepsilon) d\Omega, \qquad \varepsilon = \mathbf{Bq} \tag{2}$$

and this dependence is generally nonlinear. Equation (1) is solved iteratively

$$\mathbf{K}\delta\mathbf{q} = \mathbf{Q} - \mathbf{F}, \qquad \mathbf{q} := \mathbf{q} + \delta\mathbf{q} \tag{3}$$

with the consistent tangent stiffness matrix defined as

$$\mathbf{K} = \frac{d\mathbf{F}}{d\mathbf{q}} = \int_{\Omega} \mathbf{B}^T \mathbf{CB} d\Omega, \qquad \mathbf{C} = \frac{d\boldsymbol{\sigma}}{d\varepsilon}. \tag{4}$$

The sensitivity of a response $\Phi(\mathbf{q}, h)$ with respect to a design parameter h is expressed as

$$\frac{d\Phi}{dh} = \left.\frac{d\Phi}{dh}\right|_{\mathbf{q} \neq \mathbf{q}(h)} + \left[\frac{d\Phi}{d\mathbf{q}}\right]^T \frac{d\mathbf{q}}{dh}, \tag{5}$$

where the derivative $d\mathbf{q}/dh$ can be found as a solution of the following equation, cf. Eqs. (1), (4),

$$\mathbf{K}\frac{d\mathbf{q}}{dh} = \left.\frac{d(\mathbf{Q} - \mathbf{F})}{dh}\right|_{\mathbf{q} \neq \mathbf{q}(h)}. \tag{6}$$

Thus, the sensitivity of the response Φ requires computation of several additional vectors

$$\left.\frac{d\Phi}{dh}\right|_{\mathbf{q} \neq \mathbf{q}(h)}, \qquad \frac{d\Phi}{d\mathbf{q}}, \qquad \left.\frac{d(\mathbf{Q} - \mathbf{F})}{dh}\right|_{\mathbf{q} \neq \mathbf{q}(h)}, \tag{7}$$

and solution of a set of equations (with the same coefficient matrix as that used in equilibrium computations) against an additional right-hand side vector.

The derivative $d\mathbf{Q}/dh$ is typically zero. Computation of the explicit design derivative of **F** (cf. Eq. (2)) requires the introduction of a fictitious design-independent reference configuration (often associated with the parent configuration of an isoparametric finite element). After transformations we obtain

$$\left.\frac{d\mathbf{F}}{dh}\right|_{\mathbf{q}\neq\mathbf{q}(h)} = \int_{\Omega}\left(\left.\frac{d\mathbf{B}^T}{dh}\right|_{\mathbf{q}\neq\mathbf{q}(h)}\boldsymbol{\sigma} + \mathbf{B}^T\left.\frac{d\boldsymbol{\sigma}}{dh}\right|_{\mathbf{q}\neq\mathbf{q}(h)} + \mathbf{B}^T\boldsymbol{\sigma}\frac{1}{\bar{J}}\left.\frac{d\bar{J}}{dh}\right|_{\mathbf{q}\neq\mathbf{q}(h)}\right)d\Omega, \tag{8}$$

where $\bar{J}$ denotes the Jacobian determinant of the transformation from the design-independent to actual reference configuration of the analysed body. Note that even if the material constants do not depend on h, the derivative $d\sigma/dh_{\mathbf{q}\neq\mathbf{q}(h)}$ is not zero, i.e.

$$\left.\frac{d\boldsymbol{\sigma}}{dh}\right|_{\mathbf{q}\neq\mathbf{q}(h)} = \mathbf{C}\left.\frac{d\mathbf{B}}{dh}\right|_{\mathbf{q}\neq\mathbf{q}(h)}.$$

3. Computational examples

The formulation above has been implemented in a general-purpose finite element code. An example of a femoral part of a cementless hip implant has been analysed. Implant surface was assumed smooth in the lower part (frictionless sliding contact was assumed there) and rough (coated) on the remaining part (modelled as perfectly bound to the bone). Different material properties were assumed for cortical and spongy bone tissues, however, anisotropy of the properties was neglected.

Figure 1 presents the finite element model, while Fig. 2 shows the results of stress distribution on the lateral and medial sides of the implant together with their sensitivities with respect to a few design variables. These are: implant length, implant end width, and length of the coated (rough) part of its surface.

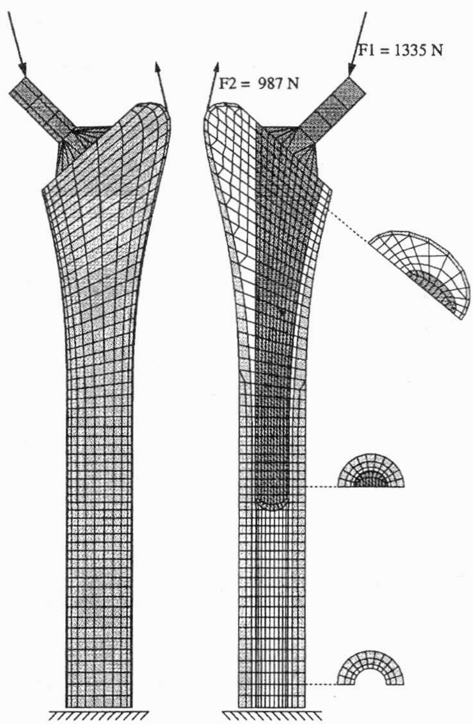

Fig. 1. Finite element model of femur with an implant. Grey denotes implant, light-grey – cortical bone, white – spongy bone, thick line indicates a sliding area

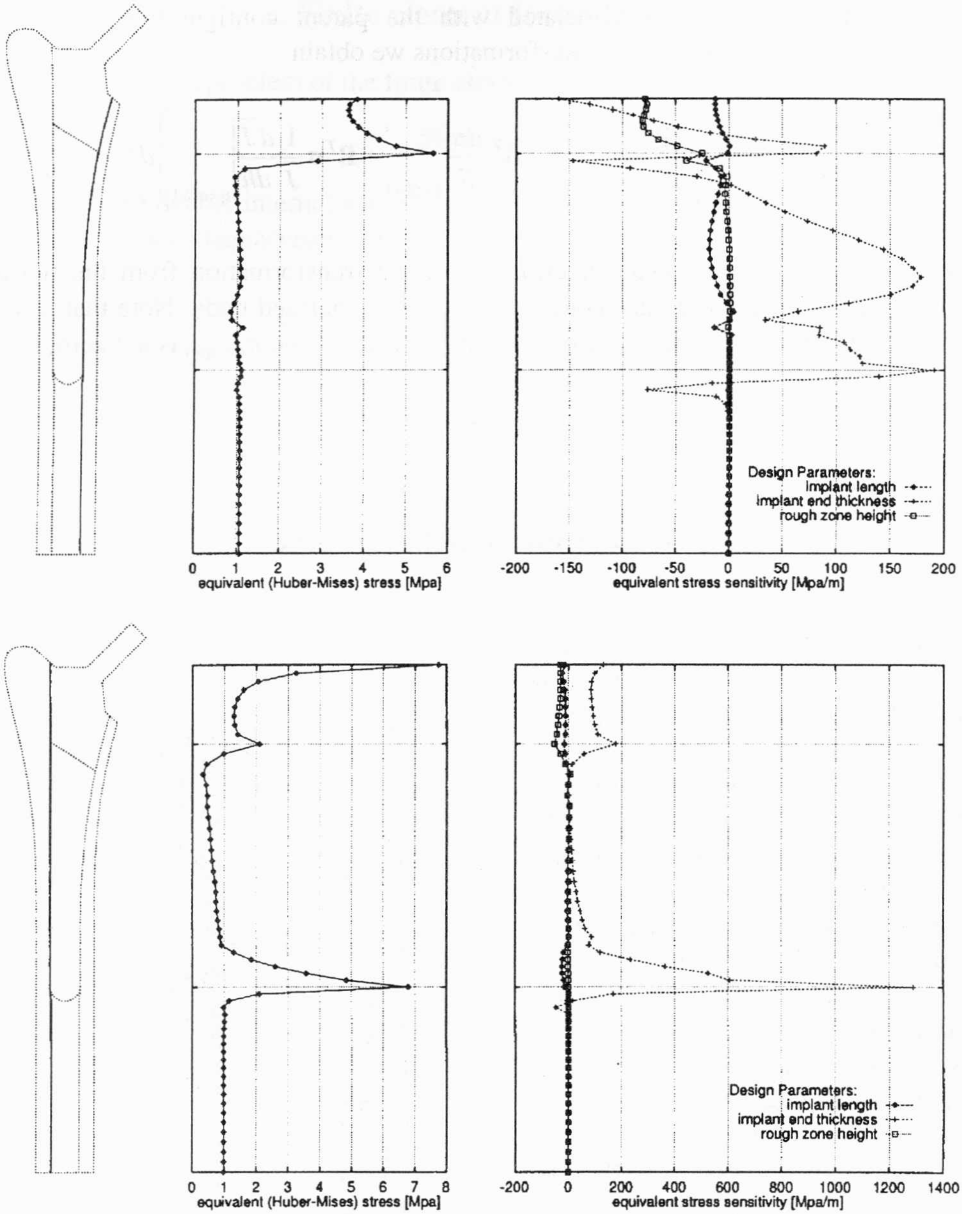

Fig. 2. Stress in bone tissue and its sensitivity on the medial and lateral bone–implant interfaces

4. Conclusions

The design sensitivity analysis appears to be relatively cheap addition to a general-purpose finite element code. Thus, the equilibrium analysis of the implant–bone system can easily be supplemented with the sensitivity gradient analysis with respect to any design parameters, including shape parameters. The gradients are of crucial importance in the optimisation of implant shape, provided that reliable failure criteria are used.

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