Modelling of viscoelastic deformation of cortical bone tissue

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The aim of the paper is to draw attention to exponential viscoelastic creep measures for the description of viscoelastic deformation of the bone tissue. The proposed two-parametric exponential time function provides a real modelling of medium-time deformation processes and brings advantage in interpretation and generalisation of experimental results. This is documented by empirical data taken from literature. The two parameters of the creep measure characterize the capability of a bone tissue to react quickly and viscoelastically to the changes of load and the rate of viscoelastic deformation increment. The model enables also a simple time-age superposition and medium-term or long-term predictions.

Key words: creep measure, viscoelastic deformation, cortical bone tissue, superposition

1. Introduction

The bone tissue is considered, in a simplified way, as a material which is anisotropic, inhomogeneous, structurally porous on all levels – from the microstructure up to the macrostructure. The pores in the form of bone channels, lacunae, blood channels and free spaces filled with pulp in the spongy bones have significant influence upon the mechanical behaviour of the bone tissue. These pores reduce the real sectional area of compact mass and seemingly aggravate the mechanical properties. Nevertheless, blood flowing through the healthy bone tissue and especially the bone marrow, thanks to their rheological properties, hydraulically strengthen long bones.

Mechanical properties of bones vary during human being life. In the phase of growth, the compact mass porosity increases together with simultaneous rise in the proportion of mineral particles. This results in growing fragility of bones which is connected with degradation of their strength and limit deformation. Dominantly, the healthy bone tissue comports itself elastically. Viscoelastic properties of bones

conditioned by the presence of collagen fibres, proteoglycans and fluids manifest themselves naturally and measurably especially during long-term load and re-modelling.

A number of authors have been examined elastic properties of bone tissues in different conditions and from different points of views. Therefore, the knowledge about elastic, momentary mechanical manifestations of bone tissues is relatively comprehensive, which has not yet hold for the behaviour linked with longer histories of load, for which the simplest mechanical model of inelastic behaviour is a viscoelastic body [1]. In the Czech Republic, the works of SOBOTKA, MAŘÍK, PETRTÝL and MELZEROVÁ [2], [3], [4] have brought the important contributions to this objectively very uneasy field of research.

In [4], for the description of viscoelastic properties of cortical tissues of the human femur diaphysis, the authoress applies the so-called Poynting–Thompson model, which together with the bone elasticity involves their time-dependent viscoelastic behaviour as well. Based on the realisation and performance of the sophisticated experimental programme, the basic momentary and time-dependent characteristics of bone tissue material are defined as the functions of discrete states of mechanical load, and the inhomogeneity of their distribution influenced by the biomaterial adaptation to external mechanical conditions is emphasized. However, the methods of determining the material characteristics of the Poynting–Thompson model are relatively complicated and a practical solution is subject to substantial simplification. For this reason, the authoress applies also a simple time logarithmic function in the description of the viscoelastic creep deformation of individual parts of the femur diaphysis.

The purpose of this work is to draw attention to the possibility of applying exponential time functions in order to describe viscoelastic deformation of asymptotic character in the linear and lightly nonlinear fields of behaviour. It is a matter of the known Kohlrausch–Williams–Wats function [5] and of its simplified form co-proposed by the author [6]. The application of this two-parametrical function brings some advantages in evaluating rheonomous processes, which will be hereinafter documented by experimental results reported in [4].

2. Time functions of viscoelastic compliance

For the description of experimental data of viscoelastic creep, the two-exponential three-parametric Kohlrausch–Williams–Wats (KWW) time function is often applied [5]. This function has the following form for the formulation of viscoelastic compliance

$$J(t) = J_0 \exp[(t/\tau)^\beta] \,. \tag{1}$$

Three free parameters of this function, i.e. J_0 [MPa^{-1}], τ and β, are indicated in the corresponding sequence as the initial viscoelastic compliance, retardation time, and shape parameter. In the case of modelling viscoelastic compliance, the first two

parameters are supposed to be positive. The third, shape parameter β determines whether the function values increase above all limits for low ($\beta < 0$) or high ($\beta > 0$) values of the independent variable. Considering the history of the viscoelastic creep deformation caused by a constant (dead) load (it is defined by the non-decreasing time function), it is obvious that the KWW function can be advantageously used only for limited time intervals. The double exponential form of the KWW function may be inconvenient for the applications. For high values of the independent variable $t \gg 1$ and low values of the positive shape parameter $\beta \ll 1$ (these conditions are fulfilled in describing the preponderance of real viscoelastic creep processes), the exponential function t^{β} can be replaced by the logarithmic function

$$t^{\beta} \approx k_1 \cdot \log t + k_2 , \tag{2}$$

where k_1 and k_2 are constants. Using equation (2), equation (1) can be modified to the term

$$\log J(t) = \log [J_0 \cdot \exp(k_2 / \tau^{\beta})] + (0.434 \cdot k_1 / \tau^{\beta}) \cdot \log t \tag{3}$$

and the new two-parametric function of viscoelastic compliance can be defined [6] in the following form

$$J(t) = P \exp[Q \log(t)]. \tag{4}$$

For two free parameters P [MPa^{-1}] and Q, the following relations are valid

$$P = J_0 \exp[k_2 / \tau^{\beta}], \qquad Q = k_1 / \tau^{\beta} . \tag{5}$$

The comparison of the applications of exponential functions (1) and (4) with a simple logarithmic function used in the work [4] in the form

$$\varepsilon(t) = a \cdot \ln(t) + b \tag{6}$$

(a and b are constants) to describe the viscoelastic deformation of bone tissue is displayed in figure 1.

The experimental data (triangular marks) in this figure represent the growth of the viscoelastic tensile deformation (without elastic response) in the medial face of the middle portion of the young man femur diaphysis in the course of physiologic load. For the unitary physiologic load, the course of the viscoelastic deformation determined in this way represents the history of viscoelastic compliance. Three regression curves, i.e. *KWW*, *PQ* and *L* represented respectively by equations (1), (4) and (6), are inset with experimental data. The values of the correlation coefficients of the regression displayed in figure 1 ($r \geq 0.994$) unambiguously prove that: (i) all three regression functions are applicable to modelling the course of viscoelastic compliance of the samples being observed in the time interval of 0.5–5 [min], (ii) relatively short initial stage of the viscoelastic creep, whose separation from the immediate (elastic) sample reaction to the load may be methodologically difficult, is described by the exponential

functions in steps, but sufficiently exactly as to the total history of the monitored process.
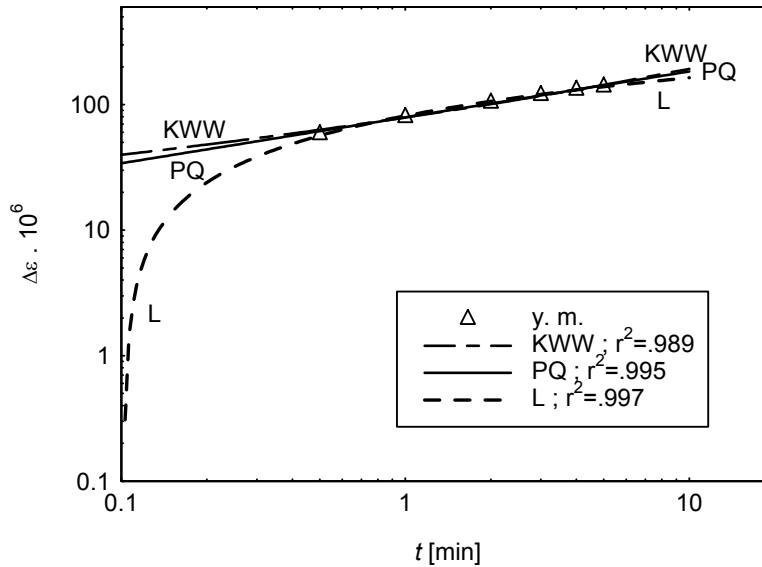

Fig. 1. Correlation of the *KWW*, *PQ* and *L* creep measures
with experimental data after [4]

The two-parametrical time function *PQ* has a considerable advantage of defining a line (see figure 1) with the gradient $k = 0.434 \cdot Q$ and the section on the ordinate axis *P* in double logarithmic scale. The parameter *P* characterises the initial viscoelastic compliance, i.e. the step modification of the viscoelastic compliance at the time $t = 0_+$ at the very beginning of the creep history, which in reality progresses continuously from the zero value in the relatively very short time period after reaching the constant level of dead load during the execution of the long-term creep test. In other words, the parameter *P* co-determines the rate at which the bone tissue is capable of the fast *viscoelastic* reaction to the changes of load. The parameter $Q(k)$ represents the rate of the change of the viscoelastic compliance (viscoelastic deformation) in the time during the medium-term and the long-term loading.

3. Experimental data
and their regression functions

As we have mentioned before, the experimental data reported by MELZEROVÁ [4] allowed application of time functions *KWW* and especially *PQ*. This concerns the values of tensile strain determined by strain gauges for the centre of the sample in the

form of a beam in three-point bending with the approximate ratio of the simple beam height to the span determined as 1:7 (i.e. with a negligible influence of the shear factor of the load). The samples, in the form of irregular cylinders, were prepared by cutting out the parts of femur (without pathologic changes) 10 cm below the centre of the hip-joint head. For the unitary physiologic load, the values of the deformation oriented along the longitudinal axis of the femur represent the history of the corresponding component of the viscoelastic compliance. The load during walking was selected as a basic physiologic load. The changes of the tensile strain with time were measured during the time interval of 5 minutes. Altogether, the tests described in [4] involved more than one hundred samples of human femurs of men and women in three age categories (with tolerance of four years), i.e. young men and women (age of 20 years), middle-aged men and women (age of 40 years) and old men and women (age of 60 years).

The results of the *PQ* regression equation (4) use are presented in figures 2 and 3. It is obvious that the values of the parameters *P* and *Q* of the behaviour model must be (except other effects) functions of sex, load and age. In figure 2, there are experimentally ascertained courses of viscoelastic deformation in the medial face of the central part of the femur diaphysis of young women (y.w.), middle-aged women (w.) and old women (o.w.) in the physiologic load (empty marks) and in its double (corresponding filled marks). The regression functions *PQ* described by equation (4) are determined by the relevant values. The analogical results for men are displayed in figure 3.
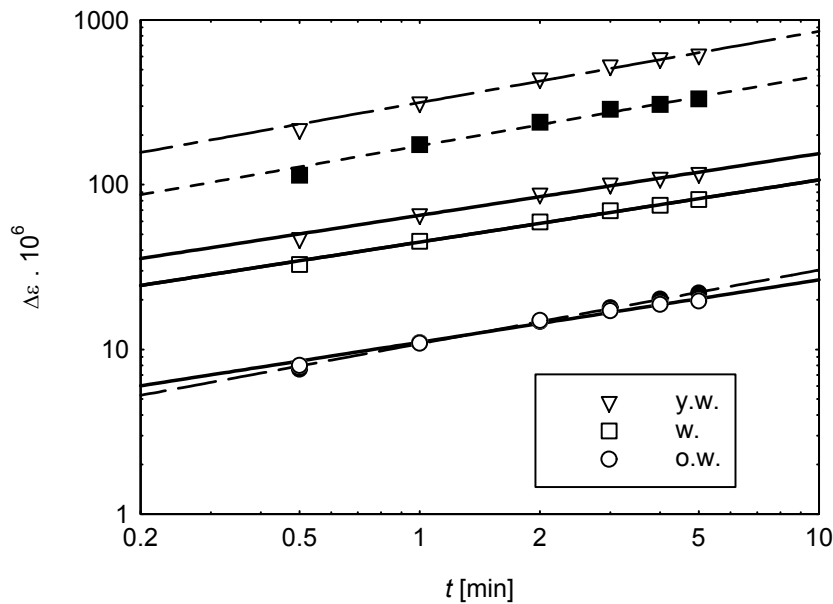
Fig. 2. Viscoelastic deformation in the medial face of a central part of femur diaphysis
of young women (y.w.), middle-aged women (w.) and old women (o.w.)
in the physiologic load (empty marks) and in its double (corresponding filled marks)
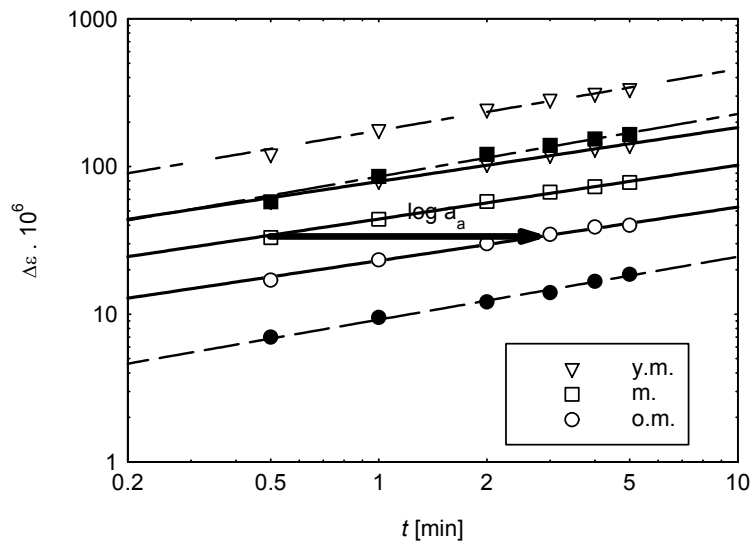

Fig. 3. Viscoelastic deformation in the medial face of a central part of femur diaphysis
of young men (y.m.), middle-aged men (m.) and old men (o.w.) in the physiologic load
(empty marks) and in its double (corresponding filled marks)

The results of the regression are sets of the parameters P [1] and Q (or $k = 0.434 \cdot Q$). The values of the parameter P for the individual categories of the experimental data are shown in figure 4.
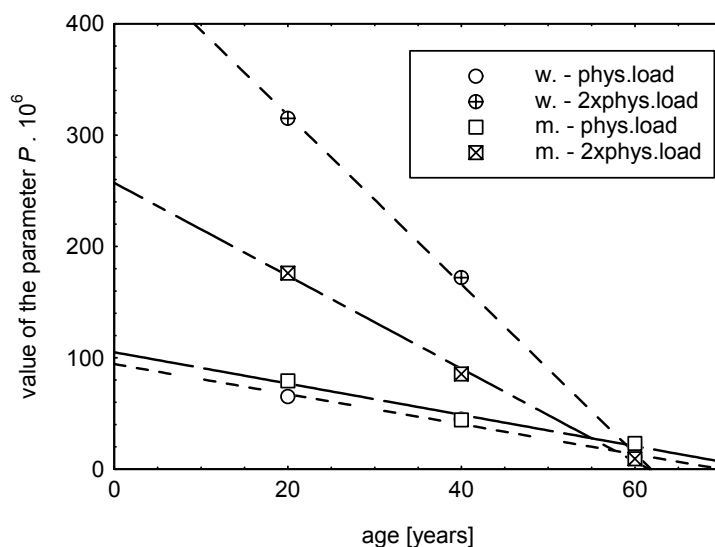
Fig. 4. Values of the parameter P for the individual categories
of the experimental data

On the basis of the results obtained (see figure 4) it is possible to draw the following conclusions: (i) in the age of 20–60 years, the capability of bone to react quickly and viscoelastically to the change of load decreases linearly with age, (ii) for the physiologic load, this capability is approximately identical for men and women, (iii) for older women and men (i.e. for the age of 60 years), it is independent of the load within the range from the physiologic load up to its double. Nevertheless, for both sexes, the change occurs with different rate which depends on the age – it is 1.6 times higher for women than for men in the case of double physiologic load.

The rate of the parameter P decrease with age (in a given age and load range) is presented in table 1.

Table 1. Rate of the parameter P decrease, depending on age [year^{-1}]

Sex	Physiologic load	Duoble physiologic load
Women	−1.35	−7.60
Men	−1.40	−4.17

As far as the rate of the viscoelastic compliance change with time (the rate of the viscoelastic deformation increase), i.e. the parameter Q, is concerned the results of regression (see the table 2) confirmed the expectation based on figures 2 and 3 – the values of the parameter Q (or $k = 0.434 \cdot Q$) do not depend upon the age in the range defined for the experiments. The average values together with probable value of the

standard deviation and the variation coefficient for men and women for both loads are gathered in table 2.

Table 2. Values of the parameter $k = 0.434 \cdot Q$

	Physiologic load	Double physiologic load	Index (Double phys.load/phys.load)
Women	0.377±0.003 (0.5%)	0.435±0.011 (2.6%)	1.153
Men	0.364±0.003 (1.0%)	0.421±0.005 (1.3%)	1.158
Index women/men	1.035	1.033	

On the basis of the values given in table 2, the following conclusions can be drawn: (i) for women, independently of the load in the range from the physiologic one to its double, the rate of time-dependent modifications of viscoelastic compliance (here the viscoelastic deformation) under a medium-term load is slightly (by 3.5%) higher compared to men, (ii) the increase in the rate of viscoelastic compliance (here the viscoelastic deformation) caused by the effect of double physiologic load, being compared with the physiologic one, is approximately 15% regardless of the sex.

The expansion of the experimental program (e.g. with more degrees of the load) by means of the aforementioned procedure could enable us to determine the quantitative functional dependence of the parameters P and Q from equation (4) upon the observed effect factors (and possibly upon other ones as well) in the form

$$P = P \text{ (sex, load, age, …)},$$

$$Q = Q \text{ (sex, load, …)}, \tag{7}$$

and to apply the model to interpolation, comparison, etc.

4. Principle of the superposition

A considerable advantage of proceeding experimental data by means of the time function PQ (4) consists in the possibility of simple applying the time–age superposition principle. From figures 2 and 3 it is apparent that the proportional viscoelastic deformation (viscoelastic compliance) in the time t for the age category v' is identical with the corresponding quantity in the time at for the age category v

$$\varepsilon_{v'}(t) = \varepsilon_v(a\,t),$$
$$J_{v'}(t) = J_v(a\,t). \tag{8}$$

The parameter a, in this case age-dependent, is indicated [7] as a shift factor. Shifting experimental data along the time axis can be compared to the time–temperature superposition applied to thermorheologically simple materials. Considering the co-linearity of individual courses of the viscoelastic deformation for the identical loads applied (figures 2 and 3) (see also table 2), we arrive at the conclusion that the time–age

superposition, expressed in a simplified way, corresponds to the horizontal shift of the experimental data of the individual courses expressed by the average value of the shifting factor, so that they are associated with the reference course. This procedure is in figures 5 and 6.
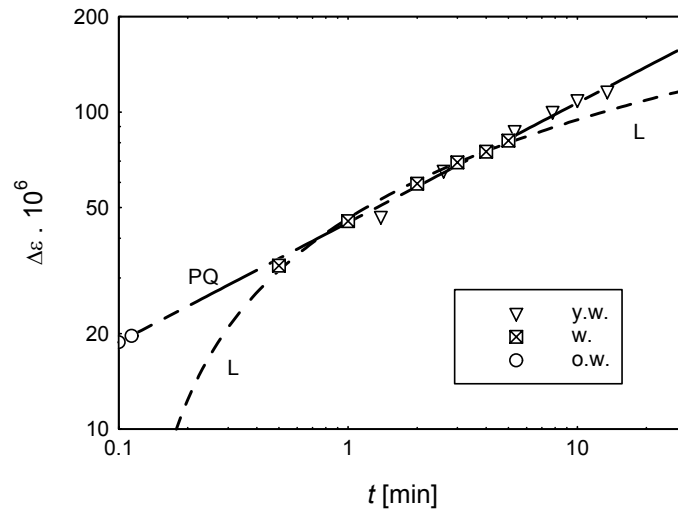

Fig. 5. Prognosis of viscoelastic deformation in the medial face of a central part of femur diaphysis of middle-aged women during the physiological load, depending on time–age superposition

In figure 5, the course of viscoelastic deformation in the medial face of the central part of femur diaphysis of middle-aged women during the physiological load was chosen as the reference course. If the time–age superposition is applied, the experimental values of the same quantity characterizing younger women are in the figure shifted towards longer times, whereas the corresponding values for older women – towards shorter times. The time period displayed in figure 4 enabled us to place only two of them (see the circular marks) towards the very beginning of the constant (dead) load action, where for the above-mentioned reasons, a real history of deformation (indicated by the course of the curve *L* according to equation (6)) is described by the time function *PQ* only in the model way.
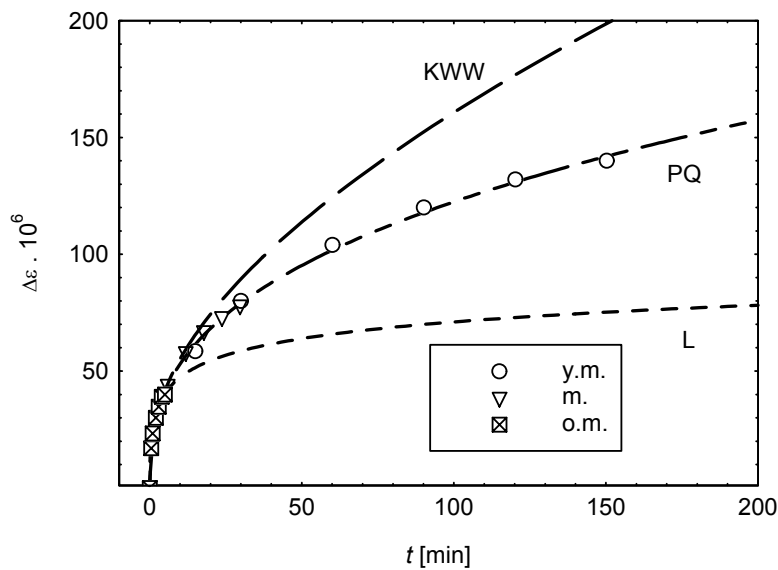
Fig. 6. Prediction of viscoelastic deformation in the medial face of a central part of femur diaphysis
of old men during the physiological load, depending on time–age superposition

Figure 6 displays the application of the time–age superposition to the viscoelastic deformation in the medial face of the central part of the femur diaphysis of old men during the physiological load. The experimental data for both remaining age categories (see figure 3) are shifted towards longer times, so that the prediction lenghtens thirty times the experiment initial time interval of 5 minutes to the value of 150 minutes. The predictions of *KWW* and *L* according to the time functions (2) and (6) are indicated in this figure as well. The above-mentioned histories as well as all medium- and long-term predictions must be considered prudently and must be subjected to experimental verification. Naturally – the experimental results from the time interval of creep longer than that used in the work [4] (if they were known) would cause the modification of the parameters of the individual time functions and consequently of the superposition process.

5. Conclusions

The two-parametric exponential time function (the measure of viscoelastic creep compliance) defined by equation (4) together with a real modelling of the history of the viscoelastic deformation of a cortical bone tissue allow a simpler interpretation and generalisation of experimental results than the Poynting–Thompson model or a time logarithmic function. Its two parameters (dependent on sex, age, loads and possibly on other factors as well) characterize the following phenomena: (i) the

capability of a bone tissue to react quickly and viscoelastically to the changes of load, (ii) a proper rate of the increment in viscoelastic deformation under medium-term (long-term) load. In the case monitored, this model of the viscoelastic behaviour enables also a simple time–age superposition and obtaining the medium-term or possibly long-term predictions.

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